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**CHAPTER 1****SIMULATION****Question 1**

Frontier Bakery keeps stock of a particular brand of cake. Daily demand based on past experience is as given below :

<b>Experience indicates :</b>						
<b>Daily demand</b>	0	15	25	35	45	50
<b>Probability</b>	0.01	0.15	0.20	0.50	0.12	0.02

Consider the sequence of random number :

48	78	09	51	56	77	15	14	68	09
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Using the sequence, simulate the demand for the next 10 days.

**(CA Final Nov. 1999)**

**Answer :**

<b>Days</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>Demand</b>	<b>35</b>	<b>35</b>	<b>15</b>	<b>35</b>	<b>35</b>	<b>35</b>	<b>15</b>	<b>15</b>	<b>35</b>	<b>15</b>

**Question 2.**

A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopeds to 204 mopeds, whose probability distribution is as given below :

<b>Production/Day</b>	<b>196</b>	<b>197</b>	<b>198</b>	<b>199</b>	<b>200</b>	<b>201</b>	<b>202</b>	<b>203</b>	<b>204</b>
<b>Probability</b>	<b>0.05</b>	<b>0.09</b>	<b>0.12</b>	<b>0.14</b>	<b>0.20</b>	<b>0.15</b>	<b>0.11</b>	<b>0.08</b>	<b>0.06</b>

The finished mopeds are transported in a specially designed three storey lorry that can accommodate only 200 mopeds. Using the following 15 numbers 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10, simulate the process to find out : what will be average number of mopeds waiting in the factory? What is the average empty space in the lorry?

**Answer :**        **2.8 mopeds per day (ii) 0.2666 mopeds per day**

**Question 3.**

A publishing house has bought out a new monthly magazine, which sells at Rs. 37.5 per copy. The cost of procuring it is Rs. 30 per copy. A newsstand estimates the sales pattern of the magazine as follows :

Demand Copies	0 < 300	300 < 600	600 < 900	900 < 1200	1200 < 1500	1500 < 1800
Probability	0.18	0.32	0.25	0.15	0.06	0.04

The newsstand has contracted for 750 copies of the magazine per month from the publisher. The unsold copies are returnable to the publisher who will take them back at cost less Rs. 4 per copy for handling charges. The newsstand manager wants to simulate of the demand and profitability. The following random number may be used for simulation :

<b>S. NO.</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>
<b>Random Number</b>	27	15	56	17	98	71	51	32	62	83	96	69

- (i). Allocate random numbers to the demand patter forecast by the newsstand.
- (ii). Simulate twelve months sales and calculate the monthly and annual profit/loss.
- (iii). Calculate the loss on lost sales. (CA Final Nov. 2005)

**Answer :**

**Profit forgone on lost sales of Rs .15750**

**Question 4.**

A car rental agency has collected the following data on the demand for five-seater vehicles over past 50 days.

<b>Daily Demand</b>	4	5	6	7	8
<b>No. of days</b>	4	10	16	14	6

The agency has only 6 cars currently. Use the following 5 random numbers to generate 5 days of demand to the rental agency. Random numbers : 15, 48, 71, 56, 90. What is the average number of cars rented per day for 5 days? (CA Final May 2011)

**Answer:**

**Average No. Of Cars Rented Per Day =29/5=5.80**

**Question 5**

The Director of finance for a farm cooperative is concerned about the yields per acre she can expect from this year's corn crop. The probability distribution of the yields for the current weather conditions is given below :

<b>Yield in Kg per acreRs.</b>	<b>120</b>	<b>140</b>	<b>160</b>	<b>180</b>
<b>ProbabilityRs.</b>	<b>0.18</b>	<b>0.26</b>	<b>0.44</b>	<b>0.12</b>

She would like to see a simulation of the yields she might expect over next 10 years for the weather conditions similar to those she is now experiencing. You may use these random numbers : 20, 72, 34, 54, 30, 22, 48, 74, 76, and 02.

She is also interested in the effect of market-price fluctuations on the co-operative farm revenue. She makes this estimate of pre-kg prices for corn.

<b>Price per Kg. (Rs.)</b>	<b>2.00</b>	<b>2.10</b>	<b>2.20</b>	<b>2.30</b>	<b>2.40</b>	<b>2.50</b>
<b>Probability Rs.</b>	<b>0.05</b>	<b>0.15</b>	<b>0.30</b>	<b>0.25</b>	<b>0.15</b>	<b>0.10</b>

Stimulate the price she might expect to observe over next 10 years using the following random numbers : 82, 95, 18, 96, 20, 84, 56, 11, 52 and 03.

Estimate the average revenue per acre over next 10 yearRs.

**(CA Final May 1991)**

**Answer: Revenue Rs. 338.60**

**Question 6.**

The occurrence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained the previous day : rain distribution is follows :

<b>Event</b>	<b>Probability</b>
No rain	0.50
1 cm. rain	0.25
2 cm. rain	0.15
3 cm. rain	0.05
4 cm. rain	0.03
5 cm. rain	0.02

If did not rain the previous day, the rain distribution is :

Event	Probability
No Rain	0.75
1 cm. rain	0.15
2 cm. rain	0.06
3 cm. rain	0.04

Stimulate the city’s weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period.

Use the following random number for simulation

67	63	39	55	29	78	70	06	78	76
----	----	----	----	----	----	----	----	----	----

Assume that for the first day of simulation it had not rained the day before.

**Answer:**

**Total rain 5 cms and 6 days will be without rain .**

**Question 7.**

The output of production line is checked by an inspector for one or more of three different types of defects, A, B and C. If defect A occurs, the item is scrapped. If defect B or C occurs, the item must be reworked. The time required to rework a B defect is 15 minutes and the time required to rework a C defect is 30 minutes. The probabilities of an A, B and C defects are 0.15, 0.20 and 0.10 respectively. For ten items coming off the assembly line, determine the number of items without any defects, the number scrapped and total minutes of rework time. Use the following random numbers :

**RN for defects A :**

48	55	91	40	93	01	83	63	47	52
----	----	----	----	----	----	----	----	----	----

**RN for defects B :**

47	36	57	04	79	55	10	13	57	09
----	----	----	----	----	----	----	----	----	----

**RN for defects C :**

82	95	18	96	20	84	56	11	52	03
----	----	----	----	----	----	----	----	----	----

(CA Final May 1994)

**Answer : (i) No Defect Items =5 (ii) Scrap = 1 item (iii) Rework = 4 Items  
(iv) 90 minutes**

**Question 8.**

Dr. Strong is dentist who schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. the following summary shown the various categories of work, their probabilities and time actually needed to complete the work :

Category	Time required	Probability of category
Filling	45 minutes	0.40
Crown	60 minutes	0.15
Cleaning	15 minutes	0.15
Extraction	45 minutes	0.10
Check up	15 minutes	0.20

Stimulate the dentist’s clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8.00 A.M., use the following random numbers for handling the above problem.

40	82	11	34	25	66	17	79
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(CA Final Nov. 1990)

**Answer :** (i) 35.625 min (ii) 0 min

**Question 9**

The management of ABC company is considering the question of marketing a new product. The fixed cost required in the project is Rs.4000. Three factors are uncertain. viz. the selling price, variable cost and the annual sales volume. The product has a life of only one year. The management has the data on these three factors as under :

Selling Price	Probability	Variable cost	Probability	Sale Units	Probability
3	0.20	1	0.30	2,000	0.30
4	0.50	2	0.60	3,000	0.30
5	0.30	3	0.10	5,000	0.40

Consider the following sequence of thirty random numbers :

81, 32, 60	04, 46, 31	67, 25, 24	10, 40, 02	39, 68, 08
59, 66, 90	12, 64, 79	31, 86, 68	82, 89, 25	11, 98, 16

Using the sequence (first 3 random for the first trial, etc.), simulate the average profit for the above project on the basis of 10 trials.

**Answer :**

**Total profit from 10 trials is Rs. 21000 and Avg Profit Rs.2100.**

**Question 10.**

A company trading in motor vehicles spares wishes to determine the level of stock it should carry for the item in its range. Demand is not certain and replenishment of stock takes 3 days. For one item X, the following information is obtained :

Demand (Units/day)	1	2	3	4	5
Probability	0.10	0.20	0.30	0.30	0.10

Each time an order is placed, the company incurs an ordering cost of Rs. 20 per order. The company also incurs carrying cost of Rs. 2.50 per unit per day. The inventory carrying cost is calculated on the basis of average stock. The manager of the company wishes to compare two options for his inventory decision :

- (i). Order 12 units when the inventory at the beginning of the day plus order outstanding is less than 12 units.
- (ii). Order 10 units when the inventory at the beginning of the day plus order outstanding is less than 10 units.

Currently (on the first day) the company has a stock of 17 units. The sequence of random number to be used is 08, 91, 25, 18, 40, 27, 85, 75, 32 and 52. You are required to carry out a simulation run over a period 10 days, recommended which option the manager should chose.

(CA Final Nov. 2004)

**Answer: (i) 261.25 (ii) 216.25**

**Question 11**

A company uses a high grade raw material. The consumption pattern is probabilistic as given below and it takes two months to replenish stocks :

Consumption/month (tons)	1	2	3	4
Probability	0.15	0.30	0.45	0.10

The cost of placing an order is Rs. 1000 and the cost of carrying stocks is Rs. 50 per month per ton. The average carrying costs are calculated on the stocks held at the end of each month.

The company has two options for the purchase of raw material as under :

Option I : Order for 5 tons when closing inventory of the month plus outstanding order is less than 8 tons.

Option II : Order for 8 tons when closing inventory of the month plus outstanding order is less than 8 tons.

Currently on 1st April, 2002, the company has a stock of 8 tons of raw materials plus 6 tons ordered two months ago. The order is to be received next month.

Using the random numbers given below, simulate 12 months consumption till 31.03.2003 and advise the company as to which purchase option should be accepted such that inventory costs are minimum. Random numbers are :

88	41	67	63	48	74	27	16	11	64	49	21
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(Similar to CA Final May 2000)

**Answer: (i) Carrying Cost = 44X50 = 2200 and Ordering Cost =5000; Total Cost =7200**

**(ii) Carrying Cost = 47X50 = 2350 and Ordering Cost = 3000; Total Cost =5350**

**It is recommended to order 8 tonnes at a time .**

**Question 12.**

With a view to improving the quality of customer services, a Bank is interested in making an assessment of the waiting time of its customers coming to one of its branches located in a residential area. This branch has only one teller’s counter. The arrival rate of the customers and the service rate of the teller are given below :

Time between two consecutive arrivals of customers (in minutes)	Probability	Service time by the teller (in minutes)	Probability
3	0.17	3	0.10
4	0.25	4	0.30



5	0.25	5	0.40
6	0.20	6	0.15
7	0.13	7	0.05

You are required to stimulate 10 arrivals of customers in the system starting from 11 AM and show the waiting time of the customers and idle time of the teller. Use the following random numbers taking the first two random numbers in two digits each for the first trial and so no : 11, 56, 23, 72, 94, 83, 83, 01, 97, 99, 83, 10, 93, 34, 33, 53, 49, 94, 37 and 97. (CA Final May 2003)

**Answer:           Waiting Time 4 Minute and Idle Time 10 Minute**

**Question 13.**

A Ltd trades in a perishable commodity. Each day, A Ltd. receives supplies of goods from a wholesaler but the quantity is a random variable, as is subsequent retail customer demand for the commodity. Both supply and demand are expressed in batches of 50 units and over the past working year (300 days) A Ltd. has kept the records of supply and demands. The records are as follows :

Wholesale Supplies	No. of days occurring	Customers' demand	No. of days occurring
50	60	50	60
100	90	100	60
150	90	150	150
200	60	200	30

A Ltd. buys the commodity at Rs. 6 per unit and sells at the rate of Rs. 10 per unit. Unsold units at the end of the day are worthless as there are no storage facilities. A Ltd. estimates that each unit of unsatisfied demand on any day costs them Rs. 2. Use the following random numbers. (8, 4), (8, 0), (3, 3), (4, 7), (9, 6), (1, 5). Simulate six days trading and estimate the annual profit.

(Adapted ICWA December 2000)

**Answer : Profit in 6 days : Rs.600 and Profit in 300 Days 30000**

**Question 14.**

ABC Cooperative Bank receives and disburses different amount of cash in each month. The Bank has an opening cash balance of Rs. 15 Crores in the first month. The pattern of receipts and disbursements from the past data is as follows:

Monthly cash receipts		Monthly cash payments	
Rs. Crores	Probability	Rs. Crores	Probability
30	0.20	33	0.15
42	0.40	60	0.20
36	0.25	39	0.40
99	0.15	57	0.25

- (i) Calculate probability that the ABC Cooperative Bank will fall short in payments.
- (ii) Calculate average monthly shortfall.
- (iii) If the ABC Cooperative Bank can get an overdraft facility of Rs. 45 Crores from some other bank, what is the probability that they will fall short in monthly payments ?

Use the following sequence of paired random numbers :

17, 78	43, 16	74, 35	31, 23	72, 44	46, 92
51, 58	68, 08	93, 58	54, 78	96, 54	09, 77

**(CA Final May 2010)**

**Answer: (i) 83.33% (ii) 39.9 (iii) 41.67%**

**Question 15**

A car manufacturing company manufactures 40 cars per day. The sale of cars depends up on the demand which has the following distribution.

<b>Sales of Cars</b>	37	38	39	40	41	42
<b>Probability</b>	0.10	0.15	0.20	0.35	0.15	0.05

The production cost and sale price of each car are Rs. 4 Lakhs and Rs. 5 Lakhs respectively. Any unsold car is to be disposed of at a loss of Rs. 2 Lakhs. There is penalty of Rs. 1 Lakh per car, if the demand is not met. Using the following random numbers, estimate the total profit/loss for the next 10 days.

09,	98,	64,	98,	94,	01,	78,	10,	15,	19
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If the company decides to produce 39 cars per day, what will be its impact on profitability?

(CA Final May 2002)

**Answer: (i) Rs. 359 Lacs (ii) Rs.359 Lacs**

**Question 16.**

A retailer deals in a perishable commodity. The daily demand and supply are variables. The data for the past 500 days show the following demand and supply :

Availability (Kg.)	Supply (No. of days)	Demand (Kg.)	Demand (No. of days)
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The retailer buys the commodity at Rs. 20 per Kg. and sells at Rs. 30 per kg. any commodity remains at the end of the day, has no sales value. Moreover the loss on unsatisfied demand is Rs. 8 per Kg. Given the following pair of random numbers, simulate 6 days sales, demand and profit : (31, 18), (63, 84), (15, 79), (07, 32), (43, 75), (81, 27). The first random number in the pair is that of supply and the second random number is for demand. (CA Final Nov. 2000)

**Answer: 400**

**Question 17.**

A refreshment centre in a railway station has two counters - (i) Self-service – opted by 60% of the customers and (ii) attended service (opted by 40% of the customers). Both counters can serve one person at a time. The arrival rate of the customers is given by the following probability distribution :

<b>No. of arrivals</b>	1	3	4	0	2
<b>Probability</b>	0.10	0.30	0.05	0.20	0.35

Formulate the **associated interval** of 2 digit random numbers for generating (i) type of service and (ii) arrival rate. (CA Final May 2012)

**Answer: Just Make Probability Distribution Series**

**Question 18.**

At a small store of readymade garments, there is one clerk at the counter who is to check bills, receive payments and place the packed garments into fancy bags. The arrival of customer at the store is random and service time varies from one minute to six minutes, the frequency distribution for which is given below :

Time between arrivals (minutes)	Frequency	Service Time (minutes)	Frequency
1	5	1	1
2	20	2	2
3	35	3	4
4	25	4	2
5	10	5	1
6	5	6	0

The store starts work at 11 a.m. and closes at 12 noon for lunch and the customers are served on the “first come first served basis.” Using Monte Carlo simulation technique, find average length of waiting line, average waiting time, average service time and total time spent by a customer in system.

<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>
64	04	02	70	03	60	16	18	36	38	07	08	59	53	01	62	36	27	97	86
30	75	38	24	57	09	12	18	65	25	11	79	61	77	10	16	55	52	59	63

You are given the following set of random numbers, first twenty for arrivals and last twenty for services :

(CA Final Nov. 2009)

**Answer : (i) 2.3customers (ii) 2.8 minutes (iii) 2.7 minutes (iv) 5.5 minutes**

**Question 19.**

A single counter ticket booking centre employs one booking clerk. A passenger on arrival immediately goes to the booking counter for being served if the counter is free. If, on the other hand, the counter is engaged, the passenger will have to wait. The passengers are served on first come first served basis. The time of arrival and the time of service varies from one minute to six minutes. The distribution of arrival and service time is as under :

Arrival/Service Time (minutes)	Arrival (Probability)	Service (Probability)
1	0.05	0.10
2	0.20	0.20
3	0.35	0.40
4	0.25	0.20
5	0.10	0.10
6	0.05	0

(i). Simulate the arrival and service of 10 passengers starting from 9 A.M. by using the following random numbers in pairs respectively for arrival and service. Random numbers 60 09, 16 12, 08 18, 36 65, 38 25, 07 11, 08 79, 59 61, 53 77, 03 10.

(ii). Determine the total duration of :

a. Idle time of booking clerk and

b. Waiting time of passengerRs.

