

CA - FINAL (RISK MODELS)

HANDOUT - II

CA - FINAL RISK MODELS

QUESTIONS

1. A junior risk analyst made the following statements during a risk-management committee meeting.
Statement 1: VAR can be difficult to estimate and different methods may yield different VAR values.
Statement 2: VAR considers only the downside risk and fails to incorporate upside benefits.
Which of the statements is most accurate?
A. Statement 1 only.
B. Statement 2 only.
C. Both are accurate.
2. Consider an investor who holds Stock X and Stock Y. The portfolio weights are 40% in X and 60% in Y. Stock X carries an expected annual return of 12% and an annual return standard deviation of 38%. Stock Y carries an expected annual return of 16% and an annual return standard deviation of 49%. The correlation between returns of Stock X and Stock Y is 0.37. The 5% VAR of annual returns is closest to:
A. -37.76%.
B. -42.69%.
C. -47.72%.
3. The following are the sorted worst 20 monthly returns of a technology stock in the past 15 years (180 months).
 1. -0.4947
 2. -0.4568
 3. -0.4363
 4. -0.4275
 5. -0.4171
 6. -0.3851
 7. -0.3845
 8. -0.3500
 9. -0.3358
 10. -0.3291
 11. -0.3254
 12. -0.3249
 13. -0.3088
 14. -0.3049

- 15. -0.3018
- 16. -0.2675
- 17. -0.2396
- 18. -0.2384
- 19. -0.2312
- 20. -0.2304

Using the historical method, the 10% monthly VAR is closest to:

- A. -32.91%
 - B. -30.18%
 - C. -23.84%
4. A portfolio has a 5% weekly VAR of \$3 million. Which of the following is most accurate?
- A. The same portfolio's 1% weekly VAR is more than \$3 million.
 - B. The same portfolio's 1% weekly VAR is \$3 million.
 - C. The same portfolio's 1% weekly VAR is less than \$3 million.
5. An investment professional makes the following statement about employing VaR techniques to measure market risk exposure:
- "My main concern is the prevalence of negative skew and positive excess kurtosis in asset class returns, neither of which are desirable from a risk perspective. The other issue with estimating risk exposure in financial markets is the 'Black Swan' effect—often crises lead to asset class returns that simply have not been witnessed before which makes them very difficult to model."Based on the information in the statement above, which of the following VaR methods is most likely to be suitable for the analyst?
- A. Variance/covariance.
 - B. Historical.
 - C. Monte Carlo.
6. What does TVAR stand for?
- A. Total VAR.
 - B. Tail VAR.
 - C. Trailing VAR.
7. What does IVAR stand for?
- A. Individual VAR.
 - B. Incremental VAR.
 - C. Internal VAR.

8. An investment fund has an estimated 1% daily value at risk (VaR) of \$150,000 using the variance/covariance method. Assuming that there are 250 independent trading days in a year, and that the mean daily return is not different from zero, which of the following values is closest to the 1% annual VaR estimate for the fund?
- A. \$1.8 million
 - B. \$2.4 million
 - C. \$3.8 million
9. A risk analyst estimates the following information regarding the risk exposure of an investment fund:
- Fund size: 1. \$52.7 million
 - 10-day 1% VaR: 2. \$2.8 million

The analyst is considering adding a new asset class with the following features:

- Position size: 1. \$5 million
- 10-day 1% VaR of new position: 2. \$0.3 million
- Correlation of new asset class with existing portfolio: 3. 0.23
- 10-day 1% VaR of portfolio with new asset class position: 4. \$3.0 million

Which of the following is closest to the incremental VaR (IVaR) of the potential new asset class?

- A. \$0.2 million
 - B. \$0.3 million
 - C. \$1.2 million
10. A risk analyst gathered the following information about annual stock index returns.

Range of stock returns	Probability
-50% to -40%	0.4%
-40% to -30%	0.6%
-30% to -20%	4.0%
-20% to -10%	10.0%
-10% to 0%	15.0%
0% to 10%	25.0%
10% to 20%	25.0%
20% to 30%	10.0%
30% to 40%	5.0%
40% to 50%	4.0%
50% to 60%	1.0%

Given the above probability distribution, the 5% VaR of the annual returns is closest to:

- A. -30%.
- B. -20%.
- C. 40%.

11.

- A. An organization's risk management function has computed that a portfolio held in one business unit has a 1 percent weekly value at risk (VAR) of £4.25 million. Describe what is meant in terms of a minimum loss.
- B. The portfolio of another business unit has a 99 percent weekly VAR of £4.25 million (stated using a confidence limit approach). Describe what is meant in terms of a maximum loss.

12. Each of the following statements about VAR is true except:

- A. VAR is the loss that would be exceeded with a given probability over a specific time period.
- B. Establishing a VAR involves several decisions, such as the probability and time period over which the VAR will be measured and the technique to be used.
- C. VAR will be larger when it is measured at 5 percent probability than when it is measured at 1 percent probability.
- D. VAR will be larger when it is measured over a month than when it is measured over a day.

13. Suppose you are given the following sample probability distribution of annual returns on a portfolio with a market value of \$10 million.

Return on Portfolio	Probability
Less than -50%	0.005
-50% to -40%	0.005
-40% to -30%	0.010
-30% to -20%	0.015
-20% to -10%	0.015
-10% to -5%	0.165
-5% to 0%	0.250
0% to 5%	0.250
5% to 10%	0.145
10% to 20%	0.075
20% to 30%	0.025
30% to 40%	0.020
40% to 50%	0.015
Greater than 50%	0.005
	1.000

Based on this probability distribution, determine the following:

- A. 1 percent yearly VAR.
- B. 5 percent yearly VAR.

