RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.9

Straight lines Ex 23.9 Q1

(i) Slope intercept form $(y = mx + c)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow y = -\sqrt{3}x - 2$

$\Rightarrow m = -\sqrt{3},\ c = -\sqrt{2}$

y-intercept $= -2$, slope $= -\sqrt{3}$

(ii) Intercept form $(\frac{x}{a} + \frac{y}{b} = 1)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow \sqrt{3}x + y = -2$

$\Rightarrow \frac{\sqrt{3}x}{-2} + \frac{y}{-2} = 1$

$\Rightarrow \frac{x}{\frac{-2}{\sqrt{3}}} + \frac{y}{-2} = 1$

$\Rightarrow x$ intercept $= \frac{-2}{\sqrt{3}}$, y intercept $= -2$

(iii) Normal form $(x\cos\alpha + y\sin\alpha = p)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow -\sqrt{3}x - y = 2$

$\Rightarrow \left(\frac{-\sqrt{3}}{2}\right)x + \left(\frac{-1}{2}\right)y = 1$

$\Rightarrow \cos\alpha = \frac{-\sqrt{3}}{2} = \cos 210^0$ and $\sin\alpha = \frac{-1}{2} = \sin 210^0$

$\Rightarrow p = 1,\ \alpha = 210^0$

Straight lines Ex 23.9 Q2(i)

$x+\sqrt{3}y-4=0$

Divide the equation by 2, we get

$\frac{1}{2}x+\frac{\sqrt{3}}{2}y=2$

$x\cos 60+y\sin 60=2$

So, p=2 and ω=60

Straight lines Ex 23.9 Q2(ii)

$x+y+\sqrt{2}=0$

$x+y=-\sqrt{2}$

Dividing each term by $\sqrt{(1)^2+(1)^2}=\sqrt{2}$

$\frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}}=-1$

$\frac{-x}{\sqrt{2}}-\frac{y}{\sqrt{2}}=1$

Comparing with $x\cos\alpha+y\sin\alpha=p$

$\cos\alpha=\frac{-1}{\sqrt{2}},\quad \sin\alpha=\frac{-1}{\sqrt{2}},\quad p=1$

Both are negative

α is in III quadrant

$\Rightarrow \alpha=\pi\frac{\pi}{4}=\frac{5\pi}{4}=225^0$

Straight lines Ex 23.9 Q2(iii)

$x - y + 2\sqrt{2} = 0$

$-x + y = 2\sqrt{2}$

Dividing each term by $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$\frac{-x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 2$

Comparing with $x \cos\alpha + y \sin\alpha = p$

$\cos\alpha = \frac{-1}{\sqrt{2}}, \quad \sin\alpha = \frac{-1}{\sqrt{2}}, \quad p = 2$

α is in II quadrant

$\Rightarrow \alpha = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4} = 135^0, \quad p = 2$

Straight lines Ex 23.9 Q2(iv)

$x - 3 = 0$

$x = 3$

Comparing with $x \cos\alpha + y \sin\alpha = p$

$\cos\alpha = 1$

$= \cos 0$

$\Rightarrow \alpha = 0$

$p = 3$

Straight lines Ex 23.9 Q2(v)

$y - 2 = 0$

$y = 2$

Comparing with $x\cos\alpha + y\sin\alpha = p$

$\sin\alpha = 1$

$\alpha = \frac{\pi}{2}, \quad p = 2$

Straight lines Ex 23.9 Q3

$\frac{x}{a} + \frac{y}{b} = 1$

The slope intercept form is

$y = mx + c$

$\frac{x}{a} + \frac{y}{b} = 1$

$bx + ay = ab$

$ay = -bx + ab$

$y = \frac{-bx}{a} + b$

Thus y-intercept is b.

Slope $= \frac{-b}{a}$

Straight lines Ex 23.9 Q4

The normal form is obtaned by dividing each term of the equation by $\sqrt{a^2+b^2}$,

a = coefficient of x

b = coefficient of y

$3x - 4y + 4 = 0 \quad ---(1)$

$3x - 4y = -4$

$-3x + 4y = 4$

Dividing each term by $\sqrt{(-3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\frac{-3}{5}x + \frac{y}{5}y = \frac{4}{5}$

$\Rightarrow p = \frac{4}{5} \qquad$ for equation (1)

Also

$2x + 4y - 5 = 0 \quad ---(2)$

$2x + 4y = 5$

Dividing each term by $\sqrt{(2)^2 + (4)^2} = \sqrt{4+16} = \sqrt{20}$

$\frac{2x}{\sqrt{20}} + \frac{4y}{\sqrt{20}} = \frac{5}{\sqrt{20}}$

$p = \frac{5}{\sqrt{20}} = \frac{5}{4.4} \qquad$ for equation (2)

Comparing P of (1) and (2)

We conclude that $3x - 4y + 4 = 0$ is nearest to origin

Straight lines Ex 23.9 Q5

Reduce $4x + 3y + 10 = 0$ to perpendicular form

$4x + 3y = -10$

$-4x - 3y = 10$

Dividing each term by $\sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\frac{-4}{5}x - \frac{3}{5}y = \frac{10}{5} = 2$

$\Rightarrow p_1 = 2 \qquad ---(1)$

$5x - 12y + 26 = 0$

$5x - 12y = -26$

Dividing each term by $\sqrt{(-5)^2 + (12)^2} = \sqrt{25+144} = \sqrt{169} = 13$

$\frac{-5}{13}x + \frac{12}{13}y = \frac{26}{13} = 2$

$\Rightarrow p_2 = 2 \qquad ---(2)$

$7x + 24y = 50$

Dividing each term by $\sqrt{(7)^2 + (24)^2} = \sqrt{49+576} = \sqrt{625} = 25$

$\frac{7x}{25} + \frac{24}{25}y = \frac{50}{25} = 2$

$\Rightarrow p_3 = 2 \qquad ---(3)$

Hence, origin is equidistant from all three lines.

Straight lines Ex 23.9 Q6

$\sqrt{3}x + y + 2 = 0$

$\sqrt{3}x + y = 2$

$-\sqrt{3}x - y = 2 \text{ - - - - - } (1)$

So,

$\cos\theta = -\sqrt{3}, \sin\theta = -1$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

$\theta = \left(\pi + \dfrac{\pi}{6}\right)$

$= 180° + 30°$

$\theta = 210°$

$p = 2 \qquad$ [From equation (1)]

Straight lines Ex 23.9 Q7

The intercept form of equation is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$3x - 2y + 6 = 0$

$3x - 2y = -6$

$$\frac{-3x}{-6} - \frac{2y}{-6} = 1$$

$$\frac{x}{\frac{-6}{3}} + \frac{y}{\frac{-6}{-2}} = 1$$

$$\frac{x}{-2} + \frac{y}{3} = 1$$

$$\Rightarrow x\text{-intercept} = a = -2$$
$$y\text{-intercept} = b = 3$$

Straight lines Ex 23.9 Q8

Perpendicular distance from the origin to the line is 5, so

$$x\cos\alpha + y\sin\alpha = 5$$
$$y\sin\alpha = -x\cos\alpha + 5$$
$$y = -\frac{\cos\alpha}{\sin\alpha}x + 5$$
$$y = -\cot\alpha x + 5$$

Comparing with $y = mx + c$

$$m = -\cot\alpha$$
$$-1 = -\cot\alpha$$
$$\cot\alpha = 1$$
$$\alpha = \frac{\pi}{4}$$

So, equation of line is

$$x\cos\frac{\pi}{4} + y\sin\frac{\pi}{4} = 5$$
$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 5$$

$$x + y = 5\sqrt{2}$$