(CA Final Nov. 2008)

**Answer :** (i) **Waiting Time 6Min** (ii) **Idle Time 6 Min**

**Question 20.**

The Everalert Ltd. which has a satisfactory preventive maintenance system in its plant, has installed a new Hot Air Generator based on electricity instead of fuel oil for drying the finished products. The Hot Air Generator requires periodic shutdown maintenance. If the shutdown is scheduled yearly, the cost of maintenance will be as under :

Maintenance Cost	Rs. 15,000	Rs. 20,000	Rs. 25,000
Probability	0.30	0.40	0.30

The costs are expected to be almost linear i.e. if the shutdown is scheduled twice per year, the maintenance cost will be double.

The probability distribution of breakdown cost is estimated as under :

Breakdown costs per annum	Rs. 75,000	Rs. 80,000	Rs. 1,00,000
Shutdown once a year	0.20	0.50	0.30

<b>Shutdown twice a year</b>	<b>0.50</b>	<b>0.30</b>	<b>0.20</b>
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Stimulate the total costs – maintenance and breakdown – and recommend whether the shutdown should be resorted once or twice a year.

**Random numbers :**

Maintenance costs (shut down once a year)	27, 44, 22, 32, 97
Maintenance costs (shut down twice a year)	42, 04, 82, 38, 91
Breakdown costs (shut down once a year)	03, 50, 73, 87, 59
Breakdown costs (shut down twice a year)	54, 65, 49, 03, 56

(ICWA Stage IV Dec. 1995)

**Answer : (i) 530000 (ii) 600000**

**Question 21.**

An international tourist company deals with the numerous personal calls each day and prides itself on its level of service. The time to deal with each caller depends on the client’s requirements which range from, say, a request for a brochure to booking a round-the-world cruise. If a client has to wait for more than 10 minutes for attention, it is company’s policy for the manager to see him personally and to give him a holiday voucher worth Rs. 15.

The company’s observations have shown that the time taken to deal with the clients and arrival pattern of their calls follow the following distribution pattern :

<b>Time to deal with clients</b>	<b>Minutes</b>	<b>2</b>	<b>4</b>	<b>6</b>	<b>10</b>	<b>14</b>	<b>20</b>	<b>30</b>
	<b>Probability</b>	<b>0.05</b>	<b>0.10</b>	<b>0.15</b>	<b>0.30</b>	<b>0.25</b>	<b>0.10</b>	<b>0.05</b>
<b>Time between call arrivals</b>	<b>Minutes</b>	<b>1</b>		<b>8</b>		<b>15</b>		<b>25</b>
	<b>Probability</b>	<b>0.2</b>		<b>0.4</b>		<b>0.3</b>		<b>0.1</b>

Required :

- (i). Describe how you would stimulate the operation of the travel agency based on the use of random number tables.
- (ii). Simulate the arrival and serving of 12 clients and show the number of clients who receive a voucher (Use line 1 of the random numbers below to derive the arrival pattern and line 2 for the serving times); and
- (iii). Calculate the weekly cost of vouchers, assuming the proportion of clients receiving vouchers derived from (ii) applies throughout a week of 75 operating hours.

Random numbers :

<b>Line 1</b>	03	47	43	73	86	36	96	47	36	61	46	98
<b>Line 2</b>	63	71	62	33	26	16	80	45	60	11	14	10

**Answer: (i) 2 customers (ii) Rs. 914.63**

**Question 22**

XYZ Co. Ltd. evaluates the investment proposals on the basis of three factors : demand, profit per unit and required amount of investment. The data for a proposal under the consideration of its Board are given below :

<b>Annual demand (Kgs)</b>	20,000	30,000	40,000	45,000	50,000	55,000	70,000
<b>Probability</b>	0.10	0.20	0.14	0.16	0.10	0.25	0.05

<b>Profit per Kg. (Rs.)</b>	10	20	30	40	50	60	70
<b>Probability</b>	0.25	0.15	0.10	0.20	0.15	0.10	0.05

<b>Investment (Rs.)</b>	20,00,000	30,00,000	50,00,000
<b>Probability</b>	0.30	0.40	0.30

Using simulation process, repeat the trial 10 times, compute the ROI for each trial and the average ROI. Use the following random numbers :

<b>Demand</b>	67, 63, 39, 55, 29, 78, 70, 06, 78, 76
<b>Profit per unit</b>	28, 57, 60, 17, 64, 20, 27, 58, 61, 30
<b>Investment</b>	76, 78, 06, 70, 78, 29, 55, 39, 63, 67

(Similar to CA Final May, 2001)

**Answer: 37.384%**





Chapter – 2

**Assignment Problems**

**Question 1.**

An Accounts officer has 4 subordinates and 4 tasks. The subordinates differ in efficiency. The tasks also differ in their intrinsic difficulty. His estimates of the time each would take to perform each task is given in the matrix below. How should the tasks be allocated one to one man, so that the total man hours are minimized ?

Subordinates	Tasks			
	I	II	III	IV
1	8	26	17	11
2	13	28	4	26
3	38	19	18	15
4	19	26	24	10

Let us apply the above steps take the above example.

**Source : Practice Manual**

**Answer :**

Subordinates	1	2	3	4
Tasks	I	III	II	IV

**Question 2.**

A manager has 5 jobs to be done. The following matrix shows the time taken by the j-th job (j = 1, 2 ... 5) on the i-th machine (i = I, II, III .....V). Assign 5 jobs to the 5 machines so that the total time taken is minimized.

Machines	Jobs				
	1	2	3	4	5
I	9	3	4	2	10
II	12	10	8	11	9
III	11	2	9	0	8
IV	8	0	10	2	1
V	7	5	6	2	9

**Source : Practice Manual**

**Answer :**

Machines	I	II	III	IV	V
Jobs	2	3	4	5	1

**Question 3.**

5 salesman are to be assigned to 5 districts. Estimates of sales revenue in thousands of rupees for each salesman are given below.

	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment pattern that maximizes the sales revenue.

**Source : Practice Manual**

**Answer:**

District	1	2	3	4	5
Salesmen	B	A	E	C	D

**Question 4.**

A firm produces four products. There are four operators who are capable of producing any of these four products. The processing time varies from operator to operator. The firm records 8 hours a day and allow 30 minutes for lunch. The processing time in minutes and the profit for each of the products are given below :

Operators	Products			
	A	B	C	D
1	15	9	10	6
2	10	6	9	6
3	25	15	15	9
4	15	9	10	10
Profit (Rs.) Per Unit	8	6	5	4

Find the optimal assignment of products to operators.

**Source : Practice Manual**

**Answer:**

<b>Operators</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>Products</b>	<b>D</b>	<b>B</b>	<b>C</b>	<b>A</b>
<b>Profit</b>	<b>300</b>	<b>450</b>	<b>150</b>	<b>240</b>

**Question 5.**

A manufacturing company has four zones A, B, C, D and four sales engineers P, Q, R, S respectively for assignment. Since the zones are not equally rich in sales potential, therefore it is estimated that a particular engineer operating in a particular zone will bring the following sales :

Zone A	:	4,20,000
Zone B	:	3,36,000
Zone C	:	2,94,000
Zone D	:	4,62,000

The engineers are having different sales ability. Working under the same conditions, their yearly sales are proportional to 14, 9, 11 and 8 respectively. The criteria of maximum expected total sales is to be met by assigning the best engineer to the richest zone, the next best to the second richest zone and so on.

Find the optimum assignment and the maximum sales.

**Source : Practice Manual**

**Answer :**

<b>Engineers</b>	<b>Zones</b>	<b>Sales (Rs.)</b>
<b>P</b>	<b>D</b>	<b>154000</b>
<b>Q</b>	<b>B</b>	<b>72000</b>
<b>R</b>	<b>A</b>	<b>110000</b>
<b>S</b>	<b>C</b>	<b>56000</b>
		<b>392000</b>

**Question 6.**

ABC company is engaged in manufacturing 5 Brands of packet snacks. It is having five manufacturing setups, each capable of manufacturing any of its brands, one at a time. The cost to make a brand on these setups vary according to following table :

	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>
B <sub>1</sub>	4	6	7	5	11
B <sub>2</sub>	7	3	6	9	5
B <sub>3</sub>	8	5	4	6	9
B <sub>4</sub>	9	12	7	11	10
B <sub>5</sub>	7	5	9	8	11

**Required**

Assuming five setups are S<sub>1</sub> S<sub>2</sub> S<sub>3</sub> S<sub>4</sub> and S<sub>5</sub> and five brands are B<sub>1</sub> B<sub>2</sub> B<sub>3</sub> B<sub>4</sub> and B<sub>5</sub>. Find the optimum assignment of the products on these setups resulting in the minimum cost.

**Source : Practice Manual**

**Answer:**

Brand	Setup	Cost
B <sub>1</sub>	S <sub>1</sub>	4
B <sub>2</sub>	S <sub>5</sub>	5
B <sub>3</sub>	S <sub>4</sub>	6
B <sub>4</sub>	S <sub>3</sub>	7
B <sub>5</sub>	S <sub>2</sub>	5
	<b>Total</b>	<b>27</b>

**Question 7.**

A factory is going to modify of a plant layout to install four new machines M<sub>1</sub> M<sub>2</sub> M<sub>3</sub> and M<sub>4</sub> there are 5 vacant places J, K, L, M and N available. Because of limited space machine M<sub>2</sub> cannot be placed at L and M<sub>3</sub> cannot be placed at J. The cost of locating machine to place in Rupees is shown below :

(Rs.)

	J	K	L	M	N
M <sub>1</sub>	18	22	30	20	22
M <sub>2</sub>	24	18	--	20	18
M <sub>3</sub>	--	22	28	22	14
M <sub>4</sub>	28	16	24	14	16

**Required**

Determine the optimal assignment schedule in such a manner that the total costs are kept at a minimum.

**Source : Practice Manual**

Answer:

Machines	Location	Costs
M <sub>1</sub>	J	18
M <sub>2</sub>	K	18
M <sub>3</sub>	N	14
M <sub>4</sub>	M	14
M <sub>5</sub> (dummy)	L	0
	<b>Total</b>	<b>64</b>

**Question 8.**

Imagine yourself to be the Executive Director of a 5-Star Hotel which has four banquet halls that can be used for all functions including weddings. The halls were all about the same size and the facilities in each hall differed. During a heavy marriage season, 4 parties approached you to reserve a hall for the marriage to be celebrated on the same day. These marriage parties were told that the first choice among these 4 halls would cost Rs. 25,000 for the day. They were also required to indicate the second, third and fourth preferences and the price that they would be willing to pay. Marriage party A & D indicated that they won't be interested in Halls 3 & 4. Other particulars are given in the following table :

**Revenue / Hall**

Marriage Party	Hall 1	Hall 2	Hall 3	Hall 4
A	Rs. 25,000	Rs. 22,500	X	X
B	Rs. 20,000	Rs. 25,000	Rs. 20,000	Rs. 12,500
C	Rs. 17,500	Rs. 25,000	Rs. 15,000	Rs. 20,000
D	Rs. 25,000	Rs. 20,000	X	X

Where X indicates that the party does not want that hall.

**Required**

Decide on an allocation that will maximize the revenue to your hotel.

Source : Practice Manual (RTP : Nov. 13)

Answer :

Marriage Party	Hall	Revenue (Rs.)
A	2	22500
B	3	20000
C	4	20000
D	1	25000
	<b>Total</b>	<b>87500</b>

**Question 9.**

A salesman has to visit five cities. He wishes to start from a particular city. Visit each city once and then return to his starting point. Cost (in Rs. '000) of travelling from one city to another is given below :

	P	Q	R	S	T
P	-	5	14	20	2
Q	17	-	8	23	5
R	23	20	-	11	20
S	35	11	17	-	14
T	2	8	5	23	-

**Required**

Find out the 'Least Cost Route'.

Source : Practice Manual (RTP : Nov. 2015)

Answer:

**P to Q to R to S to T to P with a total cost is Rs. 40000**

**Question 10.**

Methods I, II, III and IV are available for one-to-one assignment to factories A, B and C. The time taken (in hours) for implementing these methods in the factories is tabulated below with the objective of minimization.

↓ Methods	Factories →	(Time Taken - hours)		
		A	B	C
I		35	25	28
II		23	32	25
III		25	42	21
IV		35	35	28

**Required**

- (i) Show the optimal assignment by circling the cells using the assignment algorithm (description of algorithm is not required). Which method will not be implemented?
- (ii) What is the minimum savings (in hours) required over the current given duration, for preferring the implementation of the method identified in (i) above? When it so justifies, which method will it replace? Why?

**Source : Practice Manual**

**Answer:**

(i)

Method	Factory	Time Taken (hrs )
I	B	25
II	A	23
III	C	21
	<b>Total</b>	<b>69</b>

(iii) **Minimum savings 7 Hours**

**Question 11**

Answer the following independent situations relating to an assignment problem with a minimization objective.

- (i) Just after row and column minimum operations, we find that a particular row has 2 zeroes. Does this imply that the 2 corresponding numbers in the original matrix before any operation were equal? Why?
- (ii) Under the usual notation, where  $a_{32}$  means the element at the intersection of the 3rd row and 2nd column, we have, in a 4 X 4 assignment. What can you conclude about the remaining assignments? Why?

**Source : Practice Manual (Nov. 2013)**

**Question 12.**

$R_3C_2$  denotes the element at the intersection of the third row and 2nd column. Under this notation,  $R_1C_1, R_2C_1, R_3C_1, R_3C_2, R_3C_3, R_4C_3, R_4C_4$  where the only zero elements in a 4 X 4 minimization assignment problem after the row minimum and column minimum operations.

- (i) In the next step to draw lines to cover zeroes, a student drew 4 horizontal lines covering rows  $R_1, R_2, R_3, R_4$ . Will he arrive at the optimal assignment at the next step? Why? Explain the concept.

- (ii) Independent of (i), if you are given the additional information that  $R_2C_2$  element is lesser than the Row 1 and Row 2 non-zero values, how will you arrive at the optimal solution?

**Source : Practice Manual**

**Question 13.**

The ICAI decides to hold special seminars on four contemporary topics for its members: Ind. AS, Goods and Service Tax (GST), Negative list in Service tax and Direct Tax Code (DTC). Such seminars should be held once in a week in the afternoons. However, scheduling these seminars (one for each topic, and not more than one seminar per afternoon) has to be done carefully so that the number of members unable to attend is kept to a minimum. A careful study indicates that the number of members who cannot attend a particular seminar on a specific day is as follows :

	<b>Ind. AS</b>	<b>GST</b>	<b>Negative List</b>	<b>DTC</b>
Monday	40	30	50	20
Tuesday	30	20	30	30
Wednesday	50	10	20	20
Thursday	20	30	10	30
Friday	10	20	10	30

Find an optimal schedule of the seminars. Also find out the total number of members who will be missing at least one seminar.

**Source :Practice Manual and RTP Nov. 2012**

**Question 14.**

Can there be (i) more than one dummy row or column or (ii) one dummy row and a dummy column in a given problem of (a) assignment (b) transportation? Why? (In other words, state whether and why each of situations A, B, C and D is possible or not) :

	<b>Assignment</b>	<b>Transportation</b>
More than one dummy row or column	A	B
One dummy row and one dummy column	C	D

**Source : Practice Manual (May, 2014)**

**Question 15**

‘Air-Pacific’ airways operating 7 days a week has given the following time-table. Crews must have a minimum layover of 5 hours between flights. Obtain the pairing flights and minimize layover time away from home. For any given pairings the crew will be based at the city that results in the smaller layover :



Kolkata-Bangkok			Bangkok-Kolkata		
Flight Number	Depart.	Arrive	Flight Number	Depart.	Arrive
KX1	05:15 AM	07:15 AM	BX1	07:15 AM	09:15 AM
KX2	07:15 AM	09:15 AM	BX2	08:15 AM	10:15 AM
KX3	01:15 PM	03:15 AM	BX3	01:15 PM	03:15 PM
KX4	07:15 PM	09:15 AM	BX4	06:15 PM	08:15 PM

Source : RTP May, 2013

**Question 16.**

A solicitors firm employs typists on hourly piece-rate basis for their daily work. There are five typists for service and their charges and speeds are different. According to an earlier understanding only one job is given to one typist and the typist is paid for full hours even if he works for a fraction of an hour. Find the least cost allocation for the following data :

<table border="1"> <thead> <tr> <th>Typist</th> <th>Rate per hour (Rs.)</th> <th>No. of pages typed/hour</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>5</td> <td>12</td> </tr> <tr> <td>B</td> <td>6</td> <td>14</td> </tr> <tr> <td>C</td> <td>3</td> <td>8</td> </tr> <tr> <td>D</td> <td>4</td> <td>10</td> </tr> <tr> <td>E</td> <td>4</td> <td>11</td> </tr> </tbody> </table>	Typist	Rate per hour (Rs.)	No. of pages typed/hour	A	5	12	B	6	14	C	3	8	D	4	10	E	4	11	<table border="1"> <thead> <tr> <th>Jobs</th> <th>P</th> <th>Q</th> <th>R</th> <th>S</th> <th>T</th> </tr> </thead> <tbody> <tr> <td>No. of pages</td> <td>199</td> <td>175</td> <td>145</td> <td>298</td> <td>178</td> </tr> </tbody> </table>	Jobs	P	Q	R	S	T	No. of pages	199	175	145	298	178
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(CA Final Nov. 1988)

**Question 17.**

Five swimmers are eligible to compete in a relay team which is to consist of four swimmers swimming four different swimming styles : back stroke, breast stroke, free style and butterfly. The time taken by the five swimmers Anand, Bhaskar, Chandru, Dorai and Easwar – to cover a distance of 100 meters in various swimming styles are given below in minutes : seconds, Anand swims the back stroke in 1:09, breast stroke in 1:15 and has never competed in free style or butterfly. Bhaskar is a free style specialist averaging 1.01 for the 100 meters but can also swim breast stroke in 1:16 and butterfly in 1:20. Chandru swims all four styles – back 1:10; butterfly 1:12, free style 1:05 and breast stroke 1:20, Dorai swims only the butterfly 1:11 while Easwar

swims back stroke 1:20, the breast stroke 1:16, the free style 1:06 and the butterfly 1:10. Which swimmers should be assigned to which swimming style? Who will not be in relay?