14. An analyst would like to know the VAR for a portfolio consisting of two asset classes: long-term government bonds issued in the United States and long-term government bonds issued in the United Kingdom. The expected monthly return on US bonds is 0.85 percent, and the standard deviation is 3.20 percent. The expected monthly return on UK bonds, in US dollars, is 0.95 percent, and the standard deviation is 5.26 percent. The correlation between the US dollar returns of UK and US bonds is 0.35. The portfolio market value is \$100 million and is equally weighted between the two asset classes. Using the analytical or variance-covariance method, compute the following:
- A. 5 percent monthly VAR.
 - B. 1 percent monthly VAR.
 - C. 5 percent weekly VAR.
 - D. 1 percent weekly VAR.
15. You invested \$25,000 in the stock of Dell Computer Corporation in early 2011. You have compiled the monthly returns on Dell's stock during the period 2006–2010, as given below.

2006	2007	2008	2009	2010
–0.0214	–0.0347	–0.1824	–0.0723	–0.1017
–0.0106	–0.0566	–0.0070	–0.1021	0.0264
0.0262	0.0158	0.0010	0.1114	0.1344
–0.1196	0.0862	–0.0648	0.2257	0.0786
–0.0313	0.0675	0.2378	–0.0043	–0.1772
–0.0362	0.0609	–0.0512	0.1867	–0.0953
–0.1137	–0.0203	0.1229	–0.0255	0.0978
0.0401	0.0100	–0.1156	0.1831	–0.1110
0.0129	–0.0230	–0.2416	–0.0360	0.1020
0.0652	0.1087	–0.2597	–0.0531	0.1099
0.1196	–0.1980	–0.0844	–0.0228	–0.0816
–0.0789	–0.0012	–0.0833	0.0170	0.0250

Using the historical method, compute the following:

- A. 5 percent monthly VAR.
 - B. 1 percent monthly VAR.
16. Consider a \$10 million portfolio of stocks. You perform a Monte Carlo simulation to estimate the VAR for this portfolio. You choose to perform this simulation using a normal distribution of returns for the portfolio, with an expected annual return of 14.8 percent and a standard deviation of 20.5 percent. You generate 700 random outcomes of annual return for this portfolio, of which the worst 40 outcomes are given below.

-0.400	-0.320	-0.295	-0.247
-0.398	-0.316	-0.282	-0.233
-0.397	-0.314	-0.277	-0.229
-0.390	-0.310	-0.273	-0.226
-0.355	-0.303	-0.273	-0.223
-0.350	-0.301	-0.261	-0.222
-0.347	-0.301	-0.259	-0.218
-0.344	-0.300	-0.253	-0.216
-0.343	-0.298	-0.251	-0.215
-0.333	-0.296	-0.248	-0.211

Using the above information, compute the following:

- A. 5 percent annual VAR.
- B. 1 percent annual VAR.

17.

- A. A firm runs an investment portfolio consisting of stocks as well as options on stocks. Management would like to determine the VAR for this portfolio and is thinking about which technique to use. Discuss a problem with using the analytical or variance-covariance method for determining the VAR of this portfolio.
- B. Describe a situation in which an organization might logically select each of the three VAR methodologies.

18. An organization's 5 percent daily VAR shows a number fairly consistently around €3 million. A backtest of the calculation reveals that, as expected under the calculation, daily portfolio losses in excess of €3 million tend to occur about once a month. When such losses do occur, however, they typically are more than double the VAR estimate. The portfolio contains a very large short options position.

- A. Is the VAR calculation accurate?
- B. How can the VAR figure best be interpreted?
- C. What additional measures might the organization take to increase the accuracy of its overall exposure assessments?

19. A maximum acceptable probability of insolvency over a 1-year horizon of 1% would most likely indicate that an organization should hold capital reserves of no less than:

- A. 99% annual VaR.
- B. 1% annual VaR.
- C. 52 times their 1% weekly VaR.

20. The expected annual return for a \$100,000,000 portfolio is 6.0% and the historical standard deviation is 12%. Calculate VaR at 5% probability.
21. You have accumulated 100 daily returns for your \$100,000,000 portfolio. After ranking the returns from highest to lowest, you identify the lowest five returns:
-0.0019, -0.0025, -0.0034, -0.0096, -0.0101
Calculate daily VaR at 5% significance using the historical method.
22. A Monte Carlo model has generated 100 runs of possible output over 1-week periods. The average return and standard deviation are 5.7% and 2.1% respectively. The worst six outcomes are +0.5%, +1.5%, +1.6%, +0.3%, +0.7% and +0.5%. The portfolio is known to include extensive option positions.
Calculate the 1-week VaR at 5% significance for a beginning portfolio value of GBP 100 million.

DEFINITION OF VAR

23. Given a VaR of \$12.5 million at 5% for one month, which of the following statements is correct?
- A. There is a 5% chance of losing \$12.5 million over one month.
 - B. There is a 95% chance that the expected loss over the next month is less than \$12.5 million.
 - C. The minimum loss that would be expected to occur over one month 5% of the time is \$12.5 million.
24. Which of the following statements is not correct?
- A. A 1% VaR implies a downward move of 1%.
 - B. A one standard deviation downward move is equivalent to a 16% VaR.
 - C. A 5% VaR implies a move of 1.65 standard deviations less than the expected value.

PARAMETRIC VAR

25. The parameters of normal distribution required to estimate parametric VaR are:
- A. expected value and standard deviation.
 - B. skewness and kurtosis.
 - C. standard deviation and skewness.
26. Assuming a daily expected return of 0.0384% and daily standard deviation of 0.9960% (as in the example in the text), which of the following is closest to the 1% VaR for a \$150 million portfolio? Express your answer in dollars.
- A. \$3.4 million
 - B. \$2.4 million
 - C. \$1.4 million

27. Assuming a daily expected return of 0.0384% and daily standard deviation of 0.9960% (as in the example in the text), the daily 5% parametric VaR is \$2,407,530. Rounding the VaR to \$2.4 million, which of the following values is closest to the annual 5% parametric VaR? Express your answer in dollars.
- A. \$38 million
 - B. \$25 million
 - C. \$600 million

HISTORICAL SIMULATION VAR

28. Which of the following statements about the historical simulation method of estimating VaR is most correct?
- A. A 5% historical simulation VaR is the value that is 5% to the left of the expected value.
 - B. A 5% historical simulation VaR is the value that is 1.65 standard deviations to the left of the expected value.
 - C. A 5% historical simulation VaR is the fifth percentile, meaning the point on the distribution beyond which 5% of the outcomes result in larger losses.
29. Which of the following is a limitation of the historical simulation method?
- A. The past may not repeat itself.
 - B. A reliance on the normal distribution.
 - C. Estimates of the mean and variance could be biased.

