RD Sharma Solutions Class 11 Maths Chapter 30 Ex 30.4

Derivatives Ex 30.4 Q1

We have,

$$\frac{d}{dx}\left(x^3 \sin x\right)$$

$$= \sin x \frac{d\left(x^3\right)}{dx} + x^3 \frac{d\left(\sin x\right)}{dx} \quad \text{[Using product rule]}$$

$$= \sin x.3x^2 + x^3.\cos x$$

$$= x^2\left(3\sin x + x\cos x\right)$$

Derivatives Ex 30.4 Q2

We have,

$$\frac{d}{dx}\left(x^3 e^x\right)$$

$$= e^x \frac{d}{dx}\left(x^3\right) + x^3 \frac{d}{dx}\left(e^x\right) \quad \text{[Using product rule]}$$

$$= e^x 3x^2 + x^3 e^x$$

$$= x^2 e^x \left(3 + x\right)$$

Derivatives Ex 30.4 Q3

We have,

$$\frac{d}{dx}\left(x^2 e^x \log x\right)$$

$$= e^x \log x \frac{d}{dx}\left(x^2\right) + x^2 \log x \frac{d}{dx}\left(e^x\right) + x^2 e^x \frac{d}{dx}\left(\log x\right) \quad \text{[Usingproductrule]}$$

$$= e^x \log x.2x + x^2 \log x.e^x + x^2 e^x.\frac{1}{x}$$

$$= xe^x \left(2.\log x + x\log x + 1\right)$$

Derivatives Ex 30.4 Q4

We have,

$$\frac{d}{dx}\left(x^n \tan x\right)$$

$$= \tan x \frac{d}{dx}\left(x^n\right) + x^n \frac{d}{dx}\left(\tan x\right) \qquad \text{[Using product rule]}$$

$$= \tan x . n x^{n-1} + x^n \sec^2 x$$

$$= x^{n-1}\left(n . \tan x + x . \sec^2 x\right) \qquad \left[x^n = x^{n-1} x^1 = x^{n-1+1}\right]$$

Derivatives Ex 30.4 Q5

We have,

$$\frac{d}{dx}\left(x^n \log_a x\right)$$

$$= \log_a x \frac{d}{dx}\left(x^n\right) + x^n \frac{d}{dx}\left(\log_a x\right) \qquad \text{[Using product rule]}$$

$$= n x^{n-1} . \log_a x + \frac{x^n}{\log a} . \frac{1}{x} \qquad \left[\because \log_a x = \frac{\log x}{\log a}\right]$$

$$= x^{n-1}\left[n . \log_a x + \frac{1}{\log a}\right]$$

Derivatives Ex 30.4 Q6

We have,

$$\frac{d}{dx}\left(x^3 + x^2 + 1\right)\sin x$$

$$= \sin x \frac{d}{dx}\left(x^3 + x^2 + 1\right) + \left(x^3 + x^2 + 1\right)\frac{d}{dx}\left(\sin x\right) \qquad \text{[Using product rule]}$$

$$= \sin x \left(3x^2 + 2x\right) + \left(x^3 + x^2 + 1\right)\cos x$$

$$\therefore \left(x^3 + x^2 + 1\right)\cos x + \left(3x^2 + 2x\right)\sin x$$

Derivatives Ex 30.4 Q7

We have,

$$\frac{d}{dx}(\sin x \times \cos x)$$

$$\cos x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(\cos x) \qquad [\text{using product rule}]$$

$$= \cos x (\cos x) + \sin x (-\sin x)$$

$$= \cos^2 x - \sin^2 x \qquad [\because \cos 2x = \cos^2 x - \sin^2 x]$$

$$= \cos 2x$$

Derivatives Ex 30.4 Q8

We have,

$$\frac{d}{dx}\left(2^x \times \cot x \times x^{-\frac{1}{2}}\right)$$

$$= \cot x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx}\left(2^x\right) + 2^x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx}(\cot x) + 2^x \times \cot x \times \frac{d}{dx}\left(x^{-\frac{1}{2}}\right) \qquad [\text{Using product rule}]$$

$$= \frac{\cot x}{\sqrt{x}} \times 2^x \times \log 2 + \frac{2^x}{\sqrt{x}}\left(-\operatorname{cosec}^2 x\right) + 2^x \times \cot x \left(-\frac{1}{2}\right)\frac{1}{2^{\frac{3}{2}}}$$

$$= \frac{2^x}{\sqrt{x}}\left(\cot x \times \log 2 - \operatorname{cosec}^2 x - \frac{\cot x}{2x}\right)$$

