

RD Sharma
Solutions
Class 11 Maths
Chapter 30
Ex 30.2

Derivatives Ex 30.2 Q1(i)

We have,

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f \frac{2}{x+h} - \frac{2}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(2x - 2x - 2h)}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{2(x - x - h)}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{-2h}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{-2}{x(x+h)} \\ &= \frac{-2}{x^2}\end{aligned}$$

Derivatives Ex 30.2 Q1(ii)

We have,

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}h} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}h} \\ &= \lim_{h \rightarrow 0} \frac{x - (x+h)}{\sqrt{x}\sqrt{x+h}h(\sqrt{x} + \sqrt{x+h})} \\ &= \lim_{h \rightarrow 0} \frac{-h}{\sqrt{x}\sqrt{x+h}h(\sqrt{x} + \sqrt{x+h})} \\ &= \lim_{h \rightarrow 0} \frac{-1}{x\sqrt{x+h} + \sqrt{x}(\sqrt{x+h})} \\ &= \lim_{h \rightarrow 0} \frac{-1}{2x\sqrt{x}} \\ &= \frac{-1}{2} x^{-\frac{3}{2}}\end{aligned}$$

Derivatives Ex 30.2 Q1(iii)

We have,

$$f(x) = \frac{1}{x^3}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^3 - (x+h)^3}{x^3 h (x+h)^3} \\ &= \lim_{h \rightarrow 0} \frac{x^3 - (x^3 + 3x^2h + 3xh^2 + h^3)}{x^3 h (x+h)^3} \\ &= \lim_{h \rightarrow 0} \frac{x^3 - x^3 - 3x^2h - 3xh^2 - h^3}{x^3 (x+h)^3} \\ &= \lim_{h \rightarrow 0} \frac{-3x^2h - 3xh^2 - h^3}{x^3 (x+h)^3} \\ &= \frac{-3x^2}{x^6} \\ &= \frac{-3}{x^4}\end{aligned}$$

Derivatives Ex 30.2 Q1(iv)

We have,

$$f(x) = \frac{x^2 + 1}{x}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2 + 1}{x+h} - \frac{x^2 + 1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x[x^2 + h^2 + 2xh + 1] - (x^2 + 1)(x+h)}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{x^3 + xh^2 + 2x^2h + x - x^3 - x - x^2h - h}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{xh + 2x^2 - x^2 - 1}{x(x+h)} \\ &= \frac{x^2 - 1}{x^2} \\ &= 1 - \frac{1}{x^2}\end{aligned}$$

Derivatives Ex 30.2 Q1(v)

We have,

$$f(x) = \frac{x^2 - 1}{x}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2 - 1}{x+h} - \frac{x^2 - 1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x(x^2 + h^2 + 2xh - 1) - (x+h)(x^2 - 1)}{x(x+h)h} \\ &= \lim_{h \rightarrow 0} \frac{xh + 2x^2 - x^2 + 1}{x(x+h)} \\ &= \frac{x^2 + 1}{x^2} \\ &= 1 + \frac{1}{x^2}\end{aligned}$$

Derivatives Ex 30.2 Q1(vi)

We have,

$$f(x) = \frac{x+1}{x+2}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(x+h)+1}{(x+h)+2} - \frac{x+1}{x+2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+2)(x+h+1) - (x+1)(x+h+2)}{(x+h+2)(x+2)h} \\ &= \lim_{h \rightarrow 0} \frac{(x^2 + 2x + xh + 2h + 2 + x) - (x^2 + xh + 2x + x + h + 2)}{(x+h+2)(x+2)h} \\ &= \lim_{h \rightarrow 0} \frac{h}{(x+h+2)(x+2)h} \\ &= \frac{1}{(x+2)^2}\end{aligned}$$

Derivatives Ex 30.2 Q1(vii)

We have,

$$f(x) = \frac{x+2}{3x+5}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(x+h+2)}{3(x+h)+5} - \frac{x+2}{3x+5}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(3x+5)(x+h+2) - (x+2)(3x+3h+5)}{(3x+5)(3x+3h+5)h} \\ &= \lim_{h \rightarrow 0} \frac{(3x^2 + 5x + 3xh + 5h + 6x + 10) - (3x^2 + 3xh + 5x + 6x + 6h + 10)}{(3x+5)(3x+3h+5)h} \\ &= \lim_{h \rightarrow 0} \frac{-h}{(3x+5)(3x+3h+5)h} \\ &= \lim_{h \rightarrow 0} \frac{-1}{(3x+5)(3x+3h+5)} \\ &= \frac{-1}{(3x+5)^2}\end{aligned}$$

Derivatives Ex 30.2 Q1(viii)

We have,

$$f(x) = kx^n$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{k(x+h)^n - kx^n}{h} \\ &= k \lim_{h \rightarrow 0} \frac{\left(x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots\right) - x^n}{h} \quad \left[\because (x+y)^n = x^n + nx^{n-1}y + \dots\right] \\ &= k \lim_{h \rightarrow 0} nx^{n-1} + \frac{n(n-1)}{2!}x^{n-2}h + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^2 \dots \\ &= k nx^{n-1} + 0 + 0 \dots \\ &= k nx^{n-1}\end{aligned}$$

Derivatives Ex 30.2 Q1(ix)

