RD Sharma Solutions Class 12 Maths Chapter 16 Ex 16.1

Tangents and Normals Ex 16.1 Q1(i)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

Now,

$$y = \sqrt{x^3}$$

$$\therefore \quad \frac{dy}{dx} = \frac{3x^2}{2\sqrt{x^3}}$$

$\therefore$ Slope of tangent at $x = 4$ is

$$\left(\frac{dy}{dx}\right)_{x=4} = \frac{3.16}{2\sqrt{64}} = \frac{48}{16} = 3$$

Slope of normal at $x = 4$ is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = \frac{-1}{3}$$

Tangents and Normals Ex 16.1 Q1(ii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$$y = \sqrt{x}$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$\therefore$ Slope oftangent at $x = 9$.

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -6$$

Tangents and Normals Ex 16.1 Q1(iii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = x^3 - x$

$$\therefore \quad \frac{dy}{dx} = 3x^2 - 1$$

$\therefore$ Slope of tangent at $x = 2$ is

$$\left(\frac{dy}{dx}\right)_{x=2} = 3.2^2 - 1 = 11$$

Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = \frac{-1}{11}$$

Tangents and Normals Ex 16.1 Q1(iv)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = 2x^2 + 3\sin x$

$$\therefore \quad \frac{dy}{dx} = 4x + 3\cos x$$

So, slope of tangent of $x = 0$ is

$$\left(\frac{dy}{dx}\right)_{x=0} = 4.0 + 3\cos 0° = 3$$

And slope of normanl is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = \frac{-1}{3}$$

Tangents and Normals Ex 16.1 Q1(v)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x = a(\theta - \sin\theta),\ y = a(1 + \cos\theta)$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1 - \cos\theta)}$$

$\therefore$ Slope of tangent of $\theta = -\frac{\pi}{2}$

$$\left(\frac{dy}{dx}\right)_{\theta = -\frac{\pi}{2}} = \frac{-a\sin\left(-\frac{\pi}{2}\right)}{a\left(1 - \cos\left(-\frac{\pi}{2}\right)\right)}$$

$$= \frac{a}{a(1-0)} = 1$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -1$$

Tangents and Normals Ex 16.1 Q1(vi)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x = a\cos^3\theta,\ y = a\sin^3\theta$

$$\therefore \qquad \frac{dx}{d\theta} = 3a\cos^2\theta \times (-\sin\theta) = -3a\sin\theta \times \cos^2\theta$$

and $$\frac{dy}{dx} = 3a\sin^2\theta \times \cos\theta$$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta \times \cos\theta}{-3a\sin\theta \times \cos^2\theta}$$

$$= -\tan\theta$$

$\therefore$ Slope of tangent at $\theta = \frac{\pi}{4}$ is

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = 1$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -1$$

Tangents and Normals Ex 16.1 Q1(vii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$

$$\therefore \quad \frac{dx}{d\theta} = a(1 - \cos\theta),\ \frac{dy}{d\theta} = a(0 + \sin\theta) = a\sin\theta$$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1 - \cos\theta)}$$

Now, the slope of tangent at $\theta = \frac{\pi}{2}$ is

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{a\sin\frac{\pi}{2}}{a\left(1 - \cos\frac{\pi}{2}\right)} = \frac{a}{a} = 1$$

And, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = -1$$

Tangents and Normals Ex 16.1 Q1(viii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = (\sin 2x + \cot x + 2)^2$

$\therefore \quad \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \operatorname{cosec}^2 x)$

$\therefore \quad$ Slope of tangent of $x = \frac{\pi}{2}$ is

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 2\left(\sin\pi + \cos\frac{\pi}{2} + 2\right)\left(2\cos\pi - \operatorname{cosec}^2\frac{\pi}{2}\right)$$

$$= 2(0 + 0 + 2)(-2 - 1)$$

$$= -12$$

$\therefore \quad$ Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{1}{12}$$

Tangents and Normals Ex 16.1 Q1(ix)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x^2 + 3y + y^2 = 5$

Differentiating with respect to x, we get

$$2x + 3\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$\Rightarrow \quad \frac{dy}{dx}(3 + 2y) = -2x$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-2x}{3 + 2y}$

So, the slope of tangent at $(1,1)$ is

$$\frac{dy}{dx} = \frac{-2.1}{3 + 2.1} = \frac{-2}{5}$$

The slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{5}{2}$$

Tangents and Normals Ex 16.1 Q1(x)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$xy = 6$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x}$$