(CA Final Nov. 1991)

**Answer :**

Swimmer	Swimming Style	Time (seconds )
Anand	Breast Stroke	75
Bhaskar	Free Style	61
Chandru	Back Stroke	70
Dorai	Dummy	-
Eashwer	Butterfly	70
	<b>Total Minimum Time in Relay</b>	<b>276</b>

**Dorai will not participate in the relay.**

**Question 18.**

Well-done Company has taken the third floor of a multistoried building for rent with a view to locate one of their zonal offices. There are five main rooms in this floor to be assigned to five managers. Each room has its own advantages and disadvantages. Some have windows; some are closer to the washrooms or to the canteen or secretarial pool. The rooms are of different sizes and shapes. Each of the five managers was asked to rank their room preferences among the rooms 301, 302, 303, 304 and 305. Their preferences were recorded in a table as indicated below :

Manager				
M1	M2	M3	M4	M5
302	302	303	302	301
303	304	301	305	302
304	305	304	304	304
	301	305	303	
		302		

Most of the managers did not list all the five rooms since they were not satisfied with some of these rooms and they have left off these from the list. Assuming that their preferences can be quantified by

numbers, find out as to which manager should be assigned to which rooms so that their total preferences ranking is a minimum.

Answer :

Manager	Rooms	Rank
M1	302	1
M2	304	2
M3	303	1
M4	305	2
M5	301	1
	Total	7

**Question 19.**

The captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows :

Batsman	Batting Positions				
	I	II	III	IV	V
P	40	40	35	25	50
Q	42	30	16	25	27
R	50	48	40	60	50
S	20	19	20	18	25
T	58	60	59	55	53

- (i). Find the assignment of batsmen to positions, which would give the maximum number of runs.

- (ii). If another batsman ‘U’ with the following average runs in batting positions as below is added to the team, should he be included to play in the team? If so, who will be replaced by him?

Batting position	I	II	III	IV	V
Average runs	45	52	38	50	49

(CA Final May, 1992)

**Answer:**

(i)

Batsman	Position	Runs
P	V	50
Q	I	42
R	IV	60
S	III	20
T	II	60
<b>Total</b>		<b>232</b>

(ii)

Batsman	Position	Runs
P	V	50
Q	I	42
R	IV	60
S	<b>DUMMY</b>	-
T	III	59
U	II	52
	<b>TOTAL</b>	<b>263</b>

**Question 20.**

A city corporation has decided to carry out repairs on 4 main roads in the city. The government has agreed to make a special grant of Rs. 50 Lakhs towards the cost with the condition that the repairs should be carried out at the lowest cost. Five contractors have sent the bids. Only one road will be awarded to one contractor. The bids are given below :

		Cost of repairs (Rs. Lakhs)			
	Road Rs.	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>
Contractors Rs.	C <sub>1</sub>	9	14	19	15
	C <sub>2</sub>	7	17	20	19
	C <sub>3</sub>	9	18	21	18
	C <sub>4</sub>	10	12	18	19
	C <sub>5</sub>	10	15	21	16

You are informed that C2 should get R1 and C4 should get R2 to minimize the costs.

- (i). What is the minimum cost allocation?
- (ii). How much is the minimum discount that the eliminated contractor should offer for meriting a contract?
- (iii). Independent of (ii) above, if the corporation can negotiate to get a uniform discount rate from each contractor, what is the minimum rate of discount so that the cost is within the grant amount?

**(CA Final Nov. 2011)**

**Answer:**

- (i) 54 lacs      (ii) 2 ,2 ,6 and 2 lacs respectively for R<sub>3</sub> , R<sub>1</sub> , R<sub>2</sub> and R<sub>4</sub>      (iii)7.41%**

**Question 21.**

A manager was asked to assign tasks to operators (one task per operator) so as to minimize the time taken. He was given the matrix showing the hours taken by the operators for the tasks. First he performed the row minimum operation. Secondly, he did the column minimum operation. Then, he realized that there were 4 tasks and 5 operators. At the third step he introduced the dummy row and continued with the fourth step of drawing lines to cover zeros. He drew 2 vertical lines (under operator III and operator IV) and two horizontal lines (Inside task T<sub>4</sub> and dummy Task T<sub>5</sub>). At step 5 he performed the necessary operation with the uncovered element, since the number of the lines was less than the order of matrix. After this, his matrix appeared as follows :

Tasks	Operators				
	I	II	III	IV	V
T1	4	2	5	0	0
T2	6	3	3	0	3
T3	4	0	0	0	1
T4	0	0	5	3	0
T5 (Dummy)	0	0	3	3	0

- (i). What was the matrix after step II?
  - (ii). What was the most difficult task for operators I, II and V?
  - (iii). Who is the most efficient operator?
  - (iv). If you are not told anything about the manager’s errors, which operator would be denied any task? Why?
- (CA Final May 2011)**

**Quesiton 22.**

A hospital has to pay nurses for 40 hours a week. One nurse is assigned to one patient. The cost per hour for each of the nurses is given below :

Patient RS.	W	X	Y
Nurses ↓			
K	10	10	30
L	30	10	20
M	20	30	20

Suppose that a new patient Z is admitted and that a new nurse is appointed. The new patient is charged Rs. 40 per hour by each of the existing nurses. The new nurse charges Rs. 50 per hour irrespective of the patient.

- (i). Find the nurse patient combination to minimize cost to the hospital.
  - (ii). How much does each nurse earn per week?
  - (iii). What would be your revised calculations?
  - (iv). Comment on the new solution.
- (CA Final May 2010)**

Answer : (i) ,(ii)

Nurse	Patient	Cost / Earnings
K	W	400
L	X	400
M	Y	800
	<b>Total</b>	<b>1600</b>

(iii)

Nurse	Patient	Cost / Earnings
K	W	400
L	X	400
M	Y	800
New	Z	2000
	<b>Total</b>	<b>3600</b>

(iv)

The new nurse has been assigned to the new patient so the allocation of old nurses among the old patients will not change and therefore the cost of old patients and earning of old nurses will also not change.

### Question 23.

The following matrix was obtained after performing row minimum operation on rows R<sub>1</sub> and R<sub>2</sub> in an assignment problem for minimization. Entries “xx” represent some positive numbers. (It is not meant that all “xx” numbers are equal). State two circumstances under which an optimal solution is obtained just after the row minimum and column minimum operations.

Candidates may use cell references as C<sub>1</sub> R<sub>1</sub> for uniformity, e.g. C<sub>1</sub> R<sub>1</sub> represents the cell at the intersection column<sub>1</sub> (C<sub>1</sub>) and Row<sub>1</sub> (R<sub>1</sub>) etc.

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>
R <sub>1</sub>	0	xx	xx
R <sub>2</sub>	Xx	0	xx
R <sub>3</sub>	Xx	xx	xx

(CA Final May, 2012)





CHAPTER 3**TRANSPORTATION PROBLEMS****Question 1**

Cost per Unit (Rs.)

	Warehouse 1	Warehouse 2	Warehouse 3	Total Supplies
Factory 1	3	4	2	30
Factory 2	2	1	5	25
Factory 3	4	3	3	20
Total Demand	20	20	35	75

Find the initial solution by least cost method.

**Question 2**

Cost per Unit (Rs.)

	Store 1	Store 2	Store 3	Total Supplies
Plant 1	49	60	56	14
Plant 2	45	55	53	26
Plant 3	50	80	70	36
Plant 4	52	64	55	22
Total Demand	20	32	25	

Find the initial solution by least cost method. Is the initial solution feasible?

**Question 3.** Cost per Unit (Rs.)

	Store 1	Store 2	Store 3	Store 4	Total Supplies
Source 1	48	60	56	58	14
Source 2	45	55	53	60	26
Source 3	50	65	60	62	36

<b>Total Demand</b>	20	32	25	21	
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Find the initial solution by least cost method. Is the initial solution feasible?

**Question 4.**

Following is the profit matrix based on four factories and three sales depots of a company :

	Sales Depots			Availability
	S1	S2	S3	
<b>Factory 1</b>	6	6	1	10
<b>Factory 2</b>	-2	-2	-4	190
<b>Factory 3</b>	3	2	2	50
<b>Factory 4</b>	8	5	3	100
<b>Demand</b>	80	120	150	

Find the initial solution by Vogel’s Approximation method to maximize the profit. Is the initial solution feasible?

**Answer : 480**

**Question 5.**

Origin	Profit (Rs./Unit)				Supply
	Destinations				
	1	2	3	4	
<b>A</b>	40	25	22	33	100
<b>B</b>	44	35	30	30	30
<b>C</b>	38	38	28	30	70
<b>Demand</b>	40	20	60	30	

Find the initial solution by Vogel’s Approximation method to maximize the profit. Is the initial solution feasible?

**Answer : 5130**

**Question 6.**

A compressed Natural Gas company has three plants producing gas and four outlets. The cost of transporting gas from the plants to the outlets, production capacity of each plant and requirement at different outlets is shown in the following cost matrix table :

Plants	Outlets				Capacity of production
	A	B	C	D	
X	4	6	8	6	700
Y	3	5	2	5	400
Z	3	9	6	5	600
Requirements	400	450	350	500	1700

Determine a transportation schedule so that the cost is minimized. The cost in the cost matrix is in thousands of rupees.

**(CA Final Nov. 2001)**

**Answer : Rs. 7350**

**Question 7.**

A product is manufactured by four factories A, B, C and D. The unit production costs are Rs. 2, Rs. 3, Rs. 1 and Rs. 5 respectively. The daily production capacities are 50, 70, 30 and 50 units respectively. These factories supply the product to four stores P, Q, R and S. the demands made by these stores are 25, 35, 105 and 20 units respectively. Unit transportation cost in rupees from each factory to each store is given in the following table :

Factories	Stores			
	P	Q	R	S
A	2	4	6	11
B	10	8	7	5
C	13	3	9	12
D	4	6	8	3

Determine the extent of deliveries from each of the factories to each of the stores so that the total cost (Production and transportation cost together) is minimum.

**(CA Final May, 2002)**

**Answer : Rs.1465**

**Question 8.**

Consider the following data for the transportation problem :

Factories	Destination			Supply to be exhausted
	1	2	3	
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Demand	75	20	50	

Since there is not sufficient supply, some of the demands at the three destinations may not be satisfied. For the unsatisfied demands, let the penalty cost be Rs. 1, Rs. 2 and Rs. 3 for the destinations (1), (2) and (3) respectively. Find the optimal allocation that minimizes the transportation and penalty costs.

**(CA Final Nov. 1997)**

**Answer: Rs. 555**

**Question 9.**

A company has three warehouses W1, W2 and W3. It is required to deliver a product from these warehouses to three customers A, B and C. The warehouses have the following units in stock :

Warehouse	W1	W2	W3
No. of units	65	42	43

The customer requirements are :

Customer	A	B	C
No. of units	70	30	50

The table below shows the cost of transporting one unit from warehouse to the customer :

Warehouse			
Customer	W1	W2	W3
A	5	7	8
B	4	4	6
C	6	7	7

**Answer :** Rs.830

**Question 10.**

A company produces a small component for all industrial products and distributes it to five wholesalers at a fixed price of Rs. 2.50 per unit. Sales forecasts indicate that monthly deliveries will be 3000, 3000, 10000, 5000 and 4000 units to wholesalers 1, 2, 3, 4 and 5 respectively. The monthly production capability are 5000, 10000 and 12500 at plants 1, 2 and 3 respectively. The direct costs of production for each item are Rs. 1.00, 0.90 and 0.80 at plants 1, 2 and 3 respectively. The transportation costs (Rs.) of shipping a unit from a plant to wholesaler are given below :

	Wholesalers				
	1	2	3	4	5
Plant 1	0.05	0.07	0.10	0.15	0.15
Plant 2	0.08	0.06	0.09	0.12	0.14
Plant 3	0.10	0.09	0.08	0.10	0.15

Find how many components each plant supplies to each wholesaler in order to maximize profit.

**(CA Final May, 2000)**

**Answer:** Rs.38770

**Question 11.**

A company has 3 plants and 3 warehouses. The cost of sending a unit from different plants to the warehouses, production at different plants and demand at different warehouses are shown in the following cost matrix table :

Plant	Warehouses			Production
	A	B	C	
X	8	16	16	152
Y	32	48	32	164
Z	16	32	48	154
Demand	144	204	82	

- (i). Determine a transportation schedule, so that the cost is minimized. Assume that the cost in the cost matrix is given in thousands of rupees.

- (ii). How your Answer will change if the transportation cost given above is reduced by 50%?

(CA Final May 2001)

Answer: (i) Rs.9696 (ii) No change in allocation but cost will become Rs.4848

**Question 12.**

The cost conscious company requires for the next month 300, 260 and 180 tonnes of stone chips for its three constructions C1, C2 and C3.

Stone chips are produced by the company at three mineral fields taken on short lease by the company. All the available boulders must be crushed into chips. Any excess chips over the demand at sites C1, C2 and C3 will be sold ex-fields. The fields are M1, M2 and M3 which will yield 250, 320 and 280 tonnes of stone chips respectively.

Transportation costs from mineral fields to construction sites vary according to distances, which are given below in monetary units :

To	C1	C2	C3
From			
M1	8	7	6
M2	5	4	9
M3	7	5	5

Determine the optimal economic transportation plan for the company and the overall transportation cost. What the quantities to be sold from M1, M2 and M3?

(ICWA Final Dec. 1997)

Answer: Rs. 3830

**Question 13.**

Stronghold Construction Company is interested in taking loans from banks for some of its projects P, Q, R, S and T. The rates of interest and the lending capacity differ from bank to bank. All these projects are to be completed. The relevant details are provided in the following table. Advise the company so that its cost of interest is minimum.

Bank	Interest rate in percentage for projects					Max. credit (Rs. '000)
	P	Q	R	S	T	
Private	20	18	18	17	17	Any amount
Nationalized	16	16	16	15	16	400
Co-operative	15	15	15	13	14	250
<b>Amount Required (Rs. '000)</b>	200	150	200	125	75	

(CA Final Nov. 1990)

Answer :

Source	Projects	Credit (Rs. '000)	Cost (%)	Total Cost
P	R	100	18	18000
N	P	200	16	32000
N	Q	150	16	24000
N	R	50	16	8000
C	R	50	15	7500
C	S	125	13	16250
C	T	75	14	10500
COST				Rs. 1,16,250

**Question 14.**

XYZ & Co. Has provided the following data seeking your advice on optimum investment strategy:

Investments made in the beginning of the year	Net return on selected investments (paisa)				Amount Available (Rs. In Lacs)
	P	Q	R	S	
1	95	80	70	60	70
2	75	65	60	50	40

3	70	45	50	40	90
4	60	40	40	30	30
Maximum investment (Rs.)	40	50	60	60	

The following additional information is also provided :

- ☉ P, Q, R and S represent the selected investments.
- ☉ The company has decided to have four years investment plan.
- ☉ The policy of the company is that amount invested in any year will remain so until the end of the fourth year.
- ☉ The values (Paisa) in the table represent net return on investment of one rupee till the end of the planning horizon (for example, a Rupee invested in investment P at the beginning of year I will grow to Rs. 1.95 by the end of fourth year, yielding a return of 95 paisa)
- ☉ Using the above data, determine the optimum investment strategy.

(CA Final Nov. 1996)

**Answer: Total Return Rs. 130**

**Question 15.**

A particular product is manufactured in factories A, B, C and D and sold at centres 1, 2 and 3. The cost in Rs. of product per unit and capacity in Kilograms per unit of time of each plant is given below.

Factory	Cost (Rs.) per unit	Capacity (Kgms) per unit of time
A	12	100
B	15	20
C	11	60
D	13	80

The sale price in Rs. per unit and the demand in kgms per unit of time are as follows :



Sales centre	Sales prices (Rs.) per unit	Demand (Kgs) per unit of time
1	15	120
2	14	140
3	16	60

Find the Optimal sales distribution.

(CA Final Nov. 1997)

**Answer: Rs.680**

**Question 16.**

A particular product is manufactured in factories A, B and C and sold at centres 1, 2, 3 and 4. Transportation cost per unit is given in the following matrix.

Factories ↓	Centres ↓				Total Supply
	1	2	3	4	
A	13	11	15	20	2
B	17	14	12	13	6
C	18	18	15	12	7
<b>Requirement</b>	3	3	4	5	15

Solve the transportation problem for minimum cost.

