RD Sharma Solutions Class 11 Maths Chapter 30 Ex 30.1

Derivatives EX 30.1 Q1

We have,

$f(x) = 3x$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(a) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{3(2+h) - 6}{h}$$

$$= \lim_{h \to 0} \frac{3h}{h}$$

$$= \lim_{h \to 0} 3$$

$$\therefore f'(2) = 3$$

Derivatives EX 30.1 Q2

We have,

$f(x) = x^2 - 2$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(10) = \lim_{h \to 0} \frac{f(10+h) - f(10)}{h}$$

$$= \lim_{h \to 0} \frac{(10+h)^2 - 2 - 98}{h}$$

$$= \lim_{h \to 0} \frac{100 + 20h + h^2 - 100}{h}$$

$$= \lim_{h \to 0} \frac{h(20+h)}{h}$$

$$= \lim_{h \to 0} (20+h)$$

$$\therefore f'(10) = 20$$

Derivatives EX 30.1 Q3

We have,

$f(x) = 99x$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(100) = \lim_{h \to 0} \frac{f(100+h) - f(100)}{h}$$

$$= \lim_{h \to 0} \frac{99(100+h) - 9900}{h}$$

$$= \lim_{h \to 0} \frac{9900 + 99h - 9900)}{h}$$

$$= \lim_{h \to 0} 99$$

$$\therefore f'(100) = 99$$

Derivatives EX 30.1 Q4

We have,

$f(x) = x$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h) - 1}{h}$$

$$= \lim_{h \to 0} 1$$

$$\therefore f'(1) = 1$$

Derivatives EX 30.1 Q5

We have,

$f(x) = \cos x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(0+h) - \cos 0}{h}$$

$$= \lim_{h\to 0} \frac{\cos h - \cos 0}{h}$$

$$= \lim_{h\to 0} \frac{(1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \text{---}) - 1}{h} \qquad \left[\because \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right]$$

$$= \lim_{h\to 0} \frac{h(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} - \ldots)}{h}$$

$$= \lim_{h\to 0} h(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} - \ldots)$$

$$= 0$$

$\therefore f'(0) = 0$

Derivatives EX 30.1 Q6

We have,

$f(x) = \tan x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{\tan h - \tan 0}{h}$$

$$= \lim_{h\to 0} \frac{\tan h}{h}$$

$$= 1 \qquad \left[\because \lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1\right]$$

$\therefore f'(0) = 1$

Derivatives EX 30.1 Q7(i)

We have,

$f(x) = \sin x$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'\left(\frac{\pi}{2}\right) = \lim_{h \to 0} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\sin\left(\frac{\pi}{2}+h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cos h - 1}{h}$$

$$= \lim_{h \to 0} \frac{\left(1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \dots\right) - 1}{h} \qquad \left[\because \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots\right]$$

$$= \lim_{h \to 0} \frac{h\left(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} + \dots\right)}{h}$$

$$= \lim_{h \to 0} h\left(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} + \dots\right)$$

$$= 0$$

$$\therefore f'\left(\frac{\pi}{2}\right) = 0$$

Derivatives EX 30.1 Q7(ii)

We have,

$f(x) = x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{1+h-1}{h}$$

$$= \lim_{h\to 0} 1$$

$$\therefore f'(1) = 1$$

Derivatives EX 30.1 Q7(iii)

We have,

$\because f(x) = 2\cos x$

$$\therefore f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{\pi}{2}+h\right) - 2\cos\frac{\pi}{2}}{h}$$

$$= \lim_{h\to 0} \frac{-2\sin h - 0}{h}$$

$$= -2 \qquad \left[\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$\therefore f'\left(\frac{\pi}{2}\right) = -2$$

Derivatives EX 30.1 Q7(iv)

We have, $f(x)=\sin 2x$

Therefore,

$$
\begin{aligned}
f'(a) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}\\
&= \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}+h\right)-f\left(\frac{\pi}{2}\right)}{h}\\
&= \lim_{h\to 0}\frac{\sin 2\left(\frac{\pi}{2}+h\right)-\sin 2\left(\frac{\pi}{2}\right)}{h}\\
&= \lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}\times 2+2h\right)-\sin(\pi)}{h}\\
&= \lim_{h\to 0}\frac{-\cos 2h-0}{h}\\
&= -2
\end{aligned}
$$

Therefore $f'\left(\frac{\pi}{2}\right)=-2$