We have,

$$f(x) = \frac{1}{\sqrt{3-x}}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{3-(x+h)}} - \frac{1}{\sqrt{3-x}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{3-x} - \sqrt{3-(x+h)}}{\sqrt{3-x}\sqrt{3-(x+h)} \times h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{3-x} - \sqrt{3-(x+h)}}{\sqrt{3-x}\sqrt{3-(x+h)} \times h} \times \frac{\sqrt{3-x} + \sqrt{3-(x+h)}}{\sqrt{3-x} + \sqrt{3-(x+h)}} \quad [\text{Rationalising the numerator by } \sqrt{3-x} + \sqrt{3-(x+h)}] \\ &= \lim_{h \rightarrow 0} \frac{(3-x) - (3-(x+h))}{\sqrt{3-x}\sqrt{3-(x+h)} \times h (\sqrt{3-x} + \sqrt{3-(x+h)})} \\ &= \lim_{h \rightarrow 0} \frac{h}{\sqrt{3-x}\sqrt{3-(x+h)} \times h (\sqrt{3-x} + \sqrt{3-(x+h)})} \\ &= \frac{1}{(3-x) \times 2\sqrt{3-x}} \\ &= \frac{1}{2(3-x)^{\frac{3}{2}}}\end{aligned}$$

Derivatives Ex 30.2 Q1(x)

We have,

$$f(x) = x^2 + x + 3$$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{(x+h)^2 + (x+h) + 3\} - x^2 + x + 3}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + h^2 + 2xh + x + h + 3 - x^2 - x - 3}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h + 1)}{h} \\ &= 2x + 0 + 1 \\ &= 2x + 1\end{aligned}$$

Derivatives Ex 30.2 Q1(xi)

We have,

$$f(x) = (x+2)^3$$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h+2)^3 - (x+2)^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{(x+2)+h\}^3 - (x+2)^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+2)^3 + h^3 + 3h(x+2)^2 + 3(x+2)h^2 - (x+2)^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+2)^2 + 3(x+2)h + h^2}{h} \\ &= 3(x+2)^2\end{aligned}$$

Derivatives Ex 30.2 Q1(xii)

We have,

$$f(x) = x^3 + 4x^2 + 3x + 2$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 + 4(x+h)^2 + 3(x+h) + 2 - (x^3 + 4x^2 + 3x + 2)}{h}\end{aligned}$$

On solving we get,

$$\begin{aligned}&= \lim_{h \rightarrow 0} \frac{x^3 + h^3 + 3x^2h + 3h^2x + 4x^2 + 4h^2 + 8hx + 3x + 3h + 2 - x^3 - 4x^2 - 3x - 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3 + 4h^2 + 8hx + 3h}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 + 4h + 8x + 3 \\ &= 3x^2 + 8x + 3\end{aligned}$$

Derivatives Ex 30.2 Q1(xiii)

We have,

$$f(x) = x^3 - 5x^2 + x - 5$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{(x+h)^3 + (x+h) - 5(x+h)^2 - 5\} - \{x^3 - 5x^2 + x - 5\}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x^3 + h^3 + 3x^2h + 3h^2x + x + h - 5x^2 - 5h^2 - 10xh - 5) - (x^3 - 5x^2 + x - 5)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(3x^2h + 3h^2x + h^3 + h - 5h^2 - 10xh)}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 + 1 - 5h - 10x \\ &= 3x^2 - 10x + 1\end{aligned}$$

Derivatives Ex 30.2 Q1(xiv)

We have,

$$f(x) = \sqrt{2x^2 + 1}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)^2 + 1} - \sqrt{2x^2 + 1}}{h}\end{aligned}$$

Multiplying Numerator and Denominator by $\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1}$

$$\begin{aligned}&= \lim_{h \rightarrow 0} \frac{\{2(x+h)^2 + 1 - (2x^2 + 1)\}}{h(\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1})} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 2h^2 + 4xh + 1 - 2x^2 - 1}{h(\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1})} \\ &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h(\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1})} \\ &= \frac{4x}{2\sqrt{2x^2 + 1}} \\ &= \frac{2x}{\sqrt{2x^2 + 1}}\end{aligned}$$

Derivatives Ex 30.2 Q1(xv)

We have, $f(x) = \frac{2x+3}{x-2}$

Therefore,

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left(\frac{2x+2h+3}{x+h-2}\right) - \left(\frac{2x+3}{x-2}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 2hx + 3x - 4x - 4h - 6 - 2x^2 - 2hx + 4x - 3x - 3h + 6}{h(x+h-2)(x-2)} \\ &= \lim_{h \rightarrow 0} \frac{-7}{(x+h-2)(x-2)} \\ &= \boxed{\frac{-7}{(x-2)^2}}\end{aligned}$$

Derivatives Ex 30.2 Q2(i)

We have,

$$f(x) = e^{-x}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{-(x+h)} - e^{-x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{-x}(e^{-h} - 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-e^{-x}(e^{-h} - 1)}{-h} \\ &= -e^{-x}\end{aligned}$$

$$\left[\because \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1 \right]$$

Derivatives Ex 30.2 Q2(ii)

We have,

$$f(x) = e^{3x}$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{3(x+h)} - e^{3x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{3(x)} \cdot e^{3h} - e^{3x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{3(x)} (e^{3h} - 1)}{h} \end{aligned}$$