**Answer: Rs. 127**

**Question 17.**

The following table gives the unit transportation costs and the quantities demanded/supplied at different locations to a minimization problem :

Demand Supply	C1	C2	C3	C4	Total Units
R1	100	120	200	110	20000
R2	160	80	140	120	38000
R3	180	140	60	100	16000
<b>Total Units</b>	10000	18000	22000	24000	74000

You are required to find out which cell gets the 3<sup>rd</sup> allocation in the initial basic feasible solution under each of the following methods and given the cell reference, cost per unit of that and the quantity allocated to that cell :

- (i). North West Corner Rule
- (ii). Vogel’s approximation Method
- (iii). Least cost method

(The full solution is not required)

**(CA Final May 2012)**

**Answer:** (i)  $R_2C_2$  (ii)  $R_2C_2$  (iii)  $R_1C_1$

**Question 18.**

A company has three factories F1, F2 and F3 which supply the same product to 5 agencies A1, A2, A3, A4 and A5. Unit production costs, shipping costs and selling prices are given below :

	<b>F1</b>	<b>F2</b>	<b>F3</b>
<b>Production Cost/Unit (Rs.)</b>	28	35	29
<b>Production Capacity (Units)</b>	110	240	125

<b>Agencies</b>	<b>A1</b>	<b>A2</b>	<b>A3</b>	<b>A4</b>	<b>A5</b>
<b>Selling price (Rs./Unit)</b>	40	48	42	45	41
<b>Demand (No. of units)</b>	80	100	75	45	125

Shipping cost (Rs./unit)

	<b>A1</b>	<b>A2</b>	<b>A3</b>	<b>A4</b>	<b>A5</b>
<b>F1</b>	3	9	8	12	8
<b>F2</b>	6	10	6	2	5
<b>F3</b>	3	10	3	6	8

- (i). Set up the initial transportation matrix for minimization.
- (ii). After doing (i) above, you are given the following additional information :

- a. 40 units must be transported from F2 to A2. This figure is included in the figures given for total production and demand at these locations.
- b. Not more than 30 units may be sent from F1 to A1, since the transporter’s vehicle lacks space on this route.

Incorporation conditions (a) and (b) above, obtain the initial solution by Vogel’s Approximation Method. (Do not attempt to continue for the full and final solution).

- (iii) After doing the initial solutions (a) and (b) above, you are informed that the route from F2 to A1 is blocked by sudden flooding of the roads.

Without actual re-calculation, briefly explain how your solution is likely to be affected.

**(CA Final Nov. 2010)**

**Question 19.**

The following matrix is a minimization problem for transportation costs. The unit transportation costs are right at the right hand corners of the cells and the  $\Delta_{ij}$  values are encircled. Find the optimal solution(s) and the minimum costs.

	D1	D2	D3	Supply
F1	3	4	4	500
F2	8 9	6	2 7	300
F3	0 4	2 6	5	200
Demand	300	400	300	1000

**(CA Final May 2011)**

**Question 20.**

The following table shows all the necessary information on the available supply to each warehouse, the requirement of each market and the unit transportation cost from each warehouse to each market :

To	Markets				Supply
From	I	II	III	IV	
Warehouse A	5	2	4	3	22
Warehouse B	4	8	1	6	15
Warehouse C	4	6	7	5	8
Requirement	7	12	17	9	45

The shipping clerk has worked out the following schedule from his experience :

From	A	A	A	B	C	C
To	II	III	IV	III	I	III
Units	12	1	9	15	7	1

You are required to Answer the following :

- (i). Check and see if the clerk has the optimal schedule.
- (ii). Find the optimal schedule and the minimum total shipping cost.
- (iii). If the clerk is approached by a carrier of route C to II, who offers to reduce his rate in the hope of getting some business, by how much should the rate be reduced before the clerk should consider giving him an order?

**Answer:**                      (i) No                      (ii) 104                      (iii) 103

**Question 21.**

The initial feasible solution of a profit maximization transportation problem is given in the table below :

Factory	Sales Depot 1	Sales Depot 2	Sales Depot 3
F1	2	2 <input type="text" value="10"/>	7
F2	10	10 <input type="text" value="40"/>	12 <input type="text" value="110"/>
F3	5	6 <input type="text" value="50"/>	6
F4	0 <input type="text" value="80"/>	3 <input type="text" value="20"/>	5
Dummy	8	8	8 <input type="text" value="40"/>

- (i). Give the problem to which the given table is initial feasible solution.
- (ii). Find the optimal solution.
- (iii). Is there some alternative solution to the problem?

**(CA Final Nov. 2000 Adapted)**

**Question 22.**

The initial feasible solution of a minimization transportation problem is given below : Find the optimal solution. Is there some alternative optimal solution to the problem ?

Factory	Customers					Supply
	A	B	C	D	Dummy	
P	4 <input type="text" value="10"/>	19	22 <input type="text" value="60"/>	11 <input type="text" value="30"/>	0 <input type="text" value="e"/>	100
Q	0 <input type="text" value="30"/>	9	14	14	0	30
R	6	6 <input type="text" value="20"/>	16	14	0 <input type="text" value="50"/>	70
Demand	40	20	60	30	50	200

**(CA Final May 2003 Adapted)**

**Question 23.**

XYZ Company has three plants and four warehouses. The supply and demand in units and the corresponding transportation costs are given. The table below shows the details taken from the solution procedure of the transportation problem :

	Warehouses				Supply
	I	II	III	IV	
<b>Plant A</b>	5	10	4 10	5	10
<b>Plant B</b>	6 20	8	7	25 5	25
<b>Plant C</b>	4 5	2 10	5 5	7	20
<b>Demand</b>	25	10	15	5	

Answer the following questions.

Give brief reasons :

- (i). Is this solution feasible?
- (ii). Is this solution degenerate?
- (iii). Is this solution optimum?

(CA Final May 2013)

**CHAPTER 4****LEARNING CURVE****Question 1**

You company has been approached by a customer to supply four units of a new product made to the customer's individual specification. The company experiences a 90 per cent learning rate. The estimated labour time for the first unit of this product is 1.50 hours and the company's direct labour cost is Rs. 5 per hour (a) Estimate the labour cost of this order (b) After receiving the first order, if the customer places a repeat order, what will be the labour cost for the second order, (c) If the customer had ordered all eight units at the same time, calculate the labour cost per unit for the combined order.

**Answer :**        (i)        **Rs.24.30**                                (ii)        **Rs. 19.44**                                (iii)        **Rs.5.4675**

**Question 2**

Calculate the expected average unit cost of making.

(a)        4 machines; (b) 8 machines

Using the data below:

Data:

Direct labour needed to make first machine 1,000 hours

Learning curve = 90%

Direct labour cost Rs. 15/per hour

Direct materials cost Rs. 1,50,000 per machine.

Fixed cost for either size orders — Rs. 60,000

**Answer :**        (i)        **177150**                                (ii)        **168435**

**Question 3**

Debashree Ltd. has observed that a 90% learning curve ratio applies to all labour related costs each time a new model enters production. It is anticipated that 320 units of a new model will be manufactured during 1992. Direct labour costs for the first lot of 10 units amount to 1,000 hours at Rs. 8 per hour. Variable overhead cost is Rs. 2 per direct labour hour.

You are required to determine:

1.        Total Labour and labour-related variable cost to manufacture 320 units of output.
2.        Average cost of:-

- (a) The first 40 units produced.
- (b) The first 80 units produced and
- (c) The first 100 units produced

3. Incremental cost of

- (a) Units 41-80
- (b) Units 101-20

**Answer:**

- (i) **1,88,956.80**
- (ii) **Rs. 810 per unit**
- (iii) **Rs. 729 per unit**
- (iv) **Units 41-80 =  $2,592 \times \text{Rs. } 10 = \text{Rs. } 25,92$ , B: Units 101-200 Rs. 56,400**

**Question 4**

XYZ & Co. has given the following data:

80% Average – Time Curve

Cumulative Units (X)	Average Hours	Total Hours	Marginal Hours
1	100	100	100
2	80	160	60
3	?	?	?
4	64	256	?

Required: Fill in the blanks.

**Answer:                      70.2                      210.6                      50.6                      and                      45.4**

**Question 5.**



A customer has asked your company to prepare a bid on supplying 800 units of a new product. Production will be in batches of 100 units. You estimate that costs for the first batch of 100 units will average Rs. 100 a unit. You also expect that a 90% learning curve will apply to the cumulative labour cost on this contract.

**Required:**

- (a) Prepare an estimate of the labour costs of fulfilling this contract.
- (b) Estimate the incremental labour cost of extending the production run to produce an additional 800 units.
- (c) Estimate the incremental labour cost of extending the production run form 800 units to 900 units.

**Answer :**            (a)        **58320**                    (b)        **46656**                    (c)        **6102**

**Question 6.**

Company has accepted an order for making 15 items of a specialized machine at a price of Rs. 4 lacks each. The delivery is to be completed within 4 months. The company works 23 days a monthly and the normal direct wages per day amount to Rs. 10,000. However in case of need, the company can work overtime up to 8 days during the said period at double the normal rate of wages. Overheads amount to Rs. 12,000 per normal working day but no overheads are charged on overtime working days. The material cost is Rs. 2,40,000 per machine. The company has estimated that it will take 10 working days to manufacture the first machine. The company is expected to experience a learning effect of 90% ( $b = 0.152$ ). The contract stipulates a penalty of Rs. 40,000 per machine delivered beyond the schedule of 4 months.

You are required to calculate the costs and advise the company whether it is preferable to work only during normal working days and pay penalty for any delayed delivery of the machines or to work overtime to avoid paying penalty.

**Answer:**            **It is preferable to work overtime and not to pay penalty Net benefit is Rs. 96000.**

**Question 7**

Cosy comforts Ltd. makes household appliances. It is now examining a three-year old contract to make electrical bread toasters for sale through a departmental store. During the entire contract period, it will receive for its toaster a fixed price of Rs. 40 per piece for whatever quantity it can produce in the 3 yearRs. Skilled labour is the constraint and this cannot be increased above that currently available in the Company for making the toaster.

Capital investment required Rs. 50,000 payable down cash with nil scrap value.

Additional overhead	Rs. 25,000 per annum
Material	Rs. 30 per toaster

Labour (Skilled)	Rs. 5 per hour.
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The production manager envisages a learning curve effect for labour in the form of  $y = ax^{-0.3}$  where  $y$  = average labour hours per unit,  $a$  = labour hours per first unit and  $x$  = cumulative production. He estimates that the first toaster will take 10 hours to produce and the fixed amount of skilled labour available will enable 5,000 toasters to be produced in the first year. Assume all cash inflows to arise at year end and the cost of capital is 15%. What is your advice?

**Answer:**

**NPV 25,770 . NPV is positive, Hence Project should be accepted.**

**Question 8**

An electronics firm has developed a new type of fire-alarm system. A first unit assembled had a material cost of Rs. 18,000 and took 400 hours of direct labour to assemble. Labour rate is Rs. 25 per hour. This type of assembly is known to experience a learning curve effect of 80% (index of learning = 0.3219). Demonstration of this unit to potential customers resulted in an order for 20 units during the next quarter. The firm wishes to popularize this system and will therefore pass on the benefit of cost saving due to learning effect to the customers while setting the sale price.

1. Determine the price to be set for the first lot of 20 units to be sold. The initial unit will not be sold, as this is required for demonstrations. The firm follows a practice of imputing a fixed overhead at 125% of direct labour cost and will set the selling price to earn a 20% gross margin on sale price.
2. Assume that a further order for a lot of 60 units was received on a contract basis from a single customer. The price was set on the basis of the contracted total. However, after delivery of 30 units against the contract, the contract was cancelled. Determine the deferred learning cost that may have to be written off consequent to the cancellation of contract for balance not supplied.

**Answer: 32,178.51, Full lot 60:- 28,032.5, 28,597.5.**

**Question 9**

A Company with two production departments has set the following standard for the forthcoming year:-

	Department S	Department W
Direct labour hours available per period	6000	4000
Standard wage rate per hour	Rs. 6	Rs. 5
Expected learning curve	80%	70%
Standard variable overheads per hour	Rs. 9	Rs. 5

Standard fixed overheads per hour	Rs. 12	Rs. 8
Direct labour hours required for first Unit in lot of 100 units	18	9

The direct materials are introduced in Department S. The company is able to negotiate the following prices for purchase of direct materials during the year.

<b>Level of Output</b>	<b>Price of direct materials per</b>
<b>(Units)</b>	<b>Unit of output</b>
100	Rs. 72.00
200	Rs. 64.80
800	Rs. 54.00

Overtime, if required is paid at time and a half. The overhead rates as given above not include overtime premium.

It is policy of the company to add profit margin as under in quoting the prices:

<b>Department</b>	<b>Percentage on total labour &amp; Overhead cost</b>
S	25%
W	15%
Subcontracted Work	5% on subcontract price

The Company has received a special order. Special tooling costs of the order amount to Rs. 1,200. If this order is for 200 units or less, it will be executed in the period which has a workload of 3840 direct labour hours in Department S and 2100 direct labour hours in Department W. For the work, which is done in Department W, a subcontract, an associate company quotes price of Rs. 50 per unit.

**Required:**

1. If the company decided to get the work executed entirely With in the company, what price, on cost plus basis, should be quoted for the order, if it consists of- 100 units, 200 units
2. Assuming that the initial order placed by the customer is for 200 units. What lowest price should be quoted for a repeat? Order of 600 units? Assume that this order will be executed. When there are no capacity constraints.
3. State the output level at which the company should close Down Department W to get the work executed through subcontractoRs.

**Answer:** (i) 877.80, (ii) 700.71.

**Question 10**

H Ltd. produced its first 10 units of product B. The customer is enquiring about the cost of a further 30 units of product B. The total cost of the original 10 units was:

Materials	3,000
Variable labour costs (500 hours at Rs. 10 per hour)	5,000
Variable overheads*	1,000
Other overheads**	1,000
Machine tool costs***	2,000
<b>Total Cost</b>	<b>12,000</b>

\* Directly affected by variable labour costs.

\*\* Estimated at 20% of variable labour costs.

\*\*\* All machine tool can still be used although all costs recovered on first order.

Use an 80% learning curve to estimate the total costs for a new batch of 30 units of Product B.

**Answer:** 19,920.

**Question 11**

Engine Ltd. Manufacture engine mounting for wide bodied airlines. They have been asked to bid on a prospective contract for 90 engine mounting for the Jumbo Jet aircraft. They have just completed an initial run of 30 of these mountings at the following costs:

		<b>Rs.</b>
Direct materials		20,000
Direct Labour	(6,000 hours @ Rs. 4)	24,000
Tooling cost	(re-usable)	3,000
Variable Overheads	(Rs. 0.50 per Labour hour)	3,000
Fixed	(Rs. 0.50 per labour hour)	6,000

	Total	56,000
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An 80% learning curve is thought to be pertinent in this case. The marketing director believes that the quotation is unlikely to be accepted if it exceeds Rs. 1,10,000 and as the company are short of work, he believe the contract to be vital. You are required to comment whether is it worth accepting at Rs. 1,10,000. State your assumption clearly.

**ANSWER Additional Hr for 90 unit = 9360 hr. Incremental cost 90 unit = 102120**

**Question 12.**

A company has 10 direct workers, who work for 25 days a month of 8 hours per day. The estimated down time is 25% of the total available time. The company received an order for a new product. The first unit of the new product requires 40 direct labour hours to manufacture the product. The company expects 80% (index is – 0.322) learning curve for this type of work. The company uses standard adsorption consting the cost data are as under:

Direct materials	Rs. 60 per unit
Direct labour	Rs. 6 per direct labour hour
Variable Overheads	Rs. 1 per direct labour hour
Fixed overheads	Rs. 7,500 per month

**Required:**

1. Calculate the cost per unit of the first order of 30 units.
2. If the company receives a repeat order for 20 units, what prices will be quoted to yield a profit of 25% on selling price.?

**Answer: (i) 220.56 (ii) 4257.60**

**Question 13**

An electronic firm which has developed a new type of fire-alarm system has been asked to quote for a prospective contract. The customer requires separate price quotations for each of the following possible orders:

Order	Number of Fire-alarm systems
-------	------------------------------

First	100
Second	60
Third	40

The firm estimate the following cost per unit for the first order:

Direct materials Rs. 500

Direct Labour

Deptt. A (Highly automatic) 20 hours at Rs. 10 per hour.

Deptt. B (Skilled labour) 40 hours at Rs. 15 per hour.

Variable Overheads 20% of direct labour

Fixed overheads absorbed:

Deptt. A Rs. 8 per hour

Deptt B Rs. 8 per hour

Determine a price per unit for each of the three orders, assuming the firm uses a mark-up of 25% on total costs and allows for an 80% learning curve. Extract from 80% Learning Curve table:-

X	1.0	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Y(%)	100.0	91.7	89.5	87.6	86.1	84.4	83.0	81.5	80.0

X represents the cumulative total volume produced to date expressed as a multiple of the initial order. Y is the learning curve factor, for a given X value, expressed as a percentage of the cost of the initial order.

**Answer: SP = Rs. 1764.40.40.**

**Question 14**

M Ltd. Manufactures a special product purely carried out by manual labour. It has a capacity of 20,000 units. It estimates the following cost structure :

Direct material	Direct Labour	Variable overheads
Rs. 30/unit	Rs. 20/unit (1 hour/unit)	Rs. 10/unit
Fixed overheads at maximum capacity Rs. 1,50,000		

It is estimated that at current level of efficiency, each unit requires one hour for the first 5,000 units. Subsequently it is possible to achieve 80% learning curve. The market can absorb the first 5,000 units @ Rs. 100 unit. What should be the minimum selling price acceptable for an order of 15,000 units for a prospective client? (CA Final May 2008)

**Answer: 50.40**

**Question 15**

PQ Ltd. makes and sell a labour-intensive product. Its labour force has a learning curve of 80%. This rate is not applicable to variable overheads. The cost per unit of the first product is an follows :

Direct material	Direct Labour	Variable overheads
Rs. 10,000/unit	Rs. 8,000/unit (4 hour/unit)	Rs. 2,000/unit
Fixed overheads at maximum capacity Rs. 1,50,000		

The company has received an order from X Ltd. for 4 units of the product. Another customer, Y Ltd. is also interested in purchasing 4 units of the product. PQ has the capacity of fulfilling both the order. Y Ltd. presently purchases this product at Rs. 17,200/- unit and is willing to pay this price per unit of PQ's product. But X Ltd. lets PQ choose one of the following options.