$\therefore$ Slope of tangent at $(1,6)$ is

$$\frac{dy}{dx} = -6 \text{ and}$$

Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{1}{6}$$

Tangents and Normals Ex 16.1 Q2

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} + a + b\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(x + b) = -(a + y)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(a + y)}{x + b}$$

$\therefore$ Slope of tangent $= \left(\frac{dy}{dx}\right)_{x=1,\, y=1} = \frac{-(a+1)}{b+1} = 2$ [given]

$$\Rightarrow \quad -(a + 1) = 2b + 2$$

$$\Rightarrow \quad 2b + a = -3 \qquad \text{---(i)}$$

Also, $(1,1)$ lies on the curve, so x = 1, y = 1 satisfies the equation

$$xy + ax + by = 2$$

$$\Rightarrow \quad 1 + a + b = 2$$

$$\Rightarrow \quad a + b = 1 \qquad \text{---(ii)}$$

Solving (i) and (ii), we get

$$a = 5,\ b = -4$$

Tangents and Normals Ex 16.1 Q3

We have,

$$y = x^3 + ax + b \qquad ---(i)$$

$$x - y + 5 = 0 \qquad ---(ii)$$

Now,

Point $(1,-6)$ lies on (i), so,

$$-6 = 1 + a + b$$

$\Rightarrow$ $$a + b = -7 \qquad ---(iii)$$

Also,

Slope of tangent to (i) is

$$\frac{dy}{dx} = 3x^2 + a$$

$\Rightarrow$ $$\left(\frac{dy}{dx}\right)_{(1,-6)} = 3 + a$$

And slope of tangent to (ii) is

$$\frac{dy}{dx} = 1$$

According to the question slope of (i) and (ii) are parallel

$\therefore$ $$3 + a = 1$$

$\Rightarrow$ $$a = -2$$

From (iii)

$$b = -5$$

Tangents and Normals Ex 16.1 Q4

We have,

$$y = x^3 - 3x \qquad ---(i)$$

$\therefore$ Slope of (i) is

$$\frac{dy}{dx} = 3x^2 - 3 \qquad ---(ii)$$

Also,

The slope of the chord obtained by joining the points $(1,-2)$ and $(2,2)$ is

$$\frac{2-(-2)}{2-1} \qquad \left[\text{Slope } \frac{y_2 - y_1}{x_2 - x_1}\right]$$

$$= 4$$

According to the question slope of tangent to (i) and the chord are parallel

$\therefore$ $$3x^2 - 3 = 4$$

$\Rightarrow$ $$3x^2 = 7$$

$\Rightarrow$ $$x = \pm\sqrt{\frac{7}{3}}$$

From (i)

$$y = \pm\sqrt{\frac{7}{3}} \mp 3\sqrt{\frac{7}{3}}$$

$$= \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

Thus, the required point is

$$\pm\sqrt{\frac{7}{3}}, \ \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

Tangents and Normals Ex 16.1 Q5

The given equations are

$$y = x^3 - 2x^2 - 2x \quad \text{---(i)}$$

$$y = 2x - 3 \quad \text{---(ii)}$$

Slope to the tangents of (i) and (ii) are

$$\frac{dy}{dx} = 3x^2 - 4x - 2 \quad \text{---(iii)}$$

and $$\frac{dy}{dx} = 2 \quad \text{---(iv)}$$

According to the question slope to (i) and (ii) are parallel, so

$$3x^2 - 4x - 2 = 2$$

$$\Rightarrow \quad 3x^2 - 4x - 4 = 0$$

$$\Rightarrow \quad 3x^2 - 6x + 2x - 4 = 0$$

$$\Rightarrow \quad 3x(x - 2) + 2(x - 2) = 0$$

$$\Rightarrow \quad (3x + 2)(x - 2) = 0$$

$$\Rightarrow \quad x = \frac{-2}{3} \text{ or } 2$$

From (i)

$$y = \frac{4}{27} \text{ or } -4$$

Thus, the points are

$$\left(\frac{-2}{3}, \frac{4}{27}\right) \text{ and } (2,-4)$$

Tangents and Normals Ex 16.1 Q6

We have,

$$y^2 = 2x^3 \quad \text{---(i)}$$

Differentiating (i) with respect to x, we get

$$2y\frac{dy}{dx} = 6x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2}{y} \quad \text{---(ii)}$$

According to the question

$$\frac{3x^2}{y} = 3$$

$$\Rightarrow \quad x^2 = y \quad \text{---(iii)}$$

From (i) and (ii)

$$\left(x^2\right)^2 = 2x^3$$

$$\Rightarrow \quad x^4 - 2x^3 = 0$$

$$\Rightarrow \quad x^3\left(x - 2\right) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } 2$$

If $x = 0$, then

$$\frac{dy}{dx} = \frac{3x^2}{y} \Rightarrow \frac{dy}{dx} = 0$$

Which is not possible.