MONTE CARLO SIMULATION VAR

30. When will the Monte Carlo method of estimating VaR produce virtually the same results as the parametric method?
- A. When the Monte Carlo method assumes a non-normal distribution.
 - B. When the Monte Carlo method uses the historical return and distribution parameters.
 - C. When the parameters and the distribution used in the parametric method are the same as those used in the Monte Carlo method, and the Monte Carlo method uses a sufficiently large sample.
31. Which of the following is an advantage of the Monte Carlo method?
- A. The VaR is easy to calculate with a simple formula.
 - B. It is flexible enough to accommodate many types of distributions.
 - C. The number of necessary simulations is determined by the parameters.

ADVANTAGES AND LIMITATIONS OF VaR

32. Which of the following is not an advantage of VaR?
- A. It is a simple concept to communicate.
 - B. There is widespread agreement on how to calculate it.
 - C. It can be used to compare risk across portfolios or trading units.
33. Which of the following is a limitation of VaR?
- A. It requires the use of the normal distribution.
 - B. The maximum VaR is prescribed by federal securities regulators.
 - C. It focuses exclusively on potential losses, without considering potential gains.

EXTENSIONS OF VaR

34. Conditional VaR measures the:
- A. VaR over all possible losses.
 - B. VaR under normal market conditions.
 - C. average loss, given that VaR is exceeded.
35. Which of the following correctly identifies incremental VaR?
- A. The change in VaR from increasing a position in an asset.
 - B. The increase in VaR that might occur during extremely volatile markets.
 - C. The difference between the asset with the highest VaR and the asset with the second highest VaR.
36. Which of the following statements is correct about marginal VaR?
- A. The marginal VaR is the same as the incremental VaR.
 - B. The marginal VaR is the VaR required to meet margin calls.
 - C. Marginal VaR estimates the change in VaR for a small change in a given portfolio holding.

SENSITIVITY RISK MEASURES

37. Which of the following most accurately characterizes duration and convexity?
- A. Sensitivity of bond prices to interest rates
 - B. First- and second-order effects of yield changes on bond prices
 - C. Weighted-average time to maturity based on the coupon payments and principal
38. Which of the following statements about the delta of a call option is not correct?
- A. It ranges between 0 and 1.
 - B. It precisely captures the change in the call value when the underlying changes.
 - C. It approaches 1 for an in-the-money option and 0 for an out-of-the-money option.

39. Which of the following statements about gamma and vega are correct?
- A. Gamma is a second-order effect and vega is a first-order effect.
 - B. Gamma is the effect of volatility and vega is the effect of changes in volatility.
 - C. Gamma is a second-order effect arising from changes in the sensitivity of volatility to the underlying price.

SCENARIO ANALYSIS

40. Which of the following is an example of a reverse stress test?
- A. Identify the top 10 exposures in the portfolio and then generate a hypothetical stress that could adversely affect all 10 simultaneously.
 - B. Find the worst single day's performance that could have occurred for the current portfolio had it been held throughout the past five years.
 - C. Find the returns that occurred in all risk factors in the 2008 global financial crisis, reverse the sign on these, and apply them to today's portfolio.
41. Which kind of market participant is least likely to use scenario analysis as a pass/fail stress test?
- A. Bank
 - B. Long-only asset manager
 - C. Hedge fund using leverage
42. What is the most accurate approach to scenario analysis for a portfolio that uses options?
- A. Apply the scenario to option delta.
 - B. Apply the scenario to option delta + gamma.
 - C. Fully reprice the options using the market returns specified under the scenario.

LIMITATIONS OF RISK MEASURES

43. Which of the following is not a limitation of VaR?
- A. It does not adjust for bonds of different durations.
 - B. It largely relies on recent historical correlations and volatilities.
 - C. It can be inaccurate if the size of positions held is large relative to available liquidity.
44. Which of the following statements about sensitivities is true?
- A. When duration is measured as the sensitivity to a 1 bp change in interest rates, it can be biased by choice of the historical period preceding this measure.
 - B. Sensitivity measures are the best way to determine how an option can behave under extreme market movements.

C. Duration effectively assumes that the correlation between a fixed-income exposure and the risk-free rate is 1, whereas beta takes into account the historical correlation between an equity and its comparison index.

45. Which of the following is not a limitation of scenario measures?

- A. It is difficult to ascribe probability to a given scenario.
- B. Scenario measures assume a normal distribution, and market returns are not necessarily normal.
- C. They risk being an infinite task; one cannot possibly measure all of the possible future scenarios.

46. Which measures are based on market returns during a particular historical period?

- A. Hypothetical scenario analysis and duration sensitivity
- B. Historical scenario analysis and VaR
- C. Option delta and vega

47. Geometric and Arithmetic Mean Returns (1)

As a mutual fund analyst, you are examining, as of early 2013, the most recent five years of total returns for two US large-cap value equity mutual funds.

Total Returns for Two Mutual Funds, 2008–2012 Table 15.

Year	Selected American Shares (SLASX)	T. Rowe Price Equity Income (PRFDX)
2008	–39.44%	–35.75%
2009	31.64	25.62
2010	12.53	15.15
2011	–4.35	–0.72
2012	12.82	17.25

Based on the data in Table 15, address the following:

1. Calculate the geometric mean return of SLASX.
2. Calculate the arithmetic mean return of SLASX and contrast it to the fund's geometric mean return.
3. Calculate the geometric mean return of PRFDX.
4. Calculate the arithmetic mean return of PRFDX and contrast it to the fund's geometric mean return.