- (i). A price of Rs. 16,500 per unit for the 4 units it proposed to take from PQ Ltd.
- (ii). Supply X Ltd.'s idle labour force to PQ, for only 4 units of production, with PQ having to pay only Rs. 1 per hour to X Ltd.'s workers. X Ltd.'s workers will be withdrawn after the first 4 units are produced. In this case, PQ need not to use its labour for producing X Ltd.'s requirement. X Ltd. assures PQ that its labour force also has the learning rate of 80%. In this option. X Ltd. offers to buy the product from PQ at only Rs. 14,000 per unit.

X and Y shall not know of each other's offer.

It both orders came before any work started, what is the best option that PQ may choose? Present suitable calculations in favour of your argument.

**Answer: (i) Rs. 16500 (ii) 15792**

**Question 16**

The Gifts Company makes mementos for offering chief guest and other dignitaries at functions. A customer wants 4 identical pieces of a hand – crafted item. The following costs have been estimated for the 1<sup>st</sup> unit of the product :

Direct variable costs (Excluding Labour)      Rs. 2,000/- unit  
 Direct Labour (20 hours @ Rs. 50/hour)      Rs. 1,000/- unit

It is possible to achieve 90% learning curve. The company’s policy is that one Labour works for one order. (i) What is the price per piece if the targeted contribution is Rs. 1,500 per piece? If 4 different labourers made the 4 products simultaneously to ensure faster delivery, can the price at above (i) quoted? Why? (CA Final Nov. 2009)

**Answer:**

**(i) 4310            (ii) No, the company cannot quote for this this product because this learning curve concept applies only on repetitive jobs.**

**Question 17**

The A Company has designed a Prototype Electronic Starter for which the following information are available:

Direct Labour hour	260 hours
Direct material cost	Rs. 30,000
Direct labour rate	Rs. 10 per hour
Variable Overhead	130% of direct labour
Fixed Overhead	70% of direct labour

Based on the demonstration of the prototype, the company has received order for 50 unit during first six month and another 75 units during the following six month there after.

A learning curve effect of 80% is applicable. It is expected that in view of large volume a 5% discount on material cost will be available in first six month and a 10% discount thereafter. The rates of overhead will remain unchanged and the same percentage would apply.

The company sets the selling price with a 40% mark up on costs. Determine the selling price per unit for the first 50 units and the next 75 units.

(The index of learning rate of learning curve effect of 80% is 0.31219). (ICWA Final Dec. 2002)

**Answer:            (i)        45746.40            (ii)        41345.92**

**Question 18.**

BCC manufactures executive chairs They are considering a new design of a chair to launch in the market. Proposed selling price is Rs. 120. BCC wants a contribution of 20% on selling price. There are 3 direct costs : (1) frame which is bought from the market at Rs. 51 (2) leather Rs. 25.00 per chair (3) Direct labour @ Rs. 14.50 per hour. The first table will take 2 hours. Learning curve is 95%. Find the minimum number of chairs to be produced and sold so that the target is achieved. Assume the learning



improvement will stop once 128 chairs have been made and the time for the 128<sup>th</sup> unit will be the time per unit for all subsequent units. (Adapted ACCA)

**Answer: 238 Chairs**

**Question 19**

SV Ltd. which has a fairly full order book is approached by a customer with the offer of a contract for a model that is a variant, in terms of dimensions and materials used, of one of its existing products.

Though the customer expects to pay a normal price for the model he wants SV Ltd. to take account of an 80% learning curve in its price calculation; this level has been shown to be reasonable in SV Ltd's industry for relevant work.

The prospective contract is for a total 464 units made up of an initial order of 160 units, two subsequent orders of 80 units each, and three subsequent orders of 48 units each. SV Ltd. estimates the following costs for the initial order.

Direct materials:

P- 8 mtr.	At Rs. 3.50 per mtr.
Q-12 Kg.	At Rs. 1.00 per Kg.

Direct Wages:

Departments	Hours	Rs. Per hour
1	100	2
2	320	3
3	160	1.00

Variable overhead: 20% of direct wages.

Fixed overhead Department Recovery Rate per hour (Rs.)

1	2.00
2	1.00
3	2

The nature of the work in the three production departments is as follows:-

Department 1 uses highly automatic machines. Although the operators on these machines need to be fairly skilled, their efficiency only affects the quality of the work but can have little impact on the quantity of his departments output which is largely machine controlled. Department 2 and 3 the skill of operators is a major determinant of the volume of output.

You are required to calculate the cost per unit for:

- (a) the initial order of 160 units
- (b) the second, third and fourth orders, if given successively but without guarantee of further orders and
- (c) the whole contract of six orders if given from the start but on the same basis of production and delivery.

**Note:** An 80% learning curve on ordinary graph paper would show the following relationship between the X-axis (volume) and Y-axis (Cumulative average price of elements subjects to the learning curve):

X represents the cumulative total volume produced to date expressed as a multiple of the initial order. Y is the learning curve factor, for a given X value, expressed as a percentage of the cost of the initial order.

X	Y%	X	Y%
1.0	100.0	2.1	78.9
1.1	96.0	2.2	77.8
1.2	93.3	2.3	76.8
1.3	91.7	2.4	76.0
1.4	89.5	2.5	74.9
1.5	87.6	2.6	74.0
1.6	86.1	2.7	73.2
1.7	84.4	2.8	72.3
1.8	83.0	2.9	71.5
1.9	81.5	3.0	70.7
2.0	80.0	3.1	70.0

Answer: (a) 1606 (b) 576.86, 542.68, and 319.35 (c) 3652.34

**Question 20**

X represents the cumulative total volume produced to date expressed as a multiple of the initial order. Y is the learning curve factor, for a given X value, expressed as a percentage of the cost of the initial order. Time Ltd. specializes in the manufacture of electronic watches. Development on a new watch called Punctual is to start shortly. Development of the product will take 1 year. The life cycle of the product is expected to be 2 years. The sales volume is expected as follows :

Year	Sales units
1	80,000
2	2,20,000

Estimates of the new product are as follows :

Year 1: R & D	Rs. 10,50,000
Design cost	Rs. 5,00,000
Marketing cost	Rs. 11,60,000
Office cost	Rs. 1,70,000
Total	Rs. 28,80,000

Years 2 to 3

	Year 2	Year 3
Fixed Production cost	Rs. 6,00,000	Rs. 6,00,000
Fixed Marketing cost	Rs. 1,00,000	Rs. 1,10,000
Fixed Distribution cost	Rs. 1,40,000	Rs. 1,20,000
Fixed customers service cost	Rs. 8,50,000	Rs. 15,00,000
Total	Rs. 16,90,000	Rs. 23,30,000

	Year 2	Year 3
Variable Production cost/unit	Rs. 35	Rs. 37

Variable Marketing cost/unit	Rs. 7	Rs. 8
Variable Distribution cost/unit	Rs. 3	Rs. 2
Variable Customers Service cost/unit	Rs. 2	Rs. 3
	Rs. 47	Rs. 50

The labour cost is included in production costs. Production of one unit requires 2 hours of labour. Labour cost is Rs. 10 per hour in year 2 and Rs. 12 per hour in year 3.

Ignoring time value of money, (i) Find the selling price per unit if the mark is 20% of the Life cycle cost per unit. (ii) Assume that a learning of 95% is expected to occur until the 128<sup>th</sup> unit has been completed; find the revised selling price per unit, the mark-up percentage remaining unchanged. Ignore Time Value of money.

**Answer: (i) Rs.84 (ii) 75.18**

**Question 21**

A factory has a special offer to produce 4 units of a labour intensive product by using its existing facilities after the regular shift time. The product can be produced by using only overtime hours which entails normal rate plus 25%, so that usual production is not affected. Two workers are interested in taking up this additional job every evening after their usual shift is over. One is an experienced man who has been working on a similar product. His normal wage is Rs. 48 per hour. The other worker is a new person who earns Rs. 42 an hour as normal wages. He can be safely considered to have a learning curve of 90% for this work. The company wants to minimize the Labour cost for the order. Only one person is to be chosen for the job. The experienced man take 20 hours for the first unit while the new worker will take 30 hours for the first unit. Evaluate who should be chosen for the job.

**(CA Final Nov. 2010)**

**Answer: Experienced worker Rs. 4800 and New Person Rs. 5130**

**Question 22**

A company has designed and produced a prototype electronic starter for which the following information are applicable :

Direct Labour	Direct Material	Direct Labour Rate	Variable Overheads	Fixed Overheads
260 hours	Rs. 30,000 per unit	Rs. 20 per hour	130% of labour	70% of Labour

Based on the demonstration of Prototype, the company has received order for 50 units during first six months and another 75 units order for next 6 months.

Learning curve is 80%. It is expected that a discount of 5% on materials will be available for first six months and 10% for next six months. The rates of overheads will remain unchanged and the same percentages would apply. The company sets the selling price with a 40% mark up on costs. Determine the selling price per unit for first 50 units and next 75 units. (The index of learning curve rate effect of 80% is -0.3219) **(ICWA Final Dec. 2002)**

**Answer: Selling Price Rs. 45746.40 and Rs. 41345.92**

**Question 23**

An electronic firm has developed a new type of fire-alarm system. The first unit assembled had a material cost of Rs. 18,000 and took 400 hours of direct labour to assemble. Labour rate is Rs. 25 per hour. This type of product experiences a learning curve of 80% (Index of learning is 0.3219). Demonstration of this unit to potential customers resulted in a order for 20 units for the first quarter. The firm wishes to pass the benefit of cost savings due to learning effect to the customers while setting the sale price.

- (i). Determine the price to be set for first lot of 20 units to be sold. The initial unit (that has been produced) is not to be sold as this is required for demonstrations. The firm following a fixed overhead rate at 125% of direct labour cost and will set the selling price to earn a 20% profit on sale price.
- (ii). Assume that a further order for lot of 60 units was received on contract basis from a single customer. The price was set on the basis of contracted total. However, after delivery of 30 units against the contract, the contract was cancelled. Determine the deferred learning cost that may have to be written off consequent to the cancellation of contract for the balance not supplied.

**(ICWA Final Dec. 1994)**

**Answer: (i) Selling Price Rs. 32183 and 28033 (ii) 13410**

**Question 24**

Dynamo, a manufacturer of aircraft parts, has been asked to bid for 900 units for a particular type of component. The company has completed a first lot of 400 units for another customer. The cost details of this lot are given below :

		Rs.
Direct materials		30,00,000
Direct labour	8000 Hours	20,00,000
Tooling cost		4,80,000
Variable overheads	Proportional to direct labour	9,00,000

General overheads	Proportional to direct labour	12,00,000
<b>Total</b>		<b>75,80,000</b>

Repeated assembly of this type of component experiences a learning effect of 85%. The cost benefit of this will be reflected in the bid price. Dynamo follows a policy of setting the selling price to earn 30% profit. Tooling cost have been fully recovered from the first lot sold. Determine the selling price per unit for the second lot indented.

**(ICWA Final Dec. 2001)**

**Answer:                      Selling Price 20247**



CHAPTER 5.Linear Programming**Question 1.**

A manufacturer can produce two different products, A and B during a given time period. Each of these products requires four different manufacturing operations; Grinding, turning, Assembly and Testing. The Manufacturing requirements in hours per unit of product are given below for A and B:

	A	B
Grinding	1	2
Turning	3	1
Assembly	6	3
Testing	5	4

The available capacities of these operations in hours for the given time period are: Grinding, 30; Tuning 60; Assembly, 200; Testing, 200.

The contribution to profit is Rs. 2 for each unit of A and Rs. 3 for each unit of B. The firm can sell all that it produces at the prevailing market price. Formulate the problem as a linear programming model to maximize profit.

**QUESTION 2**

An agriculturist has a farm with 126 hectoRs. He produces Radish, Mutter and Potato. Whatever he raises is fully sold in the market. He gets Rs. 5 for Radish per kg. Rs. 4 for Mutter per Kg. and Rs. 5 for Potato per kg. The average yield is 1,500 kg. of Radish per hectars, 1,800 kg. of Mutter per hectars and 1,200 kg. of potato per hectoRs. To produce each 100 kg. of Raddish and Mutter and to produce each 80 kg. of potato, a sum of Rs. 12.50 has to be used for manure; Labour required for each hectars to raise the crop is 6 man-days for Radish and Potato each and 5 man days for muttar. A total of 500 man-days of labour at a rate of Rs. 40 per man-day are available. Formulate this as a Linear-Programming model to maximize the Agriculturist's total profit.

**Answer:**

Objective Function: Maximize:  $Z = 7,072.5x_1 + 6,775x_2 + 5,572.5x_3$

Subject to the constraints:  $x_1 + x_2 + x_3 \leq 126$  (land constraint)

$6x_1 + 5x_2 + 6x_3 \leq 500$  (labour constraint)

$x_1, x_2, x_3 \geq 0$



**QUESTION 3**

Electro point manufactures three products A, B and C, which use three raw materials X, Y and Z. The raw materials required for each unit product are:

Raw material	Product		
	A	B	C
X	3	4	1
Y	2	4	2
Z	1	1	1

Currently Electro point has 200 units of X, 300 units of Y and 200 units of Z in stock. The next delivery of raw materials will be in a week's time.

The labour requirement per item of product is:

Product	A	B	C
Hours	3	2	3

**Total number of labour hours available is 400.**

Current orders for the products have to be met if the company is not to run the risk of losing customers. These orders at the moment stand at 10 units of A, 10 units of B and 40 units of C. Labour is hired mainly on a weekly basis and paid at the rate of Rs. 2 per hour. Only a small proportion of the workforce is employed on a permanent basis. Other variable costs total Rs. 2 for product A, Rs. 3 for product B and Rs. 4 for product C. Fixed costs are estimated to be Rs. 300 a week. The products are sold at the following prices:

Product	A	B	C
Units price (Rs.)	13	14	17

Formulate the above situation as a linear programming problem to determine production levels for the three products A, B and C.

**Answer:**

**Maximize (total contribution)**

$$Z = (13 - (6 + 2)) x_1 + (14 - (4 + 3)) x_2 + (17 - (6 + 4)) x_3$$

Subject to the constraints

$$3x_1 + 4x_2 + x_3 \leq 200 \text{ (raw material X constraint)}$$

$$2x_1 + 4x_2 + 2x_3 \leq 300 \text{ (raw material Y constraint)}$$

$$x_1 + x_2 + x_3 \leq 200 \text{ (raw material Z constraint)}$$

$$3x_1 + 2x_2 + 3x_3 \leq 400 \text{ (labour constraint)}$$

$$x_1 \geq 10 \text{ (minimum production)}$$

$$x_1, x_2, x_3 \geq 0 \text{ (non negative constraint)}$$

**QUESTION 4**

A firm makes two types of furniture chairs and tables. The contribution for each product as calculated by the accounting department is Rs. 20 per chair and Rs. 30 per table. Both products are processed on three machines M1, M2 and M3. The times required (in hours) by each product and total time available per week on each machine are as follows:

Machine	Chair	Table	Available hours per week
M1	3	3	36
M2	5	2	50
M3	2	6	60

How should the manufacturer schedule his production in order to maximize contribution? Formulate the problem as a linear programming problem.

**ANSWER**

Let  $X_1$  and  $X_2$  are the no. of chairs and tables.

Objective function: Maximize  $Z = 20x + 30y$

Subject to the constraints:  $3x + 3y \leq 36$  (M1 Capacity)

$5x + 2y \leq 50$  (M2 Capacity)

$2x + 6y \leq 60$  (M3 Capacity)

$x, y \geq 0$  ( Non Negative Function )

**QUESTION 5**

A Company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suiting, shirting, and woolens yielding the profit Rs. 2, Rs. 4 and Rs. 3, per meter respectively. One-meter suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly one meter of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing while one-meter woolen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours of weaving, processing and packing departments respectively.

Formulate the linear programming problem to find the product max to maximize the profit.

**ANSWER**

Objective Function :            Maximize (total profit)  $Z = 2x_1 + 4x_2 + 3x_3$

Subject to the constraints:         $3x_1 + 4x_2 + 3x_3 \leq 3600$  (Weaving Capacity )

$2x_1 + x_2 + 3x_3 \leq 2400$  (Processing Capacity )

$x_1 + 3x_2 + 3x_3 \leq 4800$  (Packing Capacity )

$x_1, x_2, x_3 \geq 0$             (Non Negative Constraint )

**QUESTION 6**

A Company manufactures two models of garden roller X and Y when preparing the 1999 budget it was found that the limitations on capacity were represented by the following weekly production :

	<b>Departments</b>		
	<b>Foundary</b>	<b>Machine shop</b>	<b>Contribution per Model</b>
<b>Model X</b>	160 units	200 units	Rs. 120
<b>Or</b>			
<b>Model Y</b>	240 units	150 units	Rs. 90

In addition the material required for Model X was in short supply and only Sufficient for 140 units per week could be guaranteed for the year. Determine the optimum combination of output. Formulate the LP problem.