Multiplying Numerator and Denominator by 3

$$\begin{aligned} &= \lim_{h \rightarrow 0} e^{3(x)} \frac{(e^{3h} - 1)}{3h} \quad \left[\lim_{h \rightarrow 0} \frac{e^{3h} - 1}{3} = 1 \right] \\ &= 3e^{3x} \end{aligned}$$

Derivatives Ex 30.2 Q2(iii)

We have,

$$f(x) = e^{ax+b}$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{a(x+h)+b} - e^{ax+b}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{ax} \times e^{ah} \times e^b - e^{ax} \times e^b}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^b \times e^{ax} (e^{ah} - 1)}{h} \\ &= \lim_{h \rightarrow 0} e^{ax+b} \times \frac{a(e^{ah} - 1)}{a \cdot h} \end{aligned}$$

Multiplying Numerator and denominator by a

$$= ae^{ax+b}$$

$$\left[\lim_{h \rightarrow 0} \frac{(e^{ah} - 1)}{ah} = 1 \right]$$

Derivatives Ex 30.2 Q2(iv)

We have,

$$f(x) = xe^x$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)e^{(x+h)} - xe^x}{h} \\ &= \lim_{h \rightarrow 0} \frac{xe^x e^h + he^x e^h - xe^x}{h} \\ &= \lim_{h \rightarrow 0} xe^x \left(\frac{e^h - 1}{h} \right) + \frac{he^{x+h}}{h} \\ &= xe^x + e^x \\ &= e^x (x+1)\end{aligned}$$

Derivatives Ex 30.2 Q2(v)

Let $f(x) = -x$. Then, $f(x+h) = -(x+h)$

$$\begin{aligned}\therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{-(x+h) + (x)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{-h}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} -1 \\ \Rightarrow \frac{d}{dx}(f(x)) &= -1\end{aligned}$$

Derivatives Ex 30.2 Q2(vi)

Let $f(x) = (-x)^{-1}$. Then, $f(x+h) = -(x+h)^{-1}$

$$\begin{aligned}\therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{(-(x+h))^{-1} - (-x)^{-1}}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\frac{-x + x + h}{x(x+h)}}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{h}{hx(x+h)} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{1}{x(x+h)} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \frac{1}{x(x+0)} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \frac{1}{x^2}\end{aligned}$$

Derivatives Ex 30.2 Q2(vii)

Let $f(x) = \sin(x+1)$. Then, $f(x+h) = \sin((x+h)+1)$

$$\begin{aligned} \therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\sin((x+h)+1) - \sin(x+1)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{2 \sin \left[\frac{((x+h)+1) - (x+1)}{2} \right] \cos \left[\frac{((x+h)+1) + (x+1)}{2} \right]}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{2 \sin \left[\frac{h}{2} \right] \cos \left[\frac{2x+2+h}{2} \right]}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\sin \left[\frac{h}{2} \right]}{\frac{h}{2}} \times \lim_{h \rightarrow 0} \cos \left[\frac{2x+2+h}{2} \right] \\ \Rightarrow \frac{d}{dx}(f(x)) &= 1 \times \cos \left[\frac{2x+2+0}{2} \right] \\ \Rightarrow \frac{d}{dx}(f(x)) &= \cos(x+1) \end{aligned}$$

Derivatives Ex 30.2 Q2(viii)

Let $f(x) = \cos\left(x - \frac{\pi}{8}\right)$. Then, $f(x+h) = \cos\left(x+h - \frac{\pi}{8}\right)$

$$\begin{aligned} \therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\cos\left(x+h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{-2 \sin \left[\frac{\left(x+h - \frac{\pi}{8}\right) + \left(x - \frac{\pi}{8}\right)}{2} \right] \sin \left[\frac{\left(x+h - \frac{\pi}{8}\right) - \left(x - \frac{\pi}{8}\right)}{2} \right]}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{-2 \sin \left[\frac{2x+h - \frac{2\pi}{8}}{2} \right] \sin \left[\frac{h}{2} \right]}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \sin \left[\frac{2x+h - \frac{2\pi}{8}}{2} \right] \times \lim_{h \rightarrow 0} \frac{\sin \left[\frac{h}{2} \right]}{\frac{h}{2}} \\ \Rightarrow \frac{d}{dx}(f(x)) &= -\sin \left[\frac{2x+0 - \frac{2\pi}{8}}{2} \right] \times 1 \\ \Rightarrow \frac{d}{dx}(f(x)) &= -\sin \left(x - \frac{\pi}{8} \right) \end{aligned}$$

Derivatives Ex 30.2 Q2(ix)

We have,

$$f(x) = x \sin x$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h) \sin(x+h) - x \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{x \{ \sin(x+h) - \sin x \}}{h} + \sin(x+h) && \left[\sin c - \sin d = 2 \cos \frac{c+d}{2} \sin \frac{c-d}{2} \right] \\ &= \lim_{h \rightarrow 0} \frac{x \times 2 \cos \left(x + \frac{h}{2} \right) \sin \frac{h}{2}}{h} + \sin(x+h) && \left[\therefore \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\ &= 2x \times \cos x \times \frac{1}{2} + \sin x \\ &= x \times \cos x + \sin x \\ &= \sin x + x \cos x \end{aligned}$$

Derivatives Ex 30.2 Q2(x)