48. The table below gives statistics relating to a hypothetical 10-year record of two portfolios.

	Mean Annual Return (%)	Standard Deviation of Return (%)	Skewness
Portfolio A	8.3	19.5	-1.9
Portfolio B	8.3	18.0	3.0

Based only on the information in the above table, perform the following:

- A. Contrast the distributions of returns of Portfolios A and B.
- B. Evaluate the relative attractiveness of Portfolios A and B.

49. Is a return distribution characterized by frequent small losses and a few large gains best described as having:

	negative skew?	a mean that is greater than the median?
A	No	No
B	No	Yes
C	Yes	No

50. An analyst gathered the following information about the return distributions for two portfolios during the same time period:

	Skewness	Kurtosis
Portfolio A	-1.3	2.2
Portfolio B	0.5	3.5

The analyst stated that the distribution for Portfolio A is more peaked than a normal distribution and that the distribution for Portfolio B has a long tail on the left side of the distribution. Which of the following is most true?

- A. The statement is not correct in reference to either portfolio.
- B. The statement is correct in reference to Portfolio A, but the statement is not correct in reference to Portfolio B.
- C. The statement is not correct in reference to Portfolio A, but the statement is correct in reference to Portfolio B.

Use the following data to answer Questions 51 and 52.

The annual returns for DRG's common stock over the years 2003, 2004, 2005, and 2006 were 15%, 19%, -8%, and 14%.

51. What is the arithmetic mean return for DRG's common stock?

- A. 10.00%.
- B. 14.00%.
- C. 15.25%.

52. What is the geometric mean return for DRG's common stock?
- A. 9.45%.
 - B. 14.21%.
 - C. It cannot be determined because the 2005 return is negative.
53. The harmonic mean of 3,4, and 5 is:
- A. 3.74.
 - B. 3.83.
 - C. 4.12
54. There is an 80% chance that the economy will be good next year and a 20% chance that it will be bad. If the economy is good, there is a 60% chance that ASD Incorporated will have FGH of \$3.00 and a 40% chance that their earnings will be \$2.50. If the economy is bad, there is a 70% chance that ASD Incorporated will have FGH of \$1.50 and a 30% chance that their earnings will be \$1.00. What is the firm's expected FGH?
- A. \$2.00.
 - B. \$4.16.
 - C. \$2.51.
55. There is a 90% chance that the economy will be good next year and a 10% chance that it will be bad. If the economy is good, there is a 60% chance that ASD Incorporated will have FGH of \$4.00 and a 40% chance that their earnings will be \$3.00. If the economy is bad, there is an 80% chance that ASD Incorporated will have FGH of \$2.00 and a 20% chance that their earnings will be \$1.00. What is the firm's expected FGH?
- A. \$5.40.
 - B. \$3.42.
 - C. \$2.50.

56. CA Arghya Sen believes a stock's price in the next quarter depends on two factors: the direction of the overall market and whether the company's next earnings report is good or poor. The possible outcomes and some probabilities are illustrated in the tree diagram shown below:



Based on this tree diagram, the expected value of the stock if the market decreases is closest to:

- A. \$62.50.
 - B. \$26.00.
 - C. \$57.00.
57. The covariance:
- A. must be between -1 and +1.
 - B. must be positive.
 - C. can be positive or negative.
58. The covariance of the returns on investments X and Y is 18.17. The standard deviation of returns on X is 7%, and the standard deviation of returns on Y is 4%. What is the value of the correlation coefficient for returns on investments X and Y?
- A. +0.85.
 - B. +0.32.
 - C. +0.65.
59. The covariance of returns on two investments over a 10-year period is 0.009. If the variance of returns for investment A is 0.020 and the variance of returns for investment B is 0.033, what is the correlation coefficient for the returns?
- A. 0.444.
 - B. 0.687.
 - C. 0.350.

60. The correlation coefficient for a series of returns on two investments is equal to 0.80. Their covariance of returns is 0.06974 . Which of the following are possible variances for the returns on the two investments?
- A. 0.02 and 0.44.
 - B. 0.08 and 0.37.
 - C. 0.04 and 0.19.
61. An analyst expects that 20% of all publicly traded companies will experience a decline in earnings next year. The analyst has developed a ratio to help forecast this decline. If the company is headed for a decline, there is a 90% chance that this ratio will be negative. If the company is not headed for a decline, there is only a 10% chance that the ratio will be negative. The analyst randomly selects a company with a negative ratio. Based on Bayes' theorem, the updated probability that the company will experience a decline is:
- A. 26%.
 - B. 18%.
 - C. 69%.

CA - FINAL RISK MODELS

SOLUTIONS

1. ANSWER : C

Both statements are true. There are multiple methods to estimate VAR and the results do not coincide. Additionally, VAR examines only the lower tail of the distribution and it does not consider the upside benefits.

2. ANSWER : C

We first use portfolio theory to find the portfolio expected return and return standard deviation:

$$r_p = w_1 r_1 + w_2 r_2$$

$$= 0.40 \times 0.12 + 0.60 \times 0.16$$

$$= 14.40\%$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho \sigma_1 \sigma_2$$

$$= 0.40^2 \times 0.38^2 + 0.60^2 \times 0.49^2 + 2 \times 0.40 \times 0.60 \times 0.37 \times 0.38 \times 0.49$$

$$= 0.142609$$

$$\sigma_p = 37.76\%$$

	Stock X	Stock Y	Portfolio
Portfolio weight	0.40	0.60	1.00
Expected return	0.12	0.16	14.40%
Standard deviation	0.38	0.49	37.76%
Correlation	0.37		

Given a 5 percent lower tail, we look up the standard normal table to find the corresponding Z-score to be -1.645. We have:

$$-1.645 = \frac{\text{VAR}_{5\%} - 0.1440}{0.3776}$$

$$\text{VAR}_{5\%} = -47.12\%$$

We conclude that the 5 percent VAR of annual returns to be -47.72 percent. There is a 5 percent chance that we lose at least 47.72 percent of the investment in any given year.