**ANSWER**

**Objective Function:**

Maximize (total contribution)  $Z = 120x_1 + 90x_2$

**Subject to the constraints:**

$$1/160x_1 + 1/240x_2 \leq 1 \quad (\text{Foundary constraints})$$

$$1/200 x_1 + 1/150x_2 \leq 1 \quad (\text{Machine shop constraint})$$

$$x_1 \leq 140 \quad (\text{Material constraint})$$

$$x_1 \geq 0, x_2 \geq 0 \quad (\text{Non Negative Function})$$

**QUESTION 7**

The following matrix gives the unit cost of transporting a product from production plants P1, P2, and P3 to destinations D1 , D2 and D3. Plants P1, P2, and P3 have a maximum production of 65,24 and 111 units respectively and destinations D1 , D2 and D3 must receive atleast 60,65 and 75 units respectively:

	<b>D1</b>	<b>D2</b>	<b>D3</b>	<b>Supply</b>
<b>P1</b>	400	600	800	65
<b>P2</b>	1000	1200	1400	24
<b>P3</b>	500	900	700	111
<b>Demand</b>	60	65	75	200

**Required :**

Formulate the above as a linear programming problem. ( Only formulation is needed. Please don't solve ).

**QUESTION 8**

A Factory manufactures two articles A and B. To manufacture the article A, a certain machine has to be worked for 1.5 hour and in addition a craftsman has to work for 2 houRs. To manufacture the article B, the machine has to be worked for 2.5 hours and in addition the craftsman has to work for 1.5 houRs. In a week the factory can avail of 80 hours of machine time and 70 hours of craftsman's time. The profit on each article A is Rs. 5 and that on each article B is Rs. 4. If all the articles produced can be sold away, find how many of each kind should be produced to earn the maximum profit per week. Formulate the linear programming problem.

**ANSWER**

**Objective Function :** Maximize (total profit)  $Z = 5x_1 + 4x_2$

**Subject to the constraints:**  $1.5x_1 + 2.5x_2 \leq 80$  (Machine Capacity )

$$2x_1 + 1.5x_2 \leq 70 \quad (\text{ Craftsman Capacity})$$

$$x_1, x_2 \geq 0 \quad (\text{Non Negative Function})$$

**QUESTION 9**

The owner of Fancy Goods shop is interested to determine how many advertisement to release in the selected three magazines. A, B and C. His main purpose is to advertise in such a way that total exposure to principal buyers of his goods is maximized. Percentages of readers for each magazine are known. Exposure in any particular magazine is the number of advertisements released multiplied by the number of principal buyers. The following data are available:

Particulars	Magazines		
	A	B	C
Readers	1.0 lakh	0.6 lakh	0.4 lakh
Principal buyers	20%	15%	8%
Cost per Advertisement	8,000	6,000	5,000

The budgeted amount is at the most Rs. 1.0 lakh for the advertisements. The owner has already decided that magazine A should have no more than 15 advertisements and that B and C each gets at least 8 advertisement. Formulate a Linear Programming Model for this Problem.

**ANSWER**

Objective function :      Maximize  $Z = 20,000x_1 + 9,000x_2 + 3,200x_3$

Subject to constraints

$8,000 x_1 + 6,000 x_2 + 5,000 x_3 \leq 1,00,000$  (Budget Constraint )

$x_1 \leq 15, x_2 \geq 8, x_3 \geq 8$  (Maximum and minimum Advertisements )

Where  $x_1, x_2$  and  $x_3 \geq 0$  ( Non Negative Constraints )

**QUESTION 10**

A firm assembles and sells two different types of outboard motor, A and B using four resources. The production process can be desired as follows:

Resources	Capacity per month
Motor unit shop resource	400 Type A units or 250 Type B units or any linear combination of the two.
Type A gear and drive shop	175 Type A units
Resources	

Type B gear and drive shop                      225 Type B units

Resource

Final Assembly resource                      200 Type A units or 350 type B units or Any linear combination of the two.

Type A units bring in a profit of Rs. 90 each and type B units, Rs. 60 each, what should be the optimum product mix?

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**ANSWER**

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**Objection Function :**        Maximize (total Profit)  $Z = 90x_1 + 60x_2$

**Subject to the constraints**

$$1/400x_1 + 1/250x_2 \leq 1 \text{ or } 5x_1 + 8x_2 \leq 2000 \text{ (Motor Shop Constraint )}$$

$$x_1 \leq 175 \text{ ( Type A gear shop constraint )}$$

$$x_2 \leq 225 \text{ ( Type B gear shop constraint )}$$

$$1/200x_1 + 1/250x_2 \leq 1 \text{ or } 5x_1 + 4x_2 \leq 1400 \text{ ( Final Assembly Constraint )}$$

$$x_1 \geq 0, x_2 \geq 0 \text{ (Non Negative Function )}$$

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**QUESTION 11**

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Consider a company that must produce two products 1 or 2 over a production period of three months of duration. The company can pay for materials and labour from two sources:

1. Company funds and
2. Borrowed funds.

The firm faces three decisions:

1. How many units should it produce of product 1 ?
2. How many units should it produce of Product 2 ?
3. How much money should it borrow to support the production of the two products ?

In making these decisions, the firm wishes to maximize the profit contribution subject to the conditions stated below:

1. Since the company’s products are enjoying a seller’s market, it can sell as many units as it can produce. The company would therefore like to produce as many units as possible subject to production capacity and financial constraints. The capacity constraints, together with cost and price data, are given in table below:

**Capacity, Price and Cost Data**

Product	Selling Price	Cost of	Required hours per unit		
	(Rs. per unit)	Production	department		
		(Rs. per unit)	A	B	C
1	14	10	0.5	0.3	0.2
2	11	8	0.3	0.4	0.1
<b>Available hours per production</b>			500	400	200
<b>Period of three months</b>					

2. The available company funds during the production period will be Rs. 3 lakhs.
3. A bank will give loans up to Rs. 2 lakhs per production period at an interest rate of 20 percent per annum provided the company’s acid (quick) test ratio is at least 1 to 1 while the loan is outstanding. Take a simplified acid-test ratio given by

$$\frac{\text{Surplus cash on hand after production} + \text{Accounts Receivables}}{\text{Bank borrowing} + \text{Interest accrued thereon}}$$

4. Also make sure that the needed funds are made available for meeting the production costs. Formulate the above as a Linear Programming Problem.

**QUESTION 12**

**IMPORTANT**

A firm produces three products A, B and C. It uses two types of raw materials I and II of which 5,000 and 7,500 units respectively are available. The raw material requirements per unit of the products are given below:

Raw Material	Requirements per unit of product		
	A	B	C
I	3	4	5
II	5	3	5

The labour time for each unit of product A is twice that of product B and three times that of product C. The entire labour force of the firm can produce the equivalent of 3,000 units of product “A”. The (marks) minimum demand of the three products is 600, 650 and 500 units respectively. Also the ratios of the number of units produced must be equal to 2 : 3 : 4. Assuming the profits per unit of A, B and C as Rs. 50, 50 and 80 respectively.

Formulate the problem as a linear programming model in order to determine the number of units of each product, which will maximize the profit.

**ANSWER**

**Objective Function :** Maximize  $Z = 50x_1 + 50x_2 + 80x_3$

**Subject to Constraints:**

$$3x_1 + 4x_2 + 5x_3 \leq 5,000$$

$$5x_1 + 3x_2 + 5x_3 \leq 7,500$$

$$6x_1 + 3x_2 + 2x_3 \leq 18,000$$

$$3x_1 = 2x_2 \text{ and } 4x_2 = 3x_3$$

$$x_1 \geq 600, x_2 \geq 650 \text{ and } x_3 \geq 500.$$

**QUESTION 13**

A manufacturing company is engaged in producing three types of products A, B, and C. The production department daily produces components sufficient to make 50 units of A, 25 units of B and 30 units of C. The management is confronted with the problem of optimizing the daily production of products in assembly department where only 100 man-hours are available daily to assemble the products. The following additional information is available.

Type of Product	Profit contribution	Assembly time per
	Per unit of Product (Rs.)	product (hRs.)



A	12	0.8
B	20	1.7
C	45	2.5

The company has a daily order commitment for 20 units of product A and a total of 15 units of B and C Product. Formulate this problem as an LP model so as to maximize the total profit.

**ANSWER**

**Objective Function :** Maximize  $Z = 12x_1 + 20x_2 + 45x_3$

**Subject to constraints:**  $0.8x_1 + 1.7x_2 + 2.5x_3 \leq 100$  ( Assembly Capacity Constraint )

$x_1 \leq 50$  (Production Capacity Constraint)

$x_2 \leq 25$  (Production Capacity Constraint)

$x_3 \leq 30$  (Production Capacity Constraint)

$x_1 \geq 20$  (Minimum Production Constraint)

$x_2 + x_3 \geq 15$  (Minimum Production Constraint)

$x_1, x_2, x_3 \geq 0$  (Non Negative Constraint)

**QUESTION 14**

A small jewellery manufacturing company employs a person who is a highly skilled gem cutter, and it wishes to use this person at least 6 hours per day for this purpose. On the other hand, the polishing facilities can be used in any amount up to 8 hours per day. The company specializes in three kinds of semiprecious stones P, Q and R. Relevant cutting polishing and cost requirements are listed in the following table. How many gemstones of each type should be processed each day to minimize the cost of finished stones? What is the minimum cost?

	P	Q	R
Cutting	2 hr	1 hr	1 hr
Polishing	1 hr	1 hr	2 hr
Cost per stone	Rs. 30	Rs. 30	Rs. 10

**ANSWER**

Objective Constraint :        Minimise  $Z(\text{cost}) = 30x_1 + 30x_2 + 10x_3$

Subject to the constraints:  $2x_1 + x_2 + x_3 \leq 6$  ( Cutting Capacity )

$x_1 + x_2 + 2x_3 \leq 8$  (Polishing Capacity )

$x_1, x_2, x_3 \geq 0$         (Non Negative Function )

**QUESTION 15**

A Mutual Fund Company has Rs. 20 lakhs available for investment in Government Bonds, blue chip stocks, speculative stocks and short-tem deposits. The annual expected return and risk factor are given below:

Type of investment	Annual Expected Return (%)	Risk Factor (0 to 100)
Government Bonds	14	12
Blue Chip Stocks	19	24
Speculative Stocks	23	48
Short-term Deposits	12	6

Mutual fund is required to deep at least Rs. 2 lakhs in short-term deposits and not to exceed average risk factor of 42. Speculative stock must be at most 20 percent of the total amount invested. How should mutual fund invest the funds so as to maximize its total expected annual return? Formulate this as a Linear Programming problem. Do not solve it.

**QUESTION 16**

WELL TYPE Manufacturing Company produces three types of typewriters; Manual type-writer, Electronic Typewriters and deluxe Electronic typewriter. All the three models are required to be machined first and then assembled. The time required for the various models are as follows:

Type	Machine Time (in hour)	Assembly Time (in hour)
Manual Typewriter	15	4
Electronic Typewriter	12	3
Deluxe Electronic Typewriter	14	5

The total available machine time and assembly time are 3,000 hours and 1,200 hours respectively. The data regarding the selling price and variable costs for the three types are:

	Manual	Electronic	Deluxe Electronic
Selling price (Rs.)	4,100	7,500	14,600
Labour, Material and Other variable cost (Rs.)	2,500	4,500	9,000

The Company sells all the three types on credit basis, but will collect the amounts on the first of next month. The labour material and other variable expenses will have to be paid in cash. This company has taken a loan of Rs. 40,000 from a co-operative bank and this company will have to repay it to the bank on 1<sup>st</sup> April, 1999. The TNC bank from whom this company has borrowed Rs. 60,000 has expressed its approval to renew the loan.

The Balance Sheet of this Company as on 31.3.99 is as follows:

Liabilities	Rs.	Assets	Rs.
Equity Share capital	1,50,000	Land	90,000
Capital Reserve	15,000	Building	70,000
General Reserve	1,10,000	Plant & Machinery	1,00,000
Profit & Loss a/c	25,000	Furniture & Fixtures	15,000
Loan term loan	1,00,000	Vehicles	30,000
Loan from TNC Bank	60,000	Inventory	5,000
Loan From Co-op Bank	40,000	Receivables	50,000
		Cash	1,40,000
Total	5,00,000	Total	5,00,000

The company will have to pay a sum of Rs. 10,000 towards the salary from top management executives and other fixed overheads for the month. Interest on long-term loans is to be paid every month at 24% per annum. Interest on loans from Co-operative bank may be taken to be Rs. 1,200 for the month. Also this company has promised to deliver 2 Manual typewriters and 8 Deluxe Electronic typewriters to one of its valued customers next month.

Also make sure the level of operations in this company is subject to the availability of cash next month. This company will also be able to sell all their types of typewriter in the market. The Senior Manager of this

company desires to know as to how many units of each typewriters must be manufactured in the factory next month so as to maximize the profits of the company. Formulate this as a linear programming problem. The formulated problem need not be solved.

**ANSWER**

**Objective Function :** Maximize  $Z = 1,600x_1 + 3,000x_2 + 5,600x_3 - (\text{Rs. } 10,000 + \text{Rs. } 2,000 + \text{Rs. } 1,200)$

**Subject to the constraints:**

$$15x_1 + 12x_2 + 14x_3 \leq 3,000 \quad (\text{Machine Capacity})$$

$$4x_1 + 3x_2 + 5x_3 \leq 1,200 \quad (\text{Assembly Capacity})$$

$$2500x_1 + 4,500x_2 + 9,000x_3 \leq 1,36,800 \quad (\text{Cash Availability})$$

$$x_1 \leq 2, x_2 \geq 0, x_3 \geq 8 \quad (\text{Production units})$$

$$x_1, x_2 \text{ and } x_3 \geq 0 \quad (\text{Non Negative Constraint})$$

**QUESTION 17**

An electronics Company produces three types of parts for automatic washing machine. It purchases costing of the parts from a local foundry and then finishes the part of drilling, shaping and polishing machines.

The selling prices of part A, B and C respectively is Rs. 8, Rs. 10, and Rs. 14. All parts made can be sold. Castings for parts A, B and C respectively cost Rs. 5, Rs. 6 and Rs. 10.

The shop possesses only one of each type of machine. Costs per hour to run each of the three machines are Rs. 20 for drilling, Rs. 30 for shaping and Rs 30 for polishing. The capacities (parts per hour) for each part on each machine are shown in the following table:

Machine	Capacity per hour		
	Part A	Part B	Part C
Drilling	25	40	25
Shaping	25	20	20
Polishing	40	30	40

The management of the shop wants to know how many parts of each type it should produce per hour in order to maximize profit for an hour's run. Formulate this problem as an LP model.

**ANSWER**

**Objective Function :** Maximise Z( Profit ) =  $0.25x_1 + 1.00x_2 + 0.95x_3$

- Subject to:**
- $x_1/25 + x_2/40 + x_3/25 \leq 1$  (Drilling Capacity )
  - $x_1/25 + x_2/20 + x_3/20 \leq 1$  (Shaping Capacity )
  - $x_1/40 + x_2/30 + x_3/20 \leq 1$  (Polishing Capacity )
  - $x_1, x_2, x_3 \geq 0$  (Non Negative Function )

**QUESTION 18**

The Delhi Florist Company is planning to make up floral arrangements for the upcoming festival. The Company has available the following supply of flowers at the costs shown:

Type	Number available	Cost per flower
Red roses	800	Rs. 0.20
Gardenias	456	Rs. 0.25
Carnations	4,000	Rs. 0.15
White roses	920	Rs. 0.20
Yellow roses	422	Rs. 0.22

These flowers can be used in any of the four popular arrangements whose makeup and selling prices are as follows:

Arrangement	Requirements	Selling price
Economy	4 red roses 2 gardenias 8 Carnations	Rs. 6
May time	8 white roses 5 gardenias 10 carnations	Rs. 8

Spring color	4 Yellow roses	Rs. 10
	9 red roses	
	10 carnations	
	9 white roses	
Deluxe rose	6 Yellow roses	Rs. 12
	12 red roses	
	12 White roses	
	12 yellow roses	

Formulate a Linear-programming problem, which allows the florist company to determine how many units of each arrangement should be made up in order to maximizes profits assuming all arrangements can be sold.

The formulate LP problem is not required to be solved.

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**ANSWER**

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**Objective Function :** Maximize (total profit)  $Z = 3.5x_1 + 2.77x_2 + 3.58x_3 + 4.56x_4$

- Subject to the constraints:**
- $4x_1 + 9x_3 + 12x_4 \leq 800$  (Roses available )
  - $2x_1 + 5x_2 \leq 456$  (Gardenias Available )
  - $8x_1 + 10x_2 + 10x_3 \leq 4000$  ( Carnations Available )
  - $8x_2 + 9x_3 + 12x_4 \leq 920$  (White Roses Available )
  - $4x_2 + 6x_3 + 12x_4 \leq 422$  (Yellow Roses Available )
  - $x_j \geq 0, j = 1, 2, 3, 4$  (Non Negative Function )

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**QUESTION 19**

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A manufacture of biscuits is considering four types of gift packs containing three types of biscuits, Orange cream (OC), Chocolate cream (CC), and Wafers (W), Market research study conducted recently to assess the preference of the consumers shows the following types of assortments to be in good demand:

Assortments	Contents	Selling price per Kg. In Rs.
-------------	----------	------------------------------

A	Not less than 40% of OC	20
	Not more than 20% of CC	
	Any quality of W	
B	Not less than 20% of OC.	25
	Not more than 40% of CC	
	Any quality of W	
C	Not less than 50% of OC	22
	Not more than 10% of CC	
	Any quality of W	
D	No restrictions	12

For the biscuits, the manufacturing capacity and costs are given below:

Type of biscuits	Plant capacity (Kg/day)	Manufacturing Cost (Rs./Kg.)
OC	200	8
CC	200	9
W	150	7

Formulate the LP model to find the production schedule, which maximize the profit assuming that there are no market restrictions.