We have,

$$f(x) = x \cos x$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h) \cos(x+h) - x \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{x \cos(x+h) + h \cos(x+h) - x \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{x \{ \cos(x+h) - \cos x \}}{h} + \cos(x+h) \\ &= \lim_{h \rightarrow 0} x \cdot 2 \sin \left(x - x - \frac{h}{2} \right) \sin \left(x + \frac{h}{2} \right) + \cos(x+h) && \left[\therefore \cos A - \cos B = 2 \sin \frac{B-A}{2} \sin \frac{B+A}{2} \right] \\ &= \lim_{h \rightarrow 0} 2x \cdot \sin \left(\frac{-h}{2} \right) \sin \left(x + \frac{h}{2} \right) + \cos(x+h) \\ &= -x \sin x + \cos x \end{aligned}$$

Derivatives Ex 30.2 Q2(xi)

We have,

$$f(x) = \sin(2x - 3)$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin\{2(x+h) - 3\} - \sin(2x - 3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2 \cos \frac{(2x+2h-3) + (2x-3)}{2} \times \sin \frac{(2x+2h-3) - (2x-3)}{2}}{h} && \left[\therefore \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2} \right] \\ &= \lim_{h \rightarrow 0} 2 \cos(2x - 3 + h) \cdot \frac{\sin h}{2} && \left[\therefore \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\ &= 2 \cos(2x - 3) \end{aligned}$$

Derivatives Ex 30.2 Q3(i)

We have,

$$f(x) = \sqrt{\sin 2x}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{\sin 2(x+h)} - \sqrt{\sin 2x}}{h}\end{aligned}$$

Multiplying Numerator and Denominator by $(\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x})$

$$\begin{aligned}&= \lim_{h \rightarrow 0} \frac{\sqrt{\sin 2(x+h)} - \sqrt{\sin 2x}}{h} \times \frac{\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}}{\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}} \\ &= \lim_{h \rightarrow 0} \frac{\sin(2x+2h) - \sin 2x}{h(\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x})} \\ &= \lim_{h \rightarrow 0} \frac{\sin(2x+2h) - \sin 2x}{h(\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x})} \quad \left[\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2} \right] \\ &= \lim_{h \rightarrow 0} \frac{2 \cos(2x+h) \times \sinh}{h} \times \frac{1}{\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}} \\ &= \frac{2 \cos 2x}{2\sqrt{\sin 2x}} \\ &= \frac{\cos 2x}{\sqrt{\sin 2x}}\end{aligned}$$

Derivatives Ex 30.2 Q3(ii)

We have,

$$f(x) = \frac{\sin x}{x}$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x \sin(x+h) - (x+h) \sin x}{xh(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{x(\sin x \cdot \cosh + \cos x \cdot \sinh) - x \sin x - h \sin x}{xh(x+h)} \quad [\because \sin(A+B) = \sin A \cos B + \cos A \sin B] \\ &= \lim_{h \rightarrow 0} \frac{x \sin x (\cosh - 1) + x \cos x \sinh - h \sin x}{(x+h)xh} \quad [\because 1 - \cosh = 2 \sin^2 \frac{h}{2}] \\ &= \frac{-x \sin x}{x(x+h)} \times \frac{2 \sin^2 \frac{h}{2}}{h^2} \times \frac{h}{4} + \frac{x \cos x}{x^2} - \frac{\sin x}{x^2} \end{aligned}$$

$$\therefore h \rightarrow 0 \Rightarrow \frac{h}{2} \rightarrow 0 \quad \text{and} \quad \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\begin{aligned} &= 0 + \frac{x \cos x - \sin x}{x^2} \\ &= \frac{x \cos x - \sin x}{x^2} \end{aligned}$$

Derivatives Ex 30.2 Q3(iii)

We have,

$$f(x) = \frac{\cos x}{x}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\cos(x+h)}{x+h} - \frac{\cos x}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x \cos(x+h) - (x+h) \cos x}{(x+h)xh} \quad [\because \cos(A+B) = \cos A \cos B - \sin A \sin B] \\ &= \lim_{h \rightarrow 0} \frac{x[\cos x \cdot \cosh - \sin x \cdot \sinh] - x \cos x - h \cos x}{(x+h)xh} \\ &= \lim_{h \rightarrow 0} \frac{x \cos x (\cosh - 1) - x \cos x \sinh - h \cos x}{(x+h)xh} \\ &= \lim_{h \rightarrow 0} \frac{-x \cos x \cdot 2 \sin^2 \frac{h}{2}}{(x+h)x} \times \frac{h^2}{4} - \frac{x \sin x}{x(x+h)} - \frac{\cos x}{x(x+h)} \\ &= 0 - \frac{x \sin x}{x^2} - \frac{\cos x}{x^2} \\ &= -\frac{x \sin x - \cos x}{x^2} \end{aligned}$$

Derivatives Ex 30.2 Q3(iv)

We have,

$$f(x) = x^2 \sin x$$

$$\begin{aligned}
\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x^2 + h^2 + 2hx)(\sin x \cdot \cosh + \cos x \cdot \sinh) - x^2 \sin x}{h} \quad [\because \sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B] \\
&= \lim_{h \rightarrow 0} \frac{x^2 \sin x (\cosh - 1) + h(h+2x) \sin x \cdot \cosh + (x+h)^2 \cos x \frac{\sinh}{h}}{h} \\
&= \lim_{h \rightarrow 0} -x^2 \sin x \times \frac{2 \sin^2 \frac{h}{2}}{\left(\frac{h}{2}\right)^2} \times \frac{h^2}{4} + (h+2x) \sin x \cdot \cosh + (x+h)^2 \cos x \\
&= 0 + (2x \sin x + x^2 \cos x) \\
&= 2x \sin x + x^2 \cos x
\end{aligned}$$