3. ANSWER : C

Ten percent of 180 observations is 18 observations. The return corresponding to the 18th worst monthly return is -23.84%. So, the 10% monthly VAR of this stock is -23.84%. Note that the 10% VAR is smaller in magnitude than the 5% VAR.

4. ANSWER : A

A smaller tail probability implies a bigger loss. So the 1% weekly VAR is more than the 5% VAR of \$3 million.

5. ANSWER : C

The analyst is unlikely to find the variance/covariance model satisfactory since it assumes a normal distribution of returns and hence ignores skewness and kurtosis. It is also unlikely that the analyst will be comfortable basing risk estimates on historical values given their belief that crises lead to returns not previously witnessed. Monte Carlo is the only method that is powerful and flexible enough to model skewness and kurtosis and scenarios that have not actually occurred in the past.

6. ANSWER : B

TVAR stands for tail VAR. It's the conditional tail expectation. It is defined as the VAR plus the expected loss in excess of VAR conditional on such loss occurs. Unlike VAR which measures the minimum amount of loss, TVAR measures the expected size of loss conditional on the loss is greater than VAR.

7. ANSWER : B

IVAR stands for incremental VAR. It's the difference between the VAR of a portfolio including a specific asset and the VAR of the same portfolio without the specific asset. IVAR measures the incremental impact in VAR by including an asset in a portfolio.

8. ANSWER : B

When the mean return is assumed to be zero, under the variance/covariance method, VaR can be scaled up by multiplying by the square root of time. Hence, annual 1% VaR = $\$150,000 \times \sqrt{250} = \$2,371,708$.

9. ANSWER : A

IVaR is the incremental effect of adding a new asset class or position to a portfolio. In this case, $\text{IVaR} = \$3.0 \text{ million} - \$2.8 \text{ million} = \$0.2 \text{ million}$.

10. ANSWER : B

Five percent annual VAR is the cutoff where the lower tail captures a 5% probability. Based on the probability distribution above, -20% is the 5% VAR.

- 11.
- A. There is a 1 percent chance that the portfolio will lose at least £4.25 million in any given week.
 - B. There is a 99 percent chance that the portfolio will lose no more than £4.25 million in one week.
12. Statement A, which is the definition of VAR, is clearly correct. Statement B is also correct, because it lists the important decisions involved in measuring VAR. Statement D is correct: The longer the time period, the larger the possible losses. Statement C, however, is incorrect. The VAR number would be larger for a 1 percent probability than for a 5 percent probability. Accordingly, the correct answer is C.
- 13.
- A. The probability is 0.005 that the portfolio will lose at least 50 percent in a year. The probability is 0.005 that the portfolio will lose between 40 percent and 50 percent in a year. Cumulating these two probabilities implies that the probability is 0.01 that the portfolio will lose at least 40 percent in a year. So, the 1 percent yearly VAR is 40 percent of the market value of \$10 million, which is \$4 million.
 - B. The probability is 0.005 that the portfolio will lose at least 50 percent in a year, 0.005 that it will lose between 40 and 50 percent, 0.010 that it will lose between 30 and 40 percent, 0.015 that it will lose between 20 and 30 percent, and 0.015 that it will lose between 10 and 20 percent. Cumulating these probabilities indicates that the probability is 0.05 that the portfolio will lose at least 10 percent in a year. So, the 5 percent yearly VAR is 10 percent of the market value of \$10 million, which is \$1 million.
14. First, we must calculate the monthly portfolio expected return and standard deviation. Using "1" to indicate the US government bonds and "2" to indicate the UK government bonds, we have
- $$\mu_P = w_1\mu_1 + w_2\mu_2 = 0.50(0.0085) + 0.50(0.0095) = 0.0090$$
- $$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho$$
- $$= (0.50)^2(0.0320)^2 + (0.50)^2(0.0526)^2 + 2(0.50)(0.50)(0.0320)(0.0526)(0.35)$$
- $$= 0.001242$$
- $$\sigma_P = \sqrt{0.001242} = 0.0352$$
- A. For a 5 percent monthly VAR, we have $\mu_P - 1.65\sigma_P = 0.0090 - 1.65(0.0352) = -0.0491$. Then the VAR would be $\$100,000,000(0.0491) = \4.91 million.
 - B. For a 1 percent monthly VAR, we have $\mu_P - 2.33\sigma_P = 0.0090 - 2.33(0.0352) = -0.0730$. Then the VAR would be $\$100,000,000(0.0730) = \7.30 million.
 - C. There are 12 months or 52 weeks in a year. So, to convert the monthly return of 0.0090 to weekly return, we first multiply the monthly return by 12 to convert it to an annual return, and then we divide the annual return by 52 to convert it to a weekly return.

So, the expected weekly return is $0.0090(12/52) = 0.0021$. Similarly, we adjust the standard deviation to $0.0352(12/\sqrt{52}) = 0.01691$. The 5 percent weekly VAR would then be $\mu_P - 1.65\sigma_P = 0.0021 - 1.65(0.01691) = -0.0258$. Then the VAR in dollars would be $\$100,000,000(0.0258) = \2.58 million.

D. The 1 percent weekly VAR would be $\mu_P - 2.33\sigma_P = 0.0021 - 2.33(0.01691) = -0.0373$. Then the VAR would be $\$100,000,000(0.0373) = \3.73 million.

15.

- A. For the five-year period, there are 60 monthly returns. Of the 60 returns, the 5 percent worst are the 3 worst returns. Therefore, based on the historical method, the 5 percent VAR would be the third worst return. From the returns given, the third worst return is -0.1980 . So, the VAR in dollars is $0.1980(\$25,000) = \$4,950$.
- B. Of the 60 returns, the 1 percent worst are the 0.6 worst returns. Therefore, we would use the single worst return. From the returns given, the worst return is -0.2597 . So, the VAR in dollars is $0.2597(\$25,000) = \$6,492.50$.

16.