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**QUESTION 20**

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A Local travel agent is planning charter trip to a major sea resort. The eight-day/seven night package includes the fare for round-trip travel, surface transportation, board and lodging and selected tour options. The charter trip is restricted to 200 persons and past experience indicates that there will not be any problem for getting 200 persons. The problem for the travel agent is determining the number of Deluxe, Standard, and Economy tour packages to, offer for this charter. These three plans each differ according to seating and service for the flight, quality of accommodation, meal plans and tour options. The following table summarizes the estimated prices are

for the three packages and the corresponding expenses for the travel agent. The travel agent has hired an aircraft for the flat fee of Rs. 2,00,000 for the entire trip.

**Price and costs for four packages per person**

<b>Tour plan</b>	<b>Price (Rs.)</b>	<b>Hotel Costs (Rs.)</b>	<b>Meals &amp; Other Expenses (Rs.)</b>
Deluxe	10,000	3,000	4,750
Standard	7,000	2,200	2,500
Economy	6,500	1,900	2,200

In planning the trip the following considerations must be taken into account:

1. At least 10 percent of the packages must be of the deluxe type.
2. At least 35 percent but not more than 70 percent must be of the standard type.
3. At least 30 percent must be of the economy type.
4. The maximum number of deluxe package available in any aircraft s restricted to 60.
5. The hotel desires that at least 120 of the tourist should be on the deluxe and standard packages together.

The travel agent wishes to determine the number of package to offer in each type so as to maximize the total profit.

**A:** Formulate the above as a Linear-programming problem.

**B:** Restate the above Linear Programming problem taking advantage of the fact that 200 packages will be sold.

**QUESTION 21**

A refinery makes 3 grades of petrol (A, B, C) from 3 crude oils (d, e, f). Crude can be used in any grade but the others satisfy the following specifications.

<b>Grade</b>	<b>Specifications</b>	<b>Selling price per litre</b>
A	Not less than 50% crude d Not more than 25% crude e	8.0
B	Not less than 25% crude d	6.5



C	Not more than 50% crude e	5.5
	No Specification	

There are capacity limitations on the amount of the three cruds elements that can be used;

Crude	Capacity	Price per litre
d	500	9.5
E	500	5.5
F	300	6.5

It is required to produce the maximum profit.

**QUESTION 22**

The Fine Paper Company produces rolls of paper used in each register. Each roll of paper is 500ft. in length and can be produced in widths of 1, 2, 3 and 5 inch. The company’s production process results in 500 rolls that are 12 inches in width. Thus the company must cut its 12-inch roll to the desired width. It has six basic cutting alternatives as follows:

Cutting Alternative	No. of Rolls				Waste
	1”	2”	3”	5”	
1	6	3	0	0	0
2	0	3	2	0	0
3	1	1	1	1	1
4	0	0	2	1	1
5	0	4	1	0	1
6	4	2	1	0	1

The maximum demand requirement for the four rolls is as follows:

Roll Width (inches)	Demand Requirements (Rolls)
1	3000

2	2000
3	1500
5	1000

The Company wishes to minimize the waste generated by its production meeting its demand requirements. Formulate the LP model.

**ANSWER**

**Objective Function :** Minimize (waste produced)  $Z = 1x_3 + 1x_4 + 1x_5 + 1x_6$

**Subject to constraints :**

$$6x_1 + 1x_3 + 4x_6 \geq 3000 \quad (1''\text{rolls production})$$

$$3x_1 + 3x_2 + 1x_3 + 4x_5 + 2x_6 \geq 2000 \quad (2''\text{rolls production})$$

$$2x_2 + 1x_3 + 2x_4 + 1x_5 + 1x_6 \geq 1500 \quad (3''\text{rolls production})$$

$$1x_3 + 1x_4 \geq 1000 \quad (5''\text{rolls production})$$

$x_j \geq 0, \text{ for all } j$

**QUESTION 23**

A firm buys casting of P and Q type of parts and sells them as finished product after machining, boring and polishing. The purchasing cost for costing are Rs. 3 and Rs. 4 each for parts P and Q and selling costs are Rs. 8 and Rs. 10 respectively. The per hour capacity of machines used for machining, boring and polishing for two products is given below:

Capacity (per hour)	Parts	
	P	Q
Machining	30	50
Boring	30	45
Polishing	45	30

The running costs for machining, boring and polishing are Rs. 30, Rs. 22.5 and Rs. 22.5 per hour respectively. Formulate the linear programming problem to find out the product mix to maximize the profit.

**ANSWER**

**Objective Function :** Maximize (total Profit)  $Z = 2.75x_1 + 4.15x_2$

Subject to the constraints:

1. Machine constraint:  $1/30x_1 + 1/60x_2 \leq 1$  or  $50x_1 + 30x_2 \leq 1500$
  2. Boring constraint:  $1/30x_1 + 1/45x_2 \leq 1$  or  $45x_1 + 30x_2 \leq 1350$
  3. Polishing constraint:  $1/45x_1 + 1/30x_2 \leq 1$  or  $30x_1 + 45x_2 \leq 1350$
- and  $x_1, x_2 \geq 0$

**QUESTION 24**

The Management accountant of Atul Enterprises Ltd. has suggested that a Linear programming model might be used for selecting the best mix of five possible products, A, B, C, D and E.

1. The following information is available:

	Per unit of Product				
	A	B	C	D	E
	Rs.	Rs.	Rs.	Rs.	Rs.
Selling price	48	42	38	31	27
Costs:					
Materials	15	14	16	15	16
Direct Labour	18	16	6	4	4
Fixed overheads*	9	8	3	2	2
Total Costs	42	38	25	21	22
Net Profits	6	4	13	10	5

\*Based on 50% of direct labour cost.

1. Expected maximum unit demand per week for each product at the prices indicated:

A	B	C	D	E
1,500	1,200	900	600	600

2. Cost of Materials includes a special component, which is in short supply; it costs Rs. 3 a unit. Only 5,800 units will be available to the company during the week. The number of units of the special component needed for a unit of each product is:

A	B	C	D	E
---	---	---	---	---

1            1            3            4            5

3. Labour is paid at a rate of Rs/ 1.50 per hour and only 20,000 hours will be available in a week.
4. The management of Atul Enterprises Ltd. has ruled that expenditure on materials must not exceed a sum of Rs. 30,000
5. All other resources are freely available insufficient quantities for planned needs.

**ANSWER**

To formulate a linear programming model based on the given data, an objective function in contribution terms is required.

(The “net profit” figures per unit of product include an arbitrary absorption of fixed overheads. This will lead to a distortion of the appropriate product mix.)

Let the decision variable  $X_1, X_2, X_3, X_4$  and  $X_5$  represent the units of products A, B, C, D and E to be produced.

Then, the objective function is to maximize the contribution i.e.

$$\text{Maximize } C = 15x_1 + 12x_2 + 16x_3 + 12x_4 + 7x_5$$

Subject to the following constraints:

$$X_1 \leq 1,500$$

$$X_2 \leq 1,200 \quad (\text{expected maximum demand constraints})$$

$$X_3 \leq 900$$

$$X_4 \leq 600$$

$$X_5 \leq 600$$

$$X_1 + X_2 + 3x_3 + 4x_4 + 5x_5 \leq 5,800 \quad (\text{Special component constraint})$$

$$12x_1 + 10 - \frac{2}{3}x_2 + 4x_3 + 2 - \frac{2}{3}x_4 + 2 - \frac{2}{3}x_5 \leq 20,000 \quad (\text{labour hours constraint})$$

$$15x_1 + 14x_2 + 16x_3 + 15x_4 + 16x_5 \leq 30,000 \quad (\text{material expenditure constraint})$$

**Question 25.**

A company produces two types of presentation goods A and B that requires gold and silver. Each unit of type A requires 3 gms of silver and 1 gm of gold while that of B requires 1 gm of silver and 2 gms of gold. The company can procure 9 gms of silver and 8 gms of gold. If each unit of type A brings a profit of Rs. 40 and that of type B Rs. 50. Determine the number of units of each type that the company should produce to maximise the profit. What is the maximum amount of profit ?

**Answer :**

2 units of type A and 3 units of type B and the maximum amount of profit which could be earned is Rs. 230.

**Question 26.**

A company produces two products X and Y, each of which requires three types of processing. The length of time which each unit requires for processing and the profit per unit are given in the following table:

	<b>Product X (hrs/ per unit )</b>	<b>Product Y (hrs/per unit )</b>	<b>Available capacity (hrs)</b>
<b>Process I</b>	<b>12</b>	<b>12</b>	<b>840</b>
<b>Process II</b>	<b>3</b>	<b>6</b>	<b>300</b>
<b>Process III</b>	<b>8</b>	<b>4</b>	<b>480</b>
<b>Profit per unit</b>	<b>5</b>	<b>7</b>	

How many units of each product should the company produce per day to maximise profit ?

**Answer:**

The company should produce 40 units of X and 30 units of Y to earn maximum profit which is Rs. 410.

**Question 27.**

A company that produce soft drink has a contract that requires that a minimum of 80 units of chemical A and 60 units of chemical B into each bottle of the drink. The chemicals are available in a prepared mix from two different suppliers. Supplier X has a mix of 4 units of A and 2 units of B that costs Rs. 10 , and supplier Y has a mix of 1 unit of A and 1 unit of B that costs Rs. 4. How many mixes from company X and company Y should the company purchase to honour contract requirement and yet minimise cost ?

**Answer :**

The company should purchase 10 mixes from X and 40 mixes from Y.

**Question 28.**

An advertising firm desires to reach two types of audiences customers with annual income of more than Rs. 40000 ( target audience A ) and customers with annual income of less than Rs. 40,000 (target audience B). The total advertising budget is Rs. 2,00,000. One programme of TV advertising costs Rs 50,000 and one programme of radio advertising costs Rs. 20,000. Contract conditions ordinary require that there should be atleast 3 programmes on TV and the number of programmes on radio must not exceed 5. Survey indicates that a single TV programme reaches 7,50,000customers in target audience A and 1,50,000 in target audience B. One radio programme reaches 40,000 customers in target audience A and 2,60,000 in target audience B.

**Reuired :**

Formulate this as a linear programming problem and determine the media mix to maximise the total reach using graphical method.

**Answer :**

We should advertise in 4 programmes on TV and no programme on radio and in this way we will reach to the maximum audience of 36,00,000 customeRs.

**Question 29**

Let us assume that you have inherited Rs. 1,00,000 from your father in law that can be invested in a combination of only two stock portfolios , with the maximum investment allowed in the portfolio set at Rs 75,000. The first portfolio has an average rate of return of 10% , where as the second has 20 %. In terms of risk factors associated with this portfolios , the first has a risk rating of 4(on a scale from 0 to 10) , and the second has 9. Since you wish to maximise your return , you will not expect an average rate of return below 12% or a risk factor above 6. Hence , you then face the important question. How much should you invest in each portfolio?

**Required:**

Formulate this as a linear programming problem and solve it by graphic method.

**Answer:**

We should invest 60,000 in the first portfolio and 40,000 in the second portfolio to earn a maximum profit of Rs. 14,000.

**Question 30.**

A Farm is engaged in breeding pigs. The pigs are fed on various products grown in the farm. In view of need to ensure certain nutrients constituents ( call them X, Y and Z) ,it becomes necessary to buy two additional roduct say , A and B. One unit of product A contains 36 units of X , 3 units of Y and 20 units of Z. One unit of product B contains 6 units of X , 12 units of Y and 10 units of Z. The minimum requirement of X , Y and Z is 108 units, 36 units and 100 units respectively. Product A cost Rs. 20 per unit and product B cost Rs. 40 per unit.

**Required :**

Formulate the above as a linear programming problem to minimise the total cost and solve this problem by using graphic method.

**Answer:**

4 units of A and 2 units of B in order to minimise cost at Rs. 160.

**Question 31.**

The costs and selling prices per unit of two products manufacturing by a company are as under :

	<b>Products</b>	
	<b>A(Rs.)</b>	<b>B(Rs.)</b>
Selling Price	<u>500</u>	<u>450</u>
Variable Costs:		
Direct <a href="#">Materials @ Rs.25</a> per kg	100	100
Direct Labour @20 per hour	80	40
Painting @30 per hour	30	60
Variable Overheads	190	175
Fixed Overheads @17.50 per labour hour	<u>70</u>	<u>35</u>

Total costs	<u>470</u>	<u>410</u>
Profit	30	40

In any month maximum availability of inputs is limited to the following :

Direct Materials	480 kg
Direct Labour	400 hours
Painting hours	200 hours

Required :

- (i) Formulate a linear program to determine the production plan which maximise the profits by using graphical method.
- (ii) State the optimal product mix and the monthly profit derives from your solution in (i) above.
- (iii) If the company can sell the painting time at 40 per hour as a separate service, show what modification will be required in the formulation of the linear programming problem . ( you are required to reformulate the problem but not to solve ).

**Answer:**

80 Units of A and 40 Units of B and optimal profit is Rs. 4000.

**Question 32.**

Use simplex method to solve the following LP problem :

Maximise  $Z = 6x_1 + 8x_2$

Subject to constraints :

$$2x_1 + 3x_2 \leq 16$$

$$4x_1 + 2x_2 \leq 16$$

**Question 33.**

Use simplex method to solve the following LP Problem :

Minimise  $Z = 3x + 2.5y$

Subject to constraints :

$$2x + 4y \geq 40$$

$$5x + 2y \leq 50$$

$$x, y \geq 0$$

**Question 34.**

Use simplex method to solve the following LP problem :

Maximise  $Z = 30x + 20y$

Subject to the constraints :

$$x + y \geq 8$$

$$6x + 4y \leq 12$$

$$5x + 8y = 20$$

$$x \text{ and } y \geq 0$$

**Question 35.**

Use simplex method to solve the following LP problem :

Minimise  $Z = 30x + 20y$

Subject to the constraints :

$$x + y \geq 8$$

$$6x + 4y \leq 12$$

$$5x + 8y = 20$$

$$x \text{ and } y \geq 0$$

**Question 36.**

The following information is given relating the simplex method of a linear program with the usual notations:

Objective function:  $Z = x + 5y$  Eq 1

Subject to :  $6x + 8y \leq 12$  Eq2

$5x + 15y \geq 10$  Eq3

$X \text{ and } y \geq 10$  Eq4

Required :

If the objective is to maximise Z,

- (i) What will be the coefficient of S1 , S2 and A2 in equation 1 and 3 restated as equality?
- (ii) Identify the slack and surplus variables .
- (iii) Which variable will form part of the initial solution?
- (iv) If object is to minimise Z what will be your Answer to (i) above.

**Question 37.**

Given below is an iteration in a simplex table for a maximisation objective linear programming product mix problem for products X1 , X2 and X3.

	$C_j \rightarrow$		6	4	10	0	0	0
	Basic Variables	Quantity	X1	X2	X3	S1	S2	S3



0	S1	400	0	4/3	0	1	-1/3	0
6	X1	400	1	2/3	2	0	1/3	0
0	S3	400	0	5/3	0	0	-2/3	1
	Z <sub>j</sub>	2400	6	4	12	0	2	0
	C <sub>j</sub> -Z <sub>j</sub>		0	0	-2	0	-2	0

Answer the following questions:

1. Is the above solution feasible?
2. Perform one more iteration with X2 entering the solution to get a solution with the same value for the objective function.
3. Indicate the shadow prices.
4. If customer is prepared to pay higher price for the product X3 then by how much should the price be increased so that the company's profit remains unchanged ?
5. From the given table, derive any one original constraint inequality with the coefficients of variables in their simplest whole number forms.

**Question 38.**

Given below is an iterate in a simplex table for a maximisation objective linear programming product mix problem for product x, y and z. Each of these products is processed in three machines KA-07, KB-27 and KC-49 and each of the machine has limited available hours.

C <sub>a</sub>	C <sub>j</sub> →		30	40	20	0	0	0
	Basic Variables	Value of basic variables	X	Y	Z	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>
30	X	250	1	0	-26/16	10/16	-12/16	0
40	Y	625	0	1	31/16	-7/16	10/16	0
0	S <sub>3</sub>	125	0	0	11/16	-3/16	1/8	1

S<sub>1</sub>, S<sub>2</sub> and S<sub>3</sub> are slack variables for machine KA07, KB27 and KB49 respectively.

Answer the following questions giving reasons in brief :

- (i) Does the table above give an optimal solution?
- (ii) Are there more than one optimal solution/optimal solution?
- (iii) Is this solution feasible?
- (iv) Is this solution degenerate?
- (v) Write down the objective function of the above solution .
- (vi) Write down the optimal product mix and profit shown by the above solution .
- (vii) Which of these machines is being used to its full capacity when producing according to this solution.
- (viii) How much would you be ready to pay for another hour of capacity on each machine KA07,KB27 and KC49?
- (ix) If the company wishes to expand the production capacity , which of the three resources should be given priority?
- (x) What happens if 16 machine hours are lost due to some mechanical problem on machine KB27?
- (xi) A customer would like to have one unit of product Z and is willing to pay higher price for it. How much the price be increased that the company's profit remain unchanged ?

- (xii) A new product is proposed to be introduced which would require processing time of 4 hours on machine KA 07, 2 hours on machine KB27 and 4 hours on machine KC 49. It would yield a profit of Rs. 12 per unit. Do you think it is advisable to introduce this product?