$\therefore \quad x = 2.$

Putting $x = 2$ in the equation of the curve $y^2 = 2x^3$, we get $y = 4$.

Hence the required point is $(2, 4)$

Tangents and Normals Ex 16.1 Q7

We know that the slope to any curve is $\frac{dy}{dx} = \tan\theta$ where θ is the angle with possitive direction of x-axis.

Now,

The given curve is

$xy + 4 = 0$ ---(i)

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x} \quad \text{---(ii)}$$

Also,

$$\frac{dy}{dx} = \tan 45° = 1 \quad \text{---(iii)}$$

$\therefore$ From (ii) and (iii)

$$\frac{-y}{x} = 1$$

$$\Rightarrow \quad x = -y \quad \text{---(iv)}$$

From (i) and (iv), we get

$$-y^2 + 4 = 0$$

$$\Rightarrow \quad y = \pm 2$$

$$\therefore \quad x = \mp 2$$

Thus, the points are

$(2,-2)$ and $(-2,2)$

Tangents and Normals Ex 16.1 Q8

The given equation of the curve is

$$y = x^2 \quad \text{---(i)}$$

$\therefore$ Slope of tangent to (i) is

$$\frac{dy}{dx} = 2x \quad \text{---(ii)}$$

According to the question

$$\frac{dy}{dx} = x \quad \text{---(iii)} \quad [\text{Slope} = x\text{-coordinate}]$$

From (ii) and (iii)

$$2x = x$$

$$\Rightarrow \quad x = 0 \ \& \ y = 0$$

Thus, the required point is $(0,0)$

Tangents and Normals Ex 16.1 Q9

The given equation of the curve is

$$x^2 + y^2 - 2x - 4y + 1 = 0 \qquad ---(i)$$

Differentiating with respect is x, we get

$$2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(2y - 4) = 2 - 2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2(1-x)}{2(y-2)} \qquad ---(ii)$$

According to the question the tangent is parallel to x-axis, so $\theta = 0°$

$$\therefore \quad \text{Slope} = \tan\theta = \tan 0° = 0 \qquad ---(iii)$$

From (ii) and (iii), we get

$$\frac{1-x}{y-2} = 0$$

$$\Rightarrow \quad 1 - x = 0$$

$$\Rightarrow \quad x = 1$$

$\therefore$ from (i)

$$y = 0, 4$$

Thus, the points are $(1,0)$ and $(1,4)$

Tangents and Normals Ex 16.1 Q10

The given equation of curve is

$$y = x^2 \qquad ---(i)$$

$$\therefore \quad \text{Slope} = \frac{dy}{dx} = 2x \qquad ---(ii)$$

As per question

$$\text{slope} = \tan 45° = 1 \qquad ---(iii)$$

From (ii) and (iii), we have

$$2x = 1$$

$$\Rightarrow \quad x = \frac{1}{2}$$

$\therefore$ From (i)

$$y = \frac{1}{4}$$

Thus, the required point is

$$\left(\frac{1}{2}, \frac{1}{4}\right)$$

Tangents and Normals Ex 16.1 Q11

The given equation of the curve is

$$y = 3x^2 - 9x + 8 \qquad \text{---(i)}$$

$$\text{Slope} = \frac{dy}{dx} = 6x - 9 \qquad \text{---(ii)}$$

As per question

The tangent is equally inclined to the axes

$$\therefore \quad \theta = \frac{\pi}{4} \text{ or } \frac{-\pi}{4}$$

$$\therefore \quad \text{Slope} = \tan\theta$$

$$= \tan\frac{\pi}{4} \text{ or } \tan\left(\frac{-\pi}{4}\right)$$

$$= 1 \text{ or } -1 \qquad \text{---(iii)}$$

From (ii) and (iii), we have,

$$6x - 9 = 1 \quad \text{or} \quad 6x - 9 = -1$$

$$\Rightarrow \quad x = \frac{5}{3} \quad \text{or} \quad x = \frac{4}{3}$$

So, from (i)

$$y = \frac{4}{3} \quad \text{or} \quad y = \frac{4}{3}$$

Thus, the points are

$$\left(\frac{5}{3}, \frac{4}{3}\right) \text{ or } \left(\frac{4}{3}, \frac{4}{3}\right)$$