- A. Of the 700 outcomes, the worst 5 percent are the 35 worst returns. Therefore, the 5 percent VAR would be the 35th worst return. From the data given, the 35th worst return is -0.223 . So, the 5 percent annual VAR in dollars is $0.223(\$10,000,000) = \$2,230,000$.
- B. Of the 700 outcomes, the worst 1 percent are the 7 worst returns. Therefore, the 1 percent VAR would be the seventh worst return. From the data given, the seventh worst return is -0.347 . So, the 1 percent annual VAR in dollars is $0.347(\$10,000,000) = \$3,470,000$.

17.

- A. The analytical or variance-covariance method begins with the assumption that portfolio returns are normally distributed. A normal distribution has an unlimited upside and an unlimited downside. The assumption of a normal distribution is inappropriate when the portfolio contains options because the return distributions of options are far from normal. Call options have unlimited upside potential, as in a normal distribution, but the downside return is truncated at a loss of 100 percent. Similarly, put options have a limited upside and a large but limited downside. Likewise, covered calls and protective puts have limits in one direction or the other. Therefore, for the portfolio that has options, the assumption of a normal distribution to estimate VAR has a number of problems. In addition, it is very difficult to calculate a covariance between either two options or an option and a security with more linear characteristics—among other reasons because options have different dynamics at different points in their life cycle.
- B. Portfolios with simple, linear characteristics, particularly those with a limited budget for computing resources and analytical personnel, might select the variance/covariance method. For more complex portfolios containing options and time-sensitive bonds, the

historical method might be more appropriate. The Monte Carlo simulation method typically would not be a wise choice unless it were managed by an organization with a portfolio of complex derivatives that is willing to make and sustain a considerable investment in technology and human capital.

18.

- A. The observed outcomes are consistent with the VAR calculation's prediction on the frequency of losses exceeding the VAR. Therefore, the VAR calculation is accurate.
- B. The VAR results indicate that under "normal" market conditions that would characterize 19 out of 20 days, the portfolio ought to lose less than €3 million. It provides no other information beyond this.
- C. The portfolio certainly lends itself to scenario analysis. In this particular case, given the substantial short options position, it might be instructive to create a customized scenario under which the portfolio was analyzed in the wake of a large increase in option-implied volatility.

19. ANSWER : B

An organization that holds its 1% annual VaR as capital reserves has in theory only a 1% chance of insolvency over the next year since there is a 99% chance they will not suffer a loss equal to the VaR or worse.

20. A CA candidate would know that 5% in a single tail is associated with 1.645, or approximately 1.65, standard deviations from the mean expected return. Therefore, the 5% annual VaR is:

$$\begin{aligned}\text{VAR} &= [R_p - (z)(\sigma)] V_p \\ &= [6.0\% - 1.65(12.0\%)] (\$100,000,000) \\ &= -13.8\% (\$100,000,000) \\ &= -\$13,800,000\end{aligned}$$

where :

R_p = expected return on the portfolio

V_p = value of the portfolio

z = z - value corresponding with the desired level of significance

σ = standard deviation of returns

The interpretation is that there is 5% probability that the annual loss will exceed \$13.8 million and a 95% probability the annual loss will be less.

21. Because these are the lowest five returns, they represent the 5% lower tail of the “distribution” of 100 historical returns. The fifth lowest return (–0.0019) is the 5% daily VaR. We would say there is a 5% chance of a daily loss exceeding 0.19%, or \$190,000.
22. Option positions make the use of standard deviation inappropriate for calculating VaR. Based on the Monte Carlo simulations, the 5th percentile worst result is the 5th worst return of +1.5% for a GBP 1,500,000 gain in portfolio value over one week. 5% of the time the gain would be worse.
It appears this is a very conservative portfolio because the VaR is a gain. Typically VaR is a loss but not in this case.
23. C is correct because it is the only statement that accurately expresses the VaR. A is incorrect because VaR does not give the likelihood of losing a specific amount. B is incorrect because VaR is not an expected loss – it is a minimum loss.
24. A is correct. A 1% VaR (99% confidence) is the point on the distribution 2.33 standard deviations below the expected value. Answers B and C correctly describe a 16% and 5% VaR, respectively.
25. A is correct. The parameters of a normal distribution are the expected value and standard deviation. Skewness, as mentioned in B and C, and kurtosis as mentioned in B, are characteristics used to describe a non-normal distribution.
26. A is correct and is obtained as follows:
1. Step 1
 $2.33 \times 0.009960 = 0.023207$
 2. Step 2
 $0.000384 - 0.023207 = -0.022823$
 3. Step 3
Convert –0.022823 to 0.022823
 4. Step 4
 $0.022823 \times \$150 \text{ million} = \$3,423,450.$
- B is the estimated VaR at a 5% threshold, and C is the estimated VaR using a one-standard deviation threshold.
27. B is correct. It is found by annualizing the daily return and standard deviation and using these figures in the calculation. The annual return and standard deviation are, respectively, 0.096000 (0.000384×250) and 0.157483 ($0.009960 \times 250^{---\sqrt{}}$).
1. Step 1
 $0.157483 \times 1.65 = 0.259847$

2. Step 2

$$0.096000 - 0.259847 = -0.163847$$

3. Step 3

Convert -0.163847 to 0.163847

4. Step 4

$$0.163847 \times \$150 \text{ million} = \$24,577,050.$$

A incorrectly multiplies the daily VaR by the square root of the number of trading days in a year ($\sqrt{250}$), and C incorrectly multiplies the daily VaR by the approximate number of trading days in a year (250). Neither A nor C make the appropriate adjustment to annualize the standard deviation.