Derivatives Ex 30.4 Q9

$$\frac{d}{dx}\left(x^2 \sin x \log x\right)$$

$$= \sin x \log x \frac{d}{dx}\left(x^2\right) + x^2 \log x \frac{d}{dx}(\sin x) + x^2 \sin x \frac{d}{dx}(\log x) \qquad [\text{Using product rule}]$$

$$= \sin x \log x \times 2x + x^2 \log x \times \cos x + x^2 \sin x \times \frac{1}{x}$$

$$= 2x \times \sin x \times \log x + x^2 \times \cos x \times \log x + x \sin x$$

Derivatives Ex 30.4 Q10

We have,

$$\frac{d}{dx}\left(x^5e^x+x^6\log x\right)$$

$$=\frac{d}{dx}\left(x^5e^x\right)+\frac{d}{dx}\left(x^6\log x\right)$$

$$=e^x\frac{dx^5}{dx}+x^5\frac{de^5}{dx}+\log x\frac{d}{dx}\left(x^6\right)+x^6\frac{d}{dx}\left(\log x\right) \qquad \text{[Using product rule]}$$

$$=e^x\times5x^4+x^5\times e^x+\log x\times6x^5+x^6\times\frac{1}{x}$$

$$=5x^4\times e^x+x^5\times e^x+6x^5\times\log x+x^5$$

$$=x^4\left(5e^x+ex^x+6x\log x+x\right)$$

Derivatives Ex 30.4 Q11

We have,

$$\frac{d}{dx}\left\{\left(x\sin x+\cos x\right)\left(x\cos x-\sin x\right)\right\}$$

We will apply product rule,

$$=\left(x\cos x-\sin x\right)\frac{d}{dx}\left(x\sin x+\cos x\right)+\left(x\sin x+\cos x\right)\frac{d}{dx}\left(x\cos x-\sin x\right)$$

$$=\left(x\cos x-\sin x\right)\left\{\frac{d}{dx}\left(x\sin x\right)+\frac{d}{dx}\left(\cos x\right)\right\}+\left(x\sin x+\cos x\right)\left\{\frac{d}{dx}\left(x\cos x\right)-\frac{d}{dx}\left(\sin x\right)\right\}$$

Again apply product rule,

$$=\left(x\cos x-\sin x\right)\left\{\left(\sin x\frac{dx}{dx}+x\frac{d\sin x}{dx}\right)\right\}+\left(-\sin x\right)+\left(x\cos x+\sin x\right)\left\{\left(\sin x\frac{dx}{dx}+x\frac{d\cos x}{dx}-\cos x\right)\right\}$$

$$=\left(x\cos x-\sin x\right)\left\{\left(\sin x+x\cos x\right)-\sin x\right\}+\left(x\sin x+cosx\right)\left\{\left(\cos x-x\sin x-\cos x\right)\right\}$$

$$=\left(x\cos x-\sin x\right)\times x\cos x+\left(x\sin x+\cos x\right)\left(-x\sin x\right)$$

$$=\left(x^2\cos^2x-x\sin x\times\cos x\right)+\left(-x^2\sin^2x-x\sin x\times\cos x\right)$$

$$=x^2\left(\cos^2x-\sin^2x\right)-x\left(\sin x\cos x+\sin x\cos x\right)$$

$$=x^2-\cos2x-x\times2\sin x\cos x$$

$= x^2 \cos x 2x - x \sin 2x$

$= x\{x \times \cos 2x - \sin 2x\}$

Derivatives Ex 30.4 Q12

We have,

$\frac{d}{dx}\{(x \sin x + \cos x)(e^x + x^2 \log x)\}$

We will apply product rule,

$= (e^x + x^2 \log x)\frac{d}{dx}(x \sin x + \cos x) + (x \sin x + \cos x)\frac{d}{dx}(e^x + x^2 \log x)$

$= (e^x + x^2 \log x)\left(\frac{d}{dx}(x \sin x) + \frac{d}{dx}\cos x\right) + (x \sin x + \cos x) \times \left\{\frac{d}{dx}(e^x) + \frac{d}{dx}(x^2 \log x)\right\}$