Tangents and Normals Ex 16.1 Q12

The given equation are

$$y = 2x^2 - x + 1 \qquad \text{---(i)}$$

$$y = 3x + 4 \qquad \text{---(ii)}$$

Slope to (i) is

$$\frac{dy}{dx} = 4x - 1 \qquad \text{---(iii)}$$

Slope to (ii) is

$$\frac{dy}{dx} = 3 \qquad \text{---(iv)}$$

According to the question

$$4x - 1 = 3$$

$$\Rightarrow \quad x = 1$$

Thus from (i)

$$y = 2$$

Hence, the point is $(1, 2)$.

Tangents and Normals Ex 16.1 Q13

The given equation of curve is

$$y = 3x^2 + 4 \qquad \text{---(i)}$$

$$\text{Slope} = m_1 = \frac{dy}{dx} = 6x \qquad \text{---(ii)}$$

Now,

The given slope $m_2 = \frac{-1}{6}$

We have,

tangent to (i) is perpendicular to the tangent whose slope is $\frac{-1}{6}$

$\therefore \quad m_1 \times m_2 = -1$

$\Rightarrow \quad 6x \times \frac{-1}{6} = -1$

$\Rightarrow \quad x = 1$

From (i)

$y = 7$

Thus, the required point is $(1, 7)$.

Tangents and Normals Ex 16.1 Q14

The given equation of curve and the line is

$$x^2 + y^2 = 13 \qquad \text{---(i)}$$

and $\quad 2x + 3y = 7 \qquad \text{---(ii)}$

Slope = m_1 for (i)

$$m_1 = \frac{dy}{dx} = \frac{-x}{y} \qquad \text{---(iii)}$$

Slope = m_2 for (ii)

$$m_2 = \frac{dy}{dx} = \frac{-2}{3} \qquad \text{---(iv)}$$

According to the question

$m_1 = m_2$

$\Rightarrow \quad \frac{-x}{y} = \frac{-2}{3}$

$\Rightarrow \quad x = \frac{2}{3}y$

From (i)

$\frac{4}{9}y^2 + y^2 = 13$

$\Rightarrow \quad \frac{13y^2}{9} = 13$

$\Rightarrow \quad y = \pm 3$

$\therefore \quad x = \pm 2$

Thus, the points are $(2, 3)$ and $(-2, -3)$.

Tangents and Normals Ex 16.1 Q15

The given equation of the curve is

$$2a^2y = x^3 - 3ax^2 \qquad \text{---(i)}$$

Differentiating with respect to x, we get

$$2a^2\frac{dy}{dx} = 3x^2 - 6ax$$

$$\therefore \text{ Slope } m_1 = \frac{dy}{dx} = \frac{1}{2a^2}\left[3x^2 - 6ax\right] \qquad \text{---(ii)}$$

Also,

$$\text{Slope } m_2 = \frac{dy}{dx} = \tan\theta$$

$$= \tan 0° = 0 \qquad \left[\because \text{Slope is parallel to } x\text{-axis}\right]$$

$$\therefore \quad m_1 = m_2$$

$$\Rightarrow \quad \frac{1}{2a^2}\left[3x^2 - 6ax\right] = 0$$

$$\Rightarrow \quad 3x\left[x - 2a\right] = 0$$

$$\Rightarrow \quad x = 0 \text{ or } 2a$$

$\therefore$ From (i)

$$y = 0 \text{ or } -2a$$

Thus, the required points are $(0,0)$ or $(2a,-2a)$.

Tangents and Normals Ex 16.1 Q16

The given equations of curve and the line are

$$y = x^2 - 4x + 5 \qquad \text{---(i)}$$

$$2y + x = 7 \qquad \text{---(ii)}$$

Slope of the tangent to (i) is

$$m_1 = \frac{dy}{dx} = 2x - 4 \qquad \text{---(iii)}$$

Slope of the line (ii) is

$$m_2 = \frac{dy}{dx} = \frac{-1}{2} \qquad \text{---(iv)}$$

We have given that slope of (i) and (ii) are perpendicular to each other.

$$\therefore \quad m_1 \times m_2 = -1$$

$$\Rightarrow \quad (2x - 4)\left(\frac{-1}{2}\right) = -1$$

$$\Rightarrow \quad -2x + 4 = -2$$

$$\Rightarrow \quad x = 3$$

From (i)

$$y = 2$$

Thus, the required point is $(3,2)$.