Derivatives Ex 30.2 Q3(v)

We have,

$$f(x) = \sqrt{\sin(3x+1)}$$

$$\begin{aligned}
\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{\sin(3(x+h)+1)} - \sqrt{\sin(3x+1)}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{\sin(3x+3h)+1} - \sqrt{\sin(3x+1)}}{h} \times \frac{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}} \\
&= \lim_{h \rightarrow 0} \frac{\sin(3x+3h+1) - \sin(3x+1)}{h \left(\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)} \right)} \\
&= \lim_{h \rightarrow 0} 2 \cos \left(3x+1 + \frac{3h}{2} \right) \times \frac{\sin \frac{3h}{2}}{\frac{3h}{2}} \times \frac{3}{2} \times \frac{1}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}} \\
&= \frac{3 \cos(3x+1)}{2\sqrt{\sin(3x+1)}} \quad \left[\lim_{h \rightarrow 0} \frac{\sin \frac{3h}{2}}{\frac{3h}{2}} = 1 \right]
\end{aligned}$$

Derivatives Ex 30.2 Q3(vi)

We have,

$$f(x) = \sin x + \cos x$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{\sin(x+h) + \cos(x+h)\} - \sin x + \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{\sin(x+h) + \cos(x+h) - \sin x - \cos x\}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{\sin(x+h) - \sin x\} + \{\cos(x+h) - \cos x\}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left\{2 \sin \frac{(x+h-x)}{2} \cos \frac{(x+h+x)}{2}\right\} + \left\{-2 \sin \frac{x+h+x}{2} \sin \frac{x+h-x}{2}\right\}}{h}\end{aligned}$$

$$\left[\begin{array}{l} \because \sin A - \sin B = 2 \sin \frac{A-B}{2} \cos \frac{A+B}{2} \\ \text{and } \cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right]$$

$$\begin{aligned}&= \lim_{h \rightarrow 0} \frac{\sinh \cdot \cos \frac{2x+h}{2} - 2 \sin \left(x + \frac{h}{2}\right) \sinh}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sinh}{h} \left\{ \cos \frac{x+h}{2} - \sin \left(x + \frac{h}{2}\right) \right\} \\ &= \cos x - \sin x\end{aligned}$$

$$\left[\because \lim_{h \rightarrow 0} \frac{\sinh}{h} = 1 \right]$$

Derivatives Ex 30.2 Q3(vii)

We have,

$$f(x) = x^2 e^x$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 e^x e^h + h^2 e^x e^h + 2xh e^x e^h - x^2 e^x}{h} \\ &= \lim_{h \rightarrow 0} x^2 e^x \frac{(e^h - 1)}{h} + e^x e^h \frac{(h^2 + 2xh)}{h}\end{aligned}$$

$$\left[\because \frac{e^h - 1}{h} \rightarrow 1 \right]$$

$$\begin{aligned}\therefore &x^2 e^x + e^x (0 + 2x) \\ &= x^2 e^x + 2x e^x \\ &= e^x (x^2 + 2x)\end{aligned}$$

Derivatives Ex 30.2 Q3(viii)

We have,

$$f(x) = e^{x^2+1}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{(x+h)^2} + 1 - e^{x^2+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{x^2+h^2+2xh+1} - e^{x^2+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{x^2+1} (e^{2xh+h^2} - 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{x^2+1} (e^{2xh+h^2} - 1)}{2xh+h^2} \times \frac{2xh+h^2}{h}\end{aligned}$$

$$\therefore h \rightarrow 0$$

$$\Rightarrow 2xh + h^2 = 0$$

$$\text{and } \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1$$

$$\begin{aligned}&= \lim_{h \rightarrow 0} e^{x^2+1} \cdot 1 \times 2x + h \\ &= 2xe^{x^2+1}\end{aligned}$$

Derivatives Ex 30.2 Q3(ix)

We have,

$$f(x) = e^{\sqrt{2x}}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{\sqrt{2(x+h)}} - e^{\sqrt{2x}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{\sqrt{2x}} (e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1)}{h} \\ &= \lim_{h \rightarrow 0} e^{\sqrt{2x}} \frac{(e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1)}{\sqrt{2(x+h)} - \sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}\end{aligned}$$

Multiplying Numerator and Denominator by $\sqrt{2(x+h)} - \sqrt{2x}$

$$\therefore h \rightarrow 0 \Rightarrow \sqrt{2(x+h)} - \sqrt{2x} \Rightarrow 0$$

$$\text{and } \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1$$

$$\therefore \lim_{h \rightarrow 0} e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

Again Multiplying Numerator and Denominator by $\sqrt{2(x+h)} + \sqrt{2x}$

$$\begin{aligned}\therefore \lim_{h \rightarrow 0} e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}} \\ = e^{\sqrt{2x}} \times \frac{1}{2\sqrt{2x}}\end{aligned}$$

Derivatives Ex 30.2 Q3(x)