Again apply product rule,

$= (e^x + x^2 \log x)\left(\sin x \frac{d}{dx}(x) + x \frac{d}{dx}(\sin x)\right) - \sin x + (x \sin x + \cos x)\left\{e^x + \left(\log x \frac{d}{dx}(x^2) + x^2 \frac{d}{dx}(\log x)\right)\right\}$

$= (e^x + x^2 \log x)(\sin x + x \cos x - \sin x) + (x \sin x + \cos x)\left(e^x + \log x \times 2x + x^2 \frac{1}{x}\right)$

$= (e^x + x^2 \log x)x \cos x + (x \sin x + \cos x)(e^x + 2x \times \log x + x)$

$= x \cos x e^x + e^3 \cos x \log x + x e^x \sin x + e^x \cos x + 2x^2 \sin x \times \log x + 2x \cos x \log x + x^2 \sin x + x \cos x$

$= x \cos x (e^x + x^2 \log x) + (x \sin x + \cos x)(e^x + x + 2x \log x)$

Derivatives Ex 30.4 Q13

We have,

$\frac{d}{dx}\{(1 - 2 \tan x)(5 + 4 \sin x)\}$

$= (5 + 4 \sin x)\frac{d}{dx}(1 - 2 \tan x) + (1 - 2 \tan x)\frac{d}{dx}(5 + 4 \sin x)$ [Using product rule,]

$= (5 + 4 \sin x)(0 - 2 \sec^2 x) + (1 - 2 \tan x)(0 + 4 \cos x)$

$= -10 \sec^2 x - 8 \sin x \times \sec^2 x + 4 \cos x - 8 \cos x \times \tan x$

$= 4\left(\frac{-5}{2} \sec^2 x - 2 \sin x \times \frac{1}{\cos^2 x} + \cos x - 2 \cos x \times \frac{\sin x}{\cos x}\right)$

$= 4\left(\frac{-5}{2} \sec^2 x - 2 \tan x \sec x + \cos x - 2 \sin x\right)$

$= 4\left(\cos x - 2\sin x - 2\tan x \sec x - \frac{5}{2}\sec^2 x\right)$

Derivatives Ex 30.4 Q14

We have,

$\frac{d}{dx}\left\{\left(1+x^2\right)\cos x\right\}$

$= \cos x \frac{d}{dx}\left(1+x^2\right) + \left(1+x^2\right)\frac{d}{dx}\left(\cos x\right)$ (using product rule)

$= \cos x \times 2x + \left(1+x^2\right)\left(-\sin x\right)$

$= 2x\cos x - \left(1+x^2\right)\sin x$

Derivatives Ex 30.4 Q15

We have,

$\frac{d}{dx}\left(\sin^2 x\right)$

$= \frac{d}{dx}\left(\sin x\right)\left(\sin x\right)$

$= \sin x \frac{d}{dx}\left(\sin x\right) + \sin x \frac{d}{dx}\left(\sin x\right)$ [Using product rule]

$= \sin x \times \cos x + \sin x \times \cos x$

$= 2\sin x \cos x$

$= \sin 2x$ $\left[\because \sin 2A = 2\sin A \cos A\right]$

Derivatives Ex 30.4 Q16

We have,

$\frac{d}{dx}\left(\log_{x^2} x\right)$

$\log_{x^2} x = \frac{\log x}{\log x^2}$

$= \frac{\log x}{2\log x}$

$= \frac{1}{2}$

$\frac{d}{dx}\left(\frac{1}{2}\right) = 0$

$\therefore \frac{d}{dx}\left(\log_{x^2} x\right) = 0$

Derivatives Ex 30.4 Q17

$\frac{d}{dx}\left(e^x \log\sqrt{x} \tan x\right)$

Apply product rule,

$= \log\sqrt{x} \times \tan x \frac{d}{dx}\left(e^x\right) + e^x \times \tan x \frac{d}{dx}\left(\log\sqrt{x}\right) + e^x \log\sqrt{x} \frac{d}{dx}\left(\tan x\right)$

$= \log\sqrt{x} \times \tan e^x + e^x \tan x \frac{1}{2x} + e^x \log\sqrt{x} \times \sec^2 x$

$= \frac{1}{2}\log x \times \tan x \times e^x + \frac{1 \tan x}{2x} e^x + e^x \frac{1}{2}\log x \sec^2 x \qquad \left[\because \log\sqrt{x} = \frac{1}{2}\log x\right]$

$= \frac{1}{2} e^x \left(\log x \times \tan x + \frac{\tan x}{x} + \log x \sec^2 x\right)$