28. C is correct. In the historical method, the portfolio returns are arrayed lowest to highest and the observation at the fifth percentile (95% of the outcomes are better than this outcome) is the VaR. A is not correct because it draws a point on the distribution relative to the expected value rather than the using the 5% of the outcomes that are in the left-most of the distribution. B confuses the parametric and historical methods. In the parametric method, the 5% VaR lies 1.65 standard deviations below the mean.
29. A is correct. The historical simulation method estimates VaR based on the historical distribution of the risk factors. B is not correct; the historical simulation method does not rely on any particular distribution because it simply uses whatever distribution applied in the past. C is not correct because the historical distribution does not formally estimate the mean and variance.
30. C is correct. The Monte Carlo method simulates outcomes using whatever distribution is specified by the user. If a normal distribution is used and a sufficiently large number of simulations are run, the parameters of the Monte Carlo sample will converge with those used in the parametric method, and the overall VaR should be very close to that of the parametric method. A is incorrect because the parametric method is not well-adapted to a non-normal distribution. B is not correct because neither the Monte Carlo method nor the parametric method focus on historical outcomes.
31. B is correct. The method can handle any distribution. A is incorrect because Monte Carlo simulation is not a simple formula. C is incorrect; there is no industry-wide agreement as to the necessary number of simulations.
32. B is correct. There is no consensus on how to calculate VaR. A and C are both advantages of VaR, as we noted that VaR is fairly simple to communicate and it can show the contribution of each unit to the overall VaR.

33. C is correct. VaR deals exclusively with left-tail or adverse events. A is wrong because although parametric VaR does generally use the normal distribution, the historical simulation method uses whatever distribution occurred in the past and Monte Carlo simulation uses whatever distribution the user chooses. B is incorrect because regulators do not specify maximum VaRs, although they may encourage and require that the measure be used.
34. C is correct. Conditional VaR is the average loss conditional on exceeding the VaR. A is not correct because CVaR is not concerned with losses that do not exceed the VaR threshold, and B is incorrect as because VaR does not distinguish between normal and non-normal markets.
35. A correctly defines incremental VaR. Incremental VaR is not a change in VaR from an increase in volatility. B is not correct because incremental volatility reflects the results of intentional changes in exposure, not from uncontrollable market volatility. C is not correct because incremental VaR is not the difference in the VaRs of the assets with the greatest and second greatest VaRs.
36. C is correct. In A, marginal VaR is a similar concept to incremental VaR in that they both deal with the effect of changes in VaR, but they are not the same concept. B is incorrect because marginal VaR has nothing to do with margin calls.
37. B is correct. Duration is the first-order effect and convexity is the second-order effect of a change in interest rates on the value of a bond. A and C are correct with respect to duration, but not for convexity.
38. B is correct. A and C correctly characterize delta, whereas B states that delta is precise, which is incorrect because it gives an approximate relationship.
39. A is correct. B is not correct because Gamma does not capture the effect of volatility. Vega is the effect of volatility, but it relates to the level and not the change in volatility. C is incorrect because, although gamma is a second-order effect on the option value, it is not related to the sensitivity of volatility to the underlying price.
40. A is correct. B is not a reverse stress test because reverse stress tests focus more narrowly on trouble spots for a specific portfolio. C would illustrate how the portfolio would have performed in an extremely strong market, quite unlike what occurred in 2008.
41. B is correct. Long-only asset managers do not typically use leverage and are thus less likely to become insolvent, making a pass/fail test for solvency less relevant to them. A and C are not correct because parties that use leverage, such as hedge funds and banks, are

likely to use stress tests to determine what market movements could impair their capital and lead to insolvency.

- 42.** C is correct. Both A and B risk misestimating the actual results of the scenario because both delta and gamma estimate how an option's value might change for a small move in the underlying asset, not the large movements typically used in a scenario analysis.
- 43.** A is correct. Well-executed VaR measures do adjust for bonds of differing duration, and therefore it is not a limitation of VaR. B is incorrect because VaR ordinarily uses some period of recent history as part of the calculation, and this reliance on history is one of its limitations. C is incorrect because VaR can be inaccurate and underestimate risk if portfolio positions are too large relative to the available market liquidity, and this inability to account for the illiquidity of an individual investor's position is an additional limitation of VaR.
- 44.** C is correct. Duration assumes that all interest rates that affect a bond change by the same percentage (an effective correlation of 1). A is incorrect because the 1 bp change in rates is applied to current rates, not historical rates. B is incorrect because sensitivity measures are often too small to reveal the most extreme movements for option positions; the larger shocks used in scenario measures are preferable to reveal option characteristics.
- 45.** B is correct. Scenario measures do not assume any given distribution, and thus this is not a limitation of scenario analysis. A is incorrect because it is in fact difficult to ascribe probability to many scenarios, and thus this is a limitation of scenario analysis. C is also incorrect because it is in fact impossible to measure all possible future scenarios, and this is a limitation of scenario analysis.
- 46.** B is correct. Historical scenarios apply market returns from a particular period to the portfolio, and virtually all VaR methodologies use a historical period to underpin the VaR model, although certain methods may make adjustments if this historical period is seen to be anomalous in some way. A is incorrect because a hypothetical scenario is not based on an actual historical period, and duration sensitivity measures change in value for a given small change in rates, not for a given historical period. C is incorrect because option delta and vega measure how much an option's value will change for a given change in the price of the underlying (delta) or implied volatility (vega), and these are sensitivity measures, not measures based on a particular historical period.

47.

1. Converting the returns on SLASX to decimal form and adding 1.0 to each return produces 0.6056, 1.3164, 1.1253, 0.9565, and 1.1282. We use Equation 6 to find SLASX's geometric mean return:

$$\begin{aligned}
 R_G &= \sqrt[5]{(0.6056)(1.3164)(1.1253)(0.9565)(1.1282)} - 1 \\
 &= \sqrt[5]{0.968084} - 1 = 0.993534 - 1 = -0.006466 \\
 &= -0.65\%
 \end{aligned}$$

$$\begin{aligned}
 R_G &= \sqrt[5]{(0.6056)(1.3164)(1.1253)(0.9565)(1.1282)} - 1 \\
 &= \sqrt[5]{0.968084} - 1 = 0.993534 - 1 = -0.006466 \\
 &= -0.65\%
 \end{aligned}$$

2. For SLASX, $R^{---} = (-39.44 + 31.64 + 12.53 - 4.35 + 12.82) / 5 = 13.20 / 5 = 2.64\%$. The arithmetic mean return for SLASX exceeds the geometric mean return by $2.64 - (-0.65) = 3.29\%$ or 329 basis points.