We have,

$$f(x) = e^{\sqrt{ax+b}}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{\sqrt{a(x+h)+b}} - e^{\sqrt{ax+b}}}{h} \\ &= \lim_{h \rightarrow 0} e^{\sqrt{a(x+h)+b}} \left(\frac{e^{\sqrt{a(x+h)+b} - \sqrt{ax+b}} - 1}{h} \right) \\ &= \lim_{h \rightarrow 0} e^{\sqrt{ax+b}} \times \frac{e^{\sqrt{a(x+h)+b} - \sqrt{ax+b}} - 1}{\sqrt{a(x+h)+b} - \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{h}\end{aligned}$$

Multiplying Numerator and Denominator by $\sqrt{a(x+h)+b} + \sqrt{ax+b}$

$$\therefore h \rightarrow 0$$

$$\therefore \sqrt{a(x+h)+b} + \sqrt{ax+b} = 0$$

$$\text{and } \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1$$

$$= \lim_{h \rightarrow 0} e^{\sqrt{ax+b}} \times 1 \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{\sqrt{a(x+h)+b} + \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} + \sqrt{ax+b}}{h}$$

Again multiplied Numerator and Denominator by $\sqrt{a(x+h)+b} + \sqrt{ax+b}$

$$= \lim_{h \rightarrow 0} e^{\sqrt{ax+b}} \times \frac{a(x+h) + b - (ax+b)}{h} \times \frac{1}{(\sqrt{a(x+h)+b} + \sqrt{ax+b})}$$

$$= \frac{e^{\sqrt{ax+b}} \times a}{2\sqrt{ax+b}}$$

$$= \frac{ae^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$$

$$f(x) = a^{\sqrt{x}} = e^{\sqrt{x} \log a}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{\sqrt{x+h} \log a} - e^{\sqrt{x} \log a}}{h} \\ &= \lim_{h \rightarrow 0} e^{\sqrt{x} \log a} \frac{e^{\sqrt{x+h} \log a - \sqrt{x} \log a} - 1}{h} \\ &= \lim_{h \rightarrow 0} e^{\sqrt{x} \log a} \frac{e^{(\sqrt{x+h} - \sqrt{x}) \log a} - 1}{h} \end{aligned}$$

Multiply numerator and denominator by $(\sqrt{x+h} - \sqrt{x}) \log a$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} e^{\sqrt{x} \log a} \frac{e^{(\sqrt{x+h} - \sqrt{x}) \log a} - 1}{h(\sqrt{x+h} - \sqrt{x}) \log a} (\sqrt{x+h} - \sqrt{x}) \log a \\ &= e^{\sqrt{x} \log a} \lim_{h \rightarrow 0} \frac{e^{(\sqrt{x+h} - \sqrt{x}) \log a} - 1}{(\sqrt{x+h} - \sqrt{x}) \log a} \lim_{h \rightarrow 0} \log a \frac{(\sqrt{x+h} - \sqrt{x})}{h} \\ &= e^{\sqrt{x} \log a} \lim_{h \rightarrow 0} \log a \frac{(\sqrt{x+h} - \sqrt{x})}{h} \end{aligned}$$

Multiply numerator and denominator by $(\sqrt{x+h} + \sqrt{x})$

$$\begin{aligned} f'(x) &= e^{\sqrt{x} \log a} \lim_{h \rightarrow 0} \log a \frac{(\sqrt{x+h} - \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} (\sqrt{x+h} + \sqrt{x}) \\ &= e^{\sqrt{x} \log a} \lim_{h \rightarrow 0} \log a \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\ &= e^{\sqrt{x} \log a} \frac{\log a}{2\sqrt{x}} \\ &= \frac{a^{\sqrt{x}}}{2\sqrt{x}} \log_e a \end{aligned}$$

Derivatives Ex 30.2 Q3(xii)

We have,

$$f(x) = 3^{x^2} = e^{x^2 \log 3}$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{(x+h)^2 \log 3} - e^{x^2 \log 3}}{h} \\ &= \lim_{h \rightarrow 0} e^{x^2 \log 3} \frac{e^{((x+h)^2 - x^2) \log 3} - 1}{h} \\ &= \lim_{h \rightarrow 0} e^{x^2 \log 3} \frac{e^{(x+h)^2 - x^2} - 1}{(x+h)^2 - x^2} \times \frac{(x+h)^2 - x^2}{h} \end{aligned}$$

Multiplying Numerator and Denominator by $(x+h)^2 - x^2$

$$\begin{aligned} \therefore \lim_{h \rightarrow 0} e^{x^2 \log 3} \times \frac{(x+h+x)(x+h-x)}{h} \\ &= e^{x^2 \log 3} \times 2x \\ &= 2x e^{x^2 \log 3} \\ &= 2x 3^{x^2 \log 3} \end{aligned}$$

Derivatives Ex 30.2 Q4(i)

We have,

$$f(x) = \tan^2 x$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan^2(x+h) - \tan^2 x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{\tan(x+h) + \tan x\} \{\tan(x+h) - \tan x\}}{h} && [\because \tan^2 A - \tan^2 B = (\tan A + \tan B)(\tan A - \tan B)] \\ &= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h+x)}{\cos(x+h)\cos x} \times \frac{\sin(x+h-x)}{\cos(x+h)\cos x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(2x+h)}{h \cdot \cos(x+h)\cos x} \times \frac{\sin h}{\cos(x+h)\cos x} \\ &= \lim_{h \rightarrow 0} \frac{\sin h}{h} \times \frac{\sin 2x}{\cos^2 x \cdot \cos^2(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{\sin 2x}{\cos^2 x \cdot \cos^2 x} && \left[\because \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \right] \\ &= \lim_{h \rightarrow 0} \frac{2 \sin x \cdot \cos x}{\cos^2 x} \times \frac{1}{\cos^2 x} && [\sin 2x = 2 \sin x \cos x] \\ &= 2 \tan x \cdot \sec^2 x \end{aligned}$$