Tangents and Normals Ex 16.1 Q17

Differentiating $\frac{x^2}{4}+\frac{y^2}{25}=1$ with respect to x, we get

$$\frac{x}{2}+\frac{2y}{25}.\frac{dy}{dx}=0$$

or $$\frac{dy}{dx}=\frac{-25}{4}.\frac{x}{y}$$

(i) Now, the tangent is parallel to the x – axis if the slope of the tangent is zero.

$\therefore$ $$\frac{-25}{4}.\frac{x}{y}=0$$

This is possible if x = 0.

Then $\frac{x^2}{4}+\frac{y^2}{25}=1$ for x = 0 gives $y^2=25$

$\therefore$ $$y=\pm 5$$

Thus, the points at which the tangents are parallel to the x – axis are (0, 5) and (0, –5).

(ii) Now, the tangent is parallel to the y – axis if the slope of the normal is zero.

$\therefore$ $$\frac{4y}{25x}=0$$

This is possible if y = 0.

Then $\frac{x^2}{4}+\frac{y^2}{25}=1$ for y = 0 gives $x^2=4$

$\therefore$ $$x=\pm 2$$

Thus, the points at which the tangents are parallel to the y – axis are (2, 0) and (–2, 0).

Tangents and Normals Ex 16.1 Q18

The equation of the given curve is $x^2+y^2-2x-3=0$.

On differentiating with respect to x, we have:

$$2x+2y\frac{dy}{dx}-2=0$$

$$\Rightarrow y\frac{dy}{dx}=1-x$$

$$\Rightarrow \frac{dy}{dx}=\frac{1-x}{y}$$

Now, the tangents are parallel to the x-axis if the slope of the tangent is 0.

$$\therefore \frac{1-x}{y}=0 \Rightarrow 1-x=0 \Rightarrow x=1$$

But, $x^2+y^2-2x-3=0$ for $x=1$.

$$\Rightarrow y^2=4 \Rightarrow y=\pm 2$$

Hence, the points at which the tangents are parallel to the x-axis are (1, 2) and (1, −2)

(b)
Now, the tangents are parallel to the x-axis if the slope of the tangents is 0

$$\frac{y}{1-x}=0$$
$$y=0$$

But,

$$x^2+y^2-2x-3=0 \text{ for } y=0$$
$$x^2-2x-3=0$$
$$x=-1,3$$

Hence, the points at which the tangents are parallel to the y-axis are,
$(-1,0),(3,0)$

Tangents and Normals Ex 16.1 Q19

The equation of the given curve is $\frac{x^2}{9}+\frac{y^2}{16}=1$.

On differentiating both sides with respect to x, we have:

$$\frac{2x}{9}+\frac{2y}{16}\cdot\frac{dy}{dx}=0$$
$$\Rightarrow \frac{dy}{dx}=\frac{-16x}{9y}$$

(i) The tangent is parallel to the x-axis if the slope of the tangent is i.e., $0\frac{-16x}{9y}=0$, which is possible if $x=0$.

Then, $\frac{x^2}{9}+\frac{y^2}{16}=1$ for $x=0$

$$\Rightarrow y^2=16 \Rightarrow y=\pm 4$$

Hence, the points at which the tangents are parallel to the x-axis are

(0, 4) and (0, − 4).

(ii) The tangent is parallel to the y-axis if the slope of the normal is 0, which gives $\frac{-1}{\left(\frac{-16x}{9y}\right)} = \frac{9y}{16x} = 0 \Rightarrow y = 0.$

Then, $\frac{x^2}{9} + \frac{y^2}{16} = 1$ for $y = 0$.

$\Rightarrow x = \pm 3$

Hence, the points at which the tangents are parallel to the y-axis are

(3, 0) and (− 3, 0).

Tangents and Normals Ex 16.1 Q20

The equation of the given curve is $y = 7x^3 + 11$.

$$\therefore \frac{dy}{dx} = 21x^2$$

The slope of the tangent to a curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$.

Therefore, the slope of the tangent at the point where $x = 2$ is given by,

$$\left.\frac{dy}{dx}\right]_{x=-2} = 21(2)^2 = 84$$

It is observed that the slopes of the tangents at the points where $x = 2$ and $x = -2$ are equal.

Hence, the two tangents are parallel.