

RD Sharma
Solutions
Class 11 Maths
Chapter 23
Ex 23.17

Straight lines Ex 23.17 Q1

Let $ABCD$ be a parallelogram the equation of whose sides AB , BC , CD and DA are $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_1x + b_1y + d_1 = 0$ and $a_2x + b_2y + d_2 = 0$.

Let p_1 and p_2 be the distance between the pairs of parallel side of $ABCD$.

$$\sin \theta \frac{p_1}{AD} = \frac{p_2}{AB}$$

$$\therefore AD = \frac{p_1}{\sin \theta} \text{ and } AB = \frac{p_2}{\sin \theta}$$

$$\text{Area of } ABCD = AB \times p_1 = \frac{p_1 p_2}{\sin \theta}$$

$$\text{or } \Rightarrow AD \times p_2 = \frac{p_1 p_2}{\sin \theta}$$

Now,

$$m_1 = \text{slope of } AB = \frac{-a_1}{b_1}$$

$$m_2 = \text{slope of } AD = \frac{-a_2}{b_2}$$

Since θ is angle between AB and AD .

$$\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{\frac{-a_1}{b_1} - \frac{-a_2}{b_2}}{1 - \frac{a_1 a_2}{b_1 b_2}}$$

$$\tan \theta = \frac{a_2 b_1 - a_1 b_2}{a_1 a_2 + b_1 b_2} \Rightarrow \sin \theta = \frac{a_2 b_1 - a_1 b_2}{\sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}}$$

p_1 = Distance between AB and AD .

$$= \left| \frac{c_1 - d_1}{\sqrt{a_1^2 + b_1^2}} \right|$$

p_2 = Distance between AD and BC .

$$= \left| \frac{c_2 - d_2}{\sqrt{a_2^2 + b_2^2}} \right|$$

\therefore Area of parallelogram is

$$\frac{|c_1 - d_1| |c_2 - d_2|}{|a_2 b_1 - a_1 b_2|} \quad \text{Hence, proved.}$$

(ii) Rhombus is a parallelogram with all side equal.

$$\therefore p_1 = p_2$$

\therefore Modifying the formula of area of parallelogram derived above.

The area of rhombus

$$= \frac{p_1 p_2}{\sin \theta} \\ = \frac{2p_1 \cdot 2p_2}{2}$$

$$\begin{aligned}
 &= \frac{\vec{c} \cdot \vec{a}}{\sin \theta} = \frac{\vec{c} \cdot \vec{b}}{\sin \theta} \\
 &= 2 \left| \frac{(c_1 - d_1)}{a_2 b_1 - a_1 b_2} \right| \text{ or } 2 \left| \frac{(c_2 - d_2)}{a_2 b_1 - b_2 a_1} \right|
 \end{aligned}$$

Straight lines Ex 23.17 Q2

The area of a parallelogram is

$$\begin{aligned}
 &= \frac{|c_1 - d_1| |c_2 - d_2|}{|a_2 b_1 - b_2 a_1|} \\
 &= \frac{|-a + 2a| |3a - a|}{|3(-3) - 4(-4)|} \\
 &= \frac{a \times 2a}{7} \\
 &= \frac{2}{7} a^2
 \end{aligned}$$

Hence, proved.

Straight lines Ex 23.17 Q3

Let $ABCD$ be a parallelogram as shown in the following figure.

We observe that the following parallelogram is a rhombus, as distance between opposite sides (AB and CD) and (AD and BC) is equal = $(n - n')$.

And in a Rhombus, diagonals are perpendicular to each other.

\therefore Angle between the two diagonals is $17/2$.