Derivatives Ex 30.2 Q4(ii)

We have,

$$f(x) = \tan(2x + 1)$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan\{2(x+h) + 1\} - \tan(2x + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(2x + 2h + 1 - 2x - 1)}{h \cdot \cos\{2(x+h) + 1\} \cos(2x + 1)} && \left[\because \tan A - \tan B = \frac{\sin(A - B)}{\cos A \cdot \cos B} \right] \\ &= \lim_{h \rightarrow 0} \frac{2 \cdot \sin 2h}{2h \cdot \cos(2x + 2h + 1) \cos(2x + 1)} \end{aligned}$$

Multiplying both, Numerator and Denominator by 2.

$$\begin{aligned} \therefore \lim_{h \rightarrow 0} 2 \left(\frac{\sin 2h}{2} \right) \times \frac{1}{\cos(2x + 2h + 1) \cos(2x + 1)} \\ &= \frac{2}{\cos^2(2x + 1)} && \left[\because \lim_{h \rightarrow 0} \frac{\sin 2h}{2} = 1 \right] \\ &= 2 \sec^2(2x + 1) && \left[\because \sec^2 x = \frac{1}{\cos^2 x} \right] \\ &= 2 \sec^2(2x + 1) \end{aligned}$$

Derivatives Ex 30.2 Q4(iii)

We have,

$$f(x) = \tan 2x$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan 2(x+h) - \tan 2x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(2x+2h-2x)}{h \cdot \cos(2x+2h) \cos 2x} && \left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A \cdot \cos B} \right] \\ &= \lim_{h \rightarrow 0} \frac{\sin 2h}{h \cdot \cos(2x+2h) \cos 2x} \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin 2h}{2h} \right) \times \frac{1 \times 2}{\cos(2h+2x) \cos 2x} \\ &= \frac{2}{\cos 2x \cdot \cos 2x} && \left[\because \lim_{h \rightarrow 0} \frac{\sin 2h}{2h} = 1 \right] \\ &= 2 \sec^2 2x && \left[\because \frac{1}{\cos^2 x} = \sec^2 x \right] \end{aligned}$$

Derivatives Ex 30.2 Q4(iv)

We have,

$$f(x) = \sqrt{\tan x}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h} \end{aligned}$$

Multiplying Numerator and Denominator by $\sqrt{\tan(x+h)} + \sqrt{\tan x}$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\tan(x+h) - \tan x}{h \left(\sqrt{\tan(x+h)} + \sqrt{\tan x} \right)} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x+h-x)}{h \cdot \cos(x+h) \cos x \left(\sqrt{\tan(x+h)} + \sqrt{\tan x} \right)} \\ &= \lim_{h \rightarrow 0} \frac{\sinh}{h} \times \frac{1}{\cos(x+h) \cos x \left(\sqrt{\tan(x+h)} + \sqrt{\tan x} \right)} \\ &= \lim_{h \rightarrow 0} \frac{1}{\cos^2 x \cdot 2\sqrt{\tan x}} && \left[\lim_{h \rightarrow 0} \frac{\sinh}{h} = 1 \right] \\ &= \frac{1}{2} \frac{\sec^2 x}{\sqrt{\tan x}} && \left[\because \frac{1}{\cos^2 x} = \sec^2 x \right] \end{aligned}$$

Derivatives Ex 30.2 Q5(i)

let $f(x) = \sin \sqrt{2x}$. Then $f(x+h) = \sin \sqrt{2(x+h)}$

$$\frac{d(f(x))}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin \sqrt{2(x+h)} - \sin \sqrt{2x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin \left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right) \cos \left(\frac{\sqrt{2(x+h)} + \sqrt{2x}}{2} \right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin \left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right) (\sqrt{2(x+h)} - \sqrt{2x}) (\sqrt{2(x+h)} + \sqrt{2x}) \cos \left(\frac{\sqrt{2(x+h)} + \sqrt{2x}}{2} \right)}{\left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right) (\sqrt{2(x+h)} + \sqrt{2x}) h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin \left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right)}{\left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right)} \lim_{h \rightarrow 0} \frac{2(x+h) - 2x}{(\sqrt{2(x+h)} + \sqrt{2x}) h} \lim_{h \rightarrow 0} \cos \left(\frac{\sqrt{2(x+h)} + \sqrt{2x}}{2} \right)$$

$$= 1 \times \frac{2}{2\sqrt{2x}} \cos(\sqrt{2x})$$

$$= \frac{\cos(\sqrt{2x})}{\sqrt{2x}}$$

Derivatives Ex 30.2 Q5(ii)

We have,

$$f(x) = \cos \sqrt{x}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos \sqrt{x+h} - \cos \sqrt{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \sin \left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin \left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right) \left(\sqrt{x+h} - \sqrt{x} \right) \left(\sqrt{x+h} + \sqrt{x} \right)}{\left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right) \left(\sqrt{x+h} + \sqrt{x} \right) h} \times \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \end{aligned}$$