3. Converting the returns on PRFDX to decimal form and adding 1.0 to each return produces 0.6425, 1.2562, 1.1515, 0.9928, and 1.1725. We use Equation 6 to find PRFDX's geometric mean return:

$$\begin{aligned}
 R_G &= \sqrt[5]{(0.6425)(1.2562)(1.1515)(0.9928)(1.1725)} - 1 \\
 &= \sqrt[5]{0.081859} - 1 = 1.015861 - 1 = -0.015861 \\
 &= -1.59\%
 \end{aligned}$$

$$\begin{aligned}
 R_G &= \sqrt[5]{(0.6056)(1.3164)(1.1253)(0.9565)(1.1282)} - 1 \\
 &= \sqrt[5]{0.968084} - 1 = 0.993534 - 1 = -0.006466 \\
 &= -0.65\%
 \end{aligned}$$

4. PRFDX, $R^{---} = (-35.75 + 25.62 + 15.15 - 0.72 + 17.25) / 5 = 21.55 / 5 = 4.31\%$. The arithmetic mean for PRFDX exceeds the geometric mean return by $4.31 - 1.59 = 2.72\%$ or 272 basis points. The table below summarizes the findings.

48.

- A. With identical means, the two return distributions are similarly centered. Portfolio B's distribution has somewhat less dispersion, as measured by standard deviation. Both return distributions are asymmetric but in different ways. The return distribution for Portfolio A is negatively skewed; Portfolio B's distribution is positively skewed.
- B. Most investors would prefer the return distribution of Portfolio B, which has the same mean return as Portfolio A but less risk as measured by standard deviation of return. Furthermore, Portfolio B's returns are positively skewed, indicating a higher frequency of very large positive returns relative to Portfolio A. In contrast, Portfolio A's returns are negatively skewed.

49. B is correct. A distribution with frequent small losses and a few large gains has positive skew (long tail on the right side) and the mean is greater than the median.

50. A is correct. The analyst's statement is not correct in reference to either portfolio. Portfolio A has a kurtosis of less than 3 meaning that it is less peaked than a normal distribution (platykurtic). Portfolio B is positively skewed (long tail on the right side of the distribution).

51. Answer : A

$$(15\% + 19\% + (-8\%) + 14\%) / 4 = 10\%$$

52. Answer : A

$$(1.15 \times 1.19 \times 0.92 \times 1.14)^{0.25} - 1 = 9.45\%$$

~ Professor's Note: This question could have been answered very quickly since the geometric ~ mean must be less than the arithmetic mean computed in the preceding problem.

53. Answer : B

$$3.83$$

54. Answer : C

The expected FGH is calculated by multiplying the probability of the economic environment by the probability of the particular FGH and the FGH in each case. The expected FGH in all four outcomes are then summed to arrive at the expected FGH: $(0.80 \times 0.60 \times \$3.00) + (0.80 \times 0.40 \times \$2.50) + (0.20 \times 0.70 \times \$1.50) + (0.20 \times 0.30 \times \$1.00) = \$1.44 + \$0.80 + \$0.21 + \$0.06 = \$2.51$.

55. Answer : B

The expected FGH is calculated by multiplying the probability of the economic environment by the probability of the particular FGH and the FGH in each case. The expected FGH in all four outcomes are then summed to arrive at the expected FGH: $(0.90 \times 0.60 \times \$4.00) + (0.90 \times 0.40 \times \$3.00) + (0.10 \times 0.80 \times \$2.00) + (0.10 \times 0.20 \times \$1.00) = \$2.16 + \$1.08 + \$0.16 + \$0.02 = \$3.42$.

56. Answer : C

The expected value if the overall market decreases is $0.4(\$60) + (1 - 0.4)(\$55) = \$57$.

57. Answer : C

$\text{Cov}(a,b) = \sigma_a \sigma_b \rho_{a,b}$. Since $\rho_{a,b}$ can be positive or negative, $\text{Cov}(a,b)$ can be positive or negative.

58. Answer : C

The correlation coefficient = $\text{Cov}(X,Y) / [(\text{Std Dev. } X)(\text{Std. Dev. } Y)] = 18.17 / 28 = 0.65$

59. Answer : C

The correlation coefficient is: $\text{Cov}(A,B) / [(\text{Std Dev } A)(\text{Std Dev } B)] = 0.009 / [(\sqrt{0.02})(\sqrt{0.033})] = 0.350$.

60. Answer : C

The correlation coefficient is: $0.06974 / [(\text{Std Dev } A)(\text{Std Dev } B)] = 0.8$. $(\text{Std Dev } A)(\text{Std Dev } B) = 0.08718$. Since the standard deviation is equal to the square root of the variance, each pair of variances can be converted to standard deviations and multiplied to see if they equal 0.08718. $\sqrt{0.04} = 0.20$ and $\sqrt{0.19} = 0.43589$. The product of these equals 0.08718.

61. Answer : C

Given a set of prior probabilities for an event of interest, Bayes' formula is used to update the probability of the event, in this case that the company we have already selected will experience a decline in earnings next year. Bayes' formula says to divide the Probability of New Information given Event by the Unconditional Probability of New Information and multiply that result by the Prior Probability of the Event. In this case, $P(\text{company having a decline in earnings next year}) = 0.20$ is divided by 0.26 (which is the Unconditional Probability that a company having an earnings decline will have a negative ratio (90% have negative ratios of the 20% which have earnings declines) plus (10% have negative ratios of the 80% which do not have earnings declines) or $((0.90) \times (0.20)) + ((0.10) \times (0.80)) = 0.26$.) This result is then multiplied by the Prior Probability of the ratio being negative, 0.90. The result is $(0.20 / 0.26) \times (0.90) = 0.69$ or 69%.

NOTES