RD Sharma Solutions Class 11 Maths Chapter 30 Ex 30.5

Derivatives Ex 30.5 Q1

Using quotient rule, we have

$$\frac{d}{dx}\left(\frac{x^2+1}{x+1}\right) = \frac{(x+1)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)\times 2x-(x^2+1)\times 1}{(x+1)^2}$$

$$= \frac{2x^2+2x-x^2-1}{(x+1)^2}$$

$$= \frac{x^2+2x-1}{(x+1)^2}$$

Derivatives Ex 30.5 Q2

Using quotient rule, we have get,

$$\frac{d}{dx}\left(\frac{2x-1}{x^2+1}\right)$$

$$= \frac{(x^2+1)\frac{d}{dx}(2x-1)-(2x-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\times 2-(2x-1)\times 2x}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2+2x}{(x^2+1)^2}$$

$$= \frac{-2x^2+2x+2}{(x^2+1)^2}$$

$$= \frac{2(-x^2+x+1)}{(x^2+1)^2}$$

$$= \frac{2(1+x-x^2)}{(1+x^2)^2}$$

Derivatives Ex 30.5 Q3

By using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{x+e^x}{1+\log x}\right)$$

$$=\frac{(1+\log x)\frac{d}{dx}\left(x+e^x\right)-\left(x+e^x\right)\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$$

$$=\frac{(1+\log x)\left(1+e^x\right)-\left(x+e^x\right)\times\frac{d}{dx}}{(1+\log x)^2}$$

$$=\frac{x\left(1+\log x+e^x+e^x\log x\right)-x-e^x}{x(1+\log x)^2}$$

$$=\frac{x+x\log x+xe^x+xe^x\log x-x-e^x}{x(1+\log x)^2}$$

$$=\frac{x\log x\left(1+e^x\right)-e^x(1-x)}{x(1+\log x)^2}$$

Derivatives Ex 30.5 Q4

Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{e^x-\tan x}{\cot x-x^n}\right)$$

$$=\frac{\left(\cot x-x^n\right)\frac{d}{dx}\left(e^x-\tan x\right)-\left(e^x-\tan x\right)\frac{d}{dx}\left(\cot x-x^n\right)}{\left(\cot x-x^n\right)^2}$$

$$=\frac{\left(\cot x-x^n\right)\left(e^x-\sec^2 x\right)-\left(e^x-\tan x\right)\left(-\operatorname{cosec}^2 x-nx^{n-1}\right)}{\left(\cot x-x^n\right)^2}$$

$$=\frac{\left(\cot x-x^n\right)\left(e^x-\sec^2 x\right)+\left(e^x-\tan x\right)\left(\operatorname{cosec}^2 x+nx^{n-1}\right)}{\left(\cot x-x^n\right)^2}$$

Derivatives Ex 30.5 Q5

Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{ax^2+bx+c}{px^2+qx+r}\right)$$

$$=\frac{\left(px^2+qx+r\right)\frac{d}{dx}\left(ax^2+bx+c\right)-\left(ax^2+bx+c\right)\frac{d}{dx}\left(px^2+qx+r\right)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{\left(px^2+qx+r\right)(2ax+b)-\left(ax^2+bx+c\right)(2px+q)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{2apx^3+2aqx^2+2axr+bpx^2+bqx+br-\left(2apx^3+2pbx^2+2pcx+qax^2+bqx+cq\right)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{2apx^3-2apx^3+2aqx^2+bpx^2-2pbx^2-qax^2+2arx+bqx-2pcx-bqx+br-cq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{aqx^2-bpx^2+2arx-2cpx+br-cq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{x^2(aq-bp)+2(ar-cp)x+br-cq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{(aq-bp)x^2+2(ar-cp)x+br-cq}{\left(px^2+qx+r\right)^2}$$

Derivatives Ex 30.5 Q6

Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{x}{1+\tan x}\right)$$

$$=\frac{(1+\tan x)\frac{d}{dx}(x)-x\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$=\frac{(1+\tan x)-x\left(\sec^2 x\right)}{(1+\tan x)^2}$$

$$= \frac{1 + \tan x - x\sec^2 x}{(1+\tan x)^2}$$

Derivatives Ex 30.5 Q7

Using quotient rule, we have

$$\frac{d}{dx}\left(\frac{1}{ax^2+bx+c}\right)$$

$$= \frac{(ax^2+bx+c)\frac{d}{dx}(1) - 1\times\frac{d}{dx}(ax^2+bx+c)}{(ax^2+bx+c)^2}$$

$$= \frac{-(2ax+b)}{(ax^2+bx+c)^2}$$

$$\therefore \frac{d}{dx}\frac{1}{ax^2+bx+c} = \frac{-(2ax+b)}{(ax^2+bx+c)^2}$$

Derivatives Ex 30.5 Q8

We have,

$$\frac{d}{dx}\left(\frac{e^x}{1+x^2}\right)$$

Using quotient rule,

$$= \frac{(1+x^2)\frac{d}{dx}(e^x) - (e^x)\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)e^x - e^x \times 2x}{(1+x^2)^2}$$

$$= \frac{e^x(1+x^2-2x)}{(1+x^2)^2}$$

$$= \frac{e^x(1-x)^2}{(1+x^2)^2}$$

Derivatives Ex 30.5 Q9

We have,

$$\frac{d}{dx}\left(\frac{e^x+\sin x}{1+\log x}\right)$$

Using quotient rule, we get

$$= \frac{(1+\log x)\frac{d}{dx}(e^x+\sin x)-(e^x+\sin x)\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$$

$$= \frac{(1+\log x)(e^x+\cos x)-(e^x+\sin x)\frac{1}{x}}{(1+\log x)^2}$$

$$= \frac{x(1+\log x)(e^x+\cos x)-(e^x+\sin x)}{x(1+\log x)^2}$$

Derivatives Ex 30.5 Q10

We have,

$$\frac{d}{dx}\left(\frac{x\tan x}{\sec x+\tan x}\right)$$

Using quotient rule, we get

$$= \frac{(\sec x+\tan x)\frac{d}{dx}(x\tan x)-(x\tan x)\frac{d}{dx}(\sec x+\tan x)}{(\sec x+\tan x)^2}$$

$$= \frac{(\sec x+\tan x)(x\sec^2 x+\tan x)-(x\tan x)(\sec x\tan x+\sec^2 x)}{(\sec x+\tan x)^2} \quad \text{[Used product rule]}$$

$$= \frac{(\sec x+\tan x)(x\sec^2 x+\tan x)-x\sec x+\tan^2 x-x\tan x\sec^2 x}{(\sec x+\tan x)^2}$$

$$= \frac{(\sec x+\tan x)(x\sec^2 x+\tan x)-x\tan x(\sec x\tan x+\sec^2 x)}{(\sec x+\tan x)^2}$$

$$= \frac{(\sec x+\tan x)(x\sec^2 x+\tan x)-x\tan x\sec x(\sec x+\tan x)}{(\sec x+\tan x)^2}$$

$$= \frac{(x\sec^2 x+\tan x-x\tan x\sec x)(\sec x+\tan x)}{(\sec x+\tan x)^2}$$

$$= \frac{(x\sec^2 x+\tan x-x\tan x\sec x)}{(\sec x+\tan x)}$$

$$= \frac{x\sec x(\sec x-\tan x)+\tan x}{(\sec x+\tan x)}$$