Multiplied Numerator and Denominator by $(\sqrt{x+h} - \sqrt{x})$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{-\sin \left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)}{\left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)} \times \frac{x+h-x}{\left(\sqrt{x+h} + \sqrt{x} \right) h} \times \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \\ &= \lim_{h \rightarrow 0} -1 \frac{h}{h \left(\sqrt{x+h} + \sqrt{x} \right)} \times \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \quad \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\ &= \lim_{h \rightarrow 0} \frac{-1}{\left(\sqrt{x+h} + \sqrt{x} \right)} \sin \frac{\sqrt{x+h} + \sqrt{x}}{2} \\ &= \frac{-\sin \sqrt{x}}{2\sqrt{x}} \end{aligned}$$

We have,

$$f(x) = \tan \sqrt{x}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\tan \sqrt{x+h} - \tan \sqrt{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin \sqrt{x+h} - \sqrt{x}}{h \cdot \cos \sqrt{x+h} \cos \sqrt{x}}$$

$$\left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A \cdot \cos B} \right]$$

$$= \lim_{h \rightarrow 0} \frac{\sin \sqrt{x+h} - \sqrt{x}}{(x+h-x) \cos \sqrt{x} \cdot \cos \sqrt{x+h}}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(\sqrt{x+h} - \sqrt{x})}{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x}) \cos \sqrt{x} \cdot \cos \sqrt{x+h}}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(\sqrt{x+h} - \sqrt{x})}{(\sqrt{x+h} - \sqrt{x})} \times \frac{1}{(\sqrt{x+h} + \sqrt{x}) \cos \sqrt{x} \cdot \cos \sqrt{x+h}}$$

$$= 1 \times \frac{1}{2\sqrt{x} \cos \sqrt{x} \cos \sqrt{x+h}}$$

$$\left[\because \lim_{h \rightarrow 0} \frac{\sin \sqrt{x+h} - \sqrt{x}}{\sqrt{x+h} - \sqrt{x}} = 1 \right]$$

$$= \frac{1}{2\sqrt{x} \cos^2 x}$$

$$= \frac{\sec^2 x}{2\sqrt{x}}$$

We have,

$$f(x) = \tan x^2$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan(x+h)^2 - \tan x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h)^2}{\cos(x+h)^2} - \frac{\sin x^2}{\cos x^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h)^2 \cos x^2 - \cos(x+h)^2 \sin x^2}{\cos(x+h)^2 \cos x^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin((x+h)^2 - x^2)}{h \cdot \cos(x+h)^2 \cdot \cos x^2} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x^2 + h^2 + 2hx - x^2)}{h \cdot \cos(x+h)^2 \cdot \cos x^2} \\ &= \lim_{h \rightarrow 0} \frac{\sin(h^2 + 2hx)}{h \cdot \cos(x+h)^2 \cdot \cos x^2} \\ &= \lim_{h \rightarrow 0} \frac{\sinh}{h} \times \frac{(h+2x)}{\cos(x+h)^2 \cdot \cos x^2} \\ &= 1 \cdot \frac{2x}{\cos^2(x)^2} \quad \left[\because \lim_{h \rightarrow 0} \frac{\sinh}{h} = 1 \right] \\ &= 2x \sec^2 x^2 \end{aligned}$$

Derivatives Ex 30.2 Q6(i)

We have,

$$f(x) = (-x)$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-(x+h) + x}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x - h + x}{h} \\ &= -1 \end{aligned}$$

Derivatives Ex 30.2 Q6(ii)

We have,

$$f(x) = (-x)^{-1}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x + x + h}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{1}{x^2 + xh} \\ &= \frac{1}{x^2} \end{aligned}$$

Derivatives Ex 30.2 Q6(iii)

We have,

$$f(x) = \sin(x+1)$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x+h+1) - \sin(x+1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{x+h+1+x+1}{2}\right) \sin\left(\frac{x+h+1-x-1}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{2x+2+h}{2}\right) \sin \frac{h}{2}}{h} \quad \left[\because \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \right] \\ &= \lim_{h \rightarrow 0} \cos\left(\frac{2x+2+h}{2}\right) \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right) \\ &= \cos\left(\frac{2(x+1)}{2}\right) \quad \left[\because \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} = 1 \right] \\ &= \cos(x+1) \end{aligned}$$

Derivatives Ex 30.2 Q6(iv)

We have,

$$f(x) = \cos\left(x - \frac{\pi}{8}\right)$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos\left(x+h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2 \sin\left(\frac{2x+h - \frac{2\pi}{8}}{2}\right) \sin\left(\frac{h+x - \frac{\pi}{8} - x + \frac{\pi}{8}}{2}\right)}{h} \end{aligned}$$

$$\left[\because \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \right]$$

$$= \lim_{h \rightarrow 0} \frac{-2 \sin\left(\frac{2x+h - \frac{2\pi}{8}}{2}\right) \times \sin\left(\frac{h}{2}\right)}{2 \cdot \frac{h}{2}}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \sin\left(\frac{2x+h - \frac{2\pi}{8}}{2}\right)}{2} \times \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

$$= \lim_{h \rightarrow 0} \sin\left(\frac{2x+h - \frac{2\pi}{8}}{2}\right) \quad \left[\because \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} = 1 \right]$$

$$= \sin\left(\frac{2x - \frac{2\pi}{8}}{2}\right)$$

$$= -\sin\left(x - \frac{\pi}{8}\right)$$