**Question 39.**

Minimise  $Z = 2x_1 - 3x_2 + 4x_3$

Subject to constraints :

$$3x_1 + 2x_2 + 4x_3 \geq 9$$

$$2x_1 + 3x_2 + 2x_3 \geq 5$$

$$7x_1 - 2x_2 - 4x_3 \leq 10$$

$$6x_1 - 3x_2 + 4x_3 \geq 4$$

$$2x_1 + 5x_2 - 3x_3 = 3$$

$$x_1, x_2 \text{ and } x_3 \geq 0$$

Required :

Find the dual problem for the above problem.

**Question 40.**

Maximise  $Z = 2x_1 - 3x_2 + 4x_3$

Subject to constraints :

$$3x_1 + 2x_2 + 4x_3 \geq 9$$

$$2x_1 + 3x_2 + 2x_3 \geq 5$$

$$7x_1 - 2x_2 - 4x_3 \leq 10$$

$$6x_1 - 3x_2 + 4x_3 \geq 4$$

$$2x_1 + 5x_2 - 3x_3 = 3$$

$$x_1, x_2 \text{ and } x_3 \geq 0$$

Required :

Find the dual problem for the above problem.

**CHAPTER 6**  
**CPM and PERT**

**Question 1.**

Given the following information, develop a network.

Activity	Immediate Predecessor
A	—
B	—
C	A
D	A
E	C, B

**Question 2**

Draw a network diagram for the following data.

Task	Immediate Predecessor
A	—
B	—
C	B
D	B
E	B
F	E
G	A, D, C

**Question 3.**

Given the following information, develop a network.

Activity	Immediate Predecessor
A	—
B	—
C	—
D	A
E	B
F	C
G	D & E
H	F & G

**Question 4.**

Draw a network diagram for the following data.

Activity	Preceding Activities
A	—
B	A
C	A
D	B
E	A
F	B, E
G	C
H	D, F
I	G
J	H, I

**Question 5.**

Construct the network diagram comprising activities B, C, .....Q and N Such that the following constraints are satisfied:

$$B < E, F; C < G, L; E, G < H; L, H < I; L < M; H < N; H < J; I, J < P; P < Q.$$

The notation  $X < Y$  means that the activity X must be finished before Y can begin.

**Question 6.**

Develop a network based on the following information;

Activity	Immediate Predecessors
A	—
B	—
C	A
D	B
E	C, D
F	D
G	E
H	F

**Question 7.**

Adjoining is a list and description of activities for a project XYZ building a boat. Draw the network.

Activity	Preceding activity	Activity description
A	None	Design hull
B	None	Prepare boat shed
C	A	Design Mmst and mast mount
D	A	

E	A	Obtain hull
F	C	Design sails
G	C	Obtain mast mount
H	C	Obtain mast
J	B, D	Design rigging
K	F, J	Prepare hull
L	E, H, G, K	Fit mast mount to hull
M	E, H	Step mast
N	L, M	Obtain Sails and Rigging
		Fit sails and Rigging

**Question 8..**

An assembly is to be made from two parts A and B. Both parts must be turned on a lathe and B must be polished whereas, A need not be polished. The sequence of activities together with their predecessors is given. Draw a network diagram for the project.

Activity	Predecessor Activity
A Open work order	None
B Get material for A	A
C Get material for B	A
D Turn A on lathe	B
E Turn B on lathe	B, C
F Polish B	E
G Assemble A and B	D, f
H pack	G

**Question 9.**

Draw an arrow diagram showing the following relationship:

Activity	Immediate Predecessor	Activity	Immediate Predecessors
A	-	H	D, E, F
B	-	I	D
C	-	J	G
D	A	K	G
E	B,C	L	H, J
F	A	M	K
G	C	N	I, L

**Question 10.**

Construct the network for the following set of activities.

Activity	Predecessor	Activities	Predecessors
A	—	G	B, D
B	—	H	C, E
C	A	I	F
D	B	J	F
E	C	K	E, G, I
F		L	H, J

**Question 11.**

Activity	A	B	C	D	E	F	G
Dependence	—	A	A	A	B, C	C, D	E, F

**Question 12.** Draw network diagram from the following information :

<b>Activity</b>	A	B	C	D	E	F	G	H
<b>Dependence</b>	—	—	A	C	B	B	BC	F

**Question 13.**

A small project consists of seven activities for which the relevant data are given below:

<b>Activity</b>	<b>Preceding Activities</b>	<b>Activity Duration</b>
A	—	4
B	—	7
C	—	6
D	A, B	5
E	A, B	7
F	C, D, D	6
G	C, D, E	5

Draw the network and find the project completion time.

**Question 14.**

Draw the network for the following activities and find critical path and total duration of project :

<b>Activity</b>	<b>Dependence</b>	<b>Duration (Days)</b>	<b>Activity</b>	<b>Dependence</b>	<b>Duration (Days)</b>
A	-	9	G	E	10
B	-	4	H	E	8
C	-	7	I	DFH	6
D	BC	8	J	E	9
E	A	7	K	IJ	10



F	C	5	L	G	2
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**Question 15.**

Draw the network for the following activities and find critical path and total duration of project :

Activity	Dependence	Duration (Days)	Activity	Dependence	Duration (Days)
A	-	9	G	F, C	10
B	-	4	H	B	8
C	-	7	I	E, H	6
D	A, B	8	J	E, H	9
E	B	7	K	C, D, F, J	10
F	B	5	L	K	2

**Question 16.**

A Publisher has a contract with an author to publish a textbook. The (Simplified) activities associated with the production of the textbook are given subsequently. Develop the associated network for the project and also identify critical path and expected project duration :

	Activity	Predecessor Activity	Duration (Weeks)
A	Manuscript proof reading by editor	—	5
B	Sample pages prepared by typesetter	—	3
C	Book cover design	—	5
D	Preparation for Diagram used in a book	—	4
E	Authors approval of edited manuscript & sample pages	A, B	3
F	Book type setting	E	5
G	Author checks typeset pages	F	3
H	Author checks Art-work	D	2
I	Production of Printing plates	G, H	3

J	Book production & Binding	C, I	7
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**Question 17.**

The NRB Company is planning to design, develop and market a new racing cycle. The project is composed of the following activities :

Activity	Description	Predecessors	Time (Weeks)
A	Design frame	-	4
B	Design wheels	-	3
C	Design gears	-	3
D	Design handle bars	C	2
E	Test steering	A, B, D	1
F	Test gears	A, B, D	2
G	Performance test	E, F	3
H	Manufacturing layout	A, B, D	3
I	Manufacturing demonstration	H	5
J	Preparing advertising	G	2
K	Preparing users manual	G	4
L	Distribute to dealers	I, J, K	2

Draw the network; find critical path and total duration of project. NRB would like to complete the project in 15 weeks. Would it help if they :

- (i). Work over time to get the frame designed in only 3 weeks?
- (ii). Assign more designers to design the gears? If so, from what activity should the designers be taken from?

**Question 18.**

Draw the network for the following activities and find critical path and total duration of project and also make float analysis :

Activity	Dependence (Days)	Duration	Activity	Dependence (Days)	Duration
A	-	6	G	C, D	3
B	-	3	H	E	5
C	A	5	I	C, D	5
D	A	4	J	G, H	2
E	B	3	K	F	3
F	B	2	L	J, K	2

**Question 19.**

Draw the network for the following activities and find critical path and total duration of project also make float analysis :

Activity	Dependence (Days)	Duration	Activity	Dependence (Days)	Duration
A	-	6	G	C	3
B	A	3	H	D	5
C	A	5	I	G	5
D	A	4	J	D	2
E	B	3	K	J	3
F	E	2	L	E, H, I	2

**Question 20**

Draw network. Determine the critical path and duration of the project. Make float analysis. (Duration in days)

<b>Activity</b>	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
<b>Duration</b>	2	8	10	6	3	3	7	5	2	8

(CA Final May 1999)

**Question 21.**

The number of days of total floats (TF). Earliest start times (EST) and duration in days are given for some of the following activities :

Activity	TF	EST	Duration	Activity	TF	EST	Duration
1-2	0	0		4-6	0	12	
1-3	0			5-7	3		
1-4	5			6-7		23	
2-4	0	4		6-8	2		
2-5	1		5	7-8	0	23	
3-6	2	12		8-9		30	6

Draw the network. List the path and their durations and state when the project can be completed.

(CA Final Nov. 2011)

**Question 22.**

The following activities must be accomplished in order to complete a construction project :

Activity	A	B	C	D	E	F	G	H	I	J
<b>Duration</b>	3	8	4	2	1	7	5	6	8	9
<b>Predecessors</b>	--	--	AB	B	A	C	EF	DF	GH	I

- (i). Construct a network diagram for this project. Find the CP and the duration of the project.

- (ii). Assume that you are project manager of the project mentioned above. The project has progressed for 10 weeks and the status is follows :

Activities completed : A, B, E. other activities have not started as yet.

- If no managerial action is taken at all when will the project get completed?
- What action might you take to get the project back to a schedule that can be completed by the end of week 42?

**(CA Final May 1993)**

**Question 23.**

A company had planned its operations as follows : (Duration in days)

<b>Activity</b>	1-2	2-4	1-3	3-4	1-4	2-5	4-7	3-6	5-7	6-8	7-8
<b>Duration</b>	7	8	8	6	6	16	19	24	9	7	8

- (i). Draw the network and find the critical paths
- (ii). After 15 days of working, the following progress is noted :
  - a. Activities 1-2, 1-3 an 1-4 completed as per original schedule.
  - b. Activity 2-4 is in progress and will be completed in 4 more days.
  - c. Activity 3-6 is in progress and will be completed in 17 more days.
  - d. The staff members for activity 3-6 are specialized. They are directed to complete 3-6 and undertake an activity 6-7 which will require 7 days. This re-arrangement arose due to a modification in specialization.
  - e. Activity 6-8 will be completed in 4 days instead of originally planned 7 days.
  - f. There is no change in other activities.

Update the network diagram after 15 days of start of work based on the facts given above. Indicate the revised critical path along with duration.

**(CA Final May 2007)**

**Question 24.**

A company had planned its operations as follows : (Duration in days)

<b>Activity</b>	1-2	2-4	1-3	3-4	1-4	2-5	4-7	3-6	5-7	6-8	7-8
<b>Duration</b>	7	8	8	6	6	16	19	24	9	7	8

- (i). Draw the network and find the critical paths
- (ii). After 15 days of working, the following progress is noted :

- a. Activities 1-2, 1-3 and 1-4 completed as per original schedule.
- b. Activity 2-4 is in progress and will be completed in 4 more days.
- c. Activity 3-6 is in progress and will be completed in 17 more days.
- d. The staff members for activity 3-6 are specialized. They are directed to complete 3-6 and undertake an activity 6-7 which will require 7 days. This re-arrangement arose due to a modification in specialization.
- e. Activity 6-8 will be completed in 4 days instead of originally planned 7 days.
- f. There is no change in other activities.

Update the network diagram after 15 days of start of work based on the facts given above. Indicate the revised critical path along with duration. (CA Final May 2007)

**Question 25.**

The following information is available :

Activity	A 1-2	B 1-3	C 1-4	D 2-6	E 3-5	F 5-6	G 4-6
No. of days	4	2	8	6	4	1	1
No. of men required per day	2	3	5	3	2	3	8

- (i). Draw the network and find the critical path.
- (ii). What is the peak requirement of manpower? On which day(s) will this occur?
- (iii). If the maximum number of labour available on any day is only 10, when can the project be completed?

(CA Final May 2008)

**Question 26.**

The following table lists the activities of a maintenance project. (Duration in months)

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-7	5-8	6-8	7-9	8-9
Duration	2	2	1	4	5	8	3	1	4	5	3

- (i). Draw the project network.
- (ii). Find the critical path. Determine the duration of the project.
- (iii). Suppose a piece of special equipment is required on activities 1-3, 3-6, 2-5, 5-8 and 8-9, one at a time. Will it affect the duration of the project?

**Question 27.**

Data relevant to a particular project are given below :

Activity	1-2	1-3	1-5	2-5	3-4	3-5
Duration	1	2	2	2	3	1
No. of workers	10	2	8	4	5	6

What will be the optimum schedule if not more than 12 workers are available on any day ?

**Question 28.**

The Madras Construction Company is bidding on a contract to install a line of microwave towers. It has identified the following activities, along with their expected times, predecessor restrictions and worker requirements :

Activity	Duration (Weeks)	Predecessors	Crew size, workers
A	4	-	4
B	7	-	2
C	3	A	2
D	3	A	4
E	2	B	6
F	2	B	3
G	2	D, E	3
H	3	F, G	4

The contract specifies the work to be completed in 14 weeks. The company will assign a fixed number of workers to the project for its entire duration and so it would like to ensure that the minimum number of workers is assigned and that the project is completed in the 14 weeks. Find a schedule which will do this.

(CA Final May, 1995)

**Question 29.**

The Madras Construction Company is bidding on a contract to install a line of microwave towers. It has identified the following activities, along with their expected times, predecessor restrictions and worker requirements :

Activity	Duration (Weeks)	Predecessors	Crew size, workers
A	6	-	3
B	3	-	2
C	2	-	2
D	2	C	1
E	1	B	2
F	1	D	1

The contract specifies the work to be completed in 6 weeks. The company will assign a fixed number of workers to the project for its entire duration and so it would like to ensure that the minimum number of workers is assigned and that the project is completed in the 6 weeks. Find a schedule which will do this.

Question 30.

The following information is available :

Activity	No. of days	No. of men required per day
A (1-2)	4	2
B (1-3)	2	3
C (1-4)	8	5
D (2-6)	6	3
E (3-5)	4	2
F (5-6)	1	3
G (4-6)	1	7

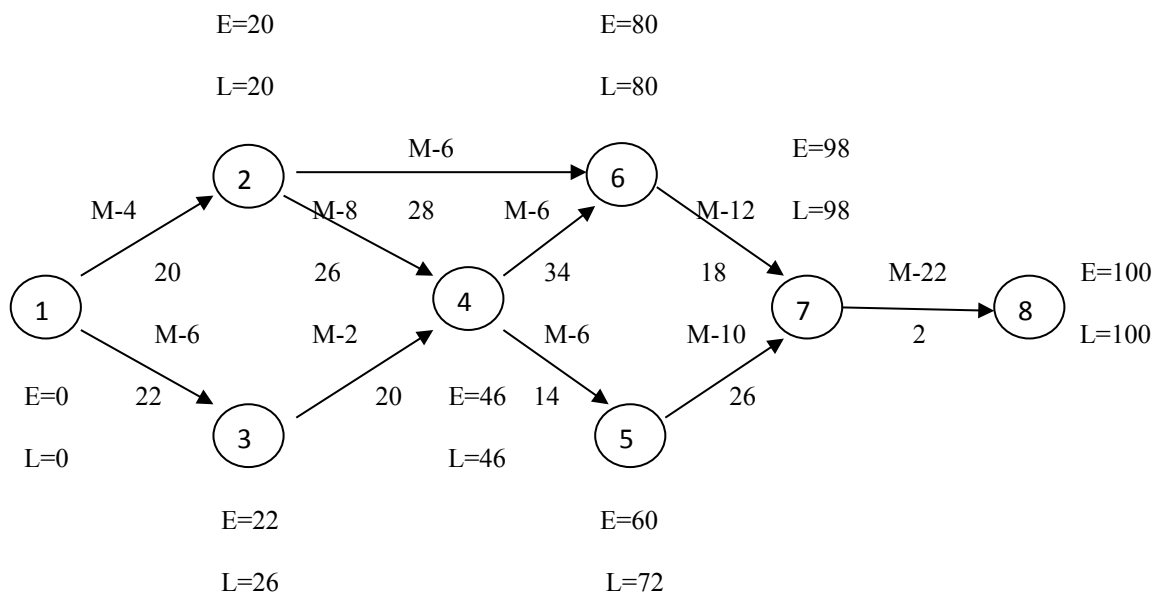
Required :

- (i) Draw the network and find the critical path.
- (ii) Find out the different type of float associated with each activity.
- (iii) Prepare time scale diagram .
- (iv) What is peak requirement of manpower? On which day(s) will this occur ?
- (v) If the maximum labour available on any day is only 10, when can the project be completed ?

Question 31.

Rearrange the activities suitably for levelling the audit executives with the help of time scale diagram if during first 52 weeks only 8 to 10 audit executives and during remaining days 16 to 22 audit executives can be made available.





\*M means no. of audit assistants required.

Question 32.

A project consists of 7 activities with the following information :

Activity	Normal time (days)	Crash time (days)	Normal Cost (Rs.)	Crash cost (Rs.)
1-2	4	3	600	800
1-3	2	2	400	400
1-4	5	4	750	900
2-3	7	5	400	600
2-5	7	6	800	1000
3-5	2	1	500	650
4-5	5	4	600	850

Indirect cost is Rs. 200 per day :

- Draw the network and find normal project duration and associated costs.
- Find the cost if the duration is to be reduced to 12 days.
- Find the cost if the duration is to be reduced to 11 days.
- Find the cost if the duration is to be reduced to 10 days.
- Find out the optimum duration with associated cost.

(CA Final May, 1995)

Question 33.

A small project consists of six activities with the following information :

Activity	Normal duration	Crash duration	Reduction cost per day (Cost slope)
1-2	9	6	20
1-3	8	5	25
1-4	15	10	30
2-4	5	3	10
3-4	10	6	15
4-5	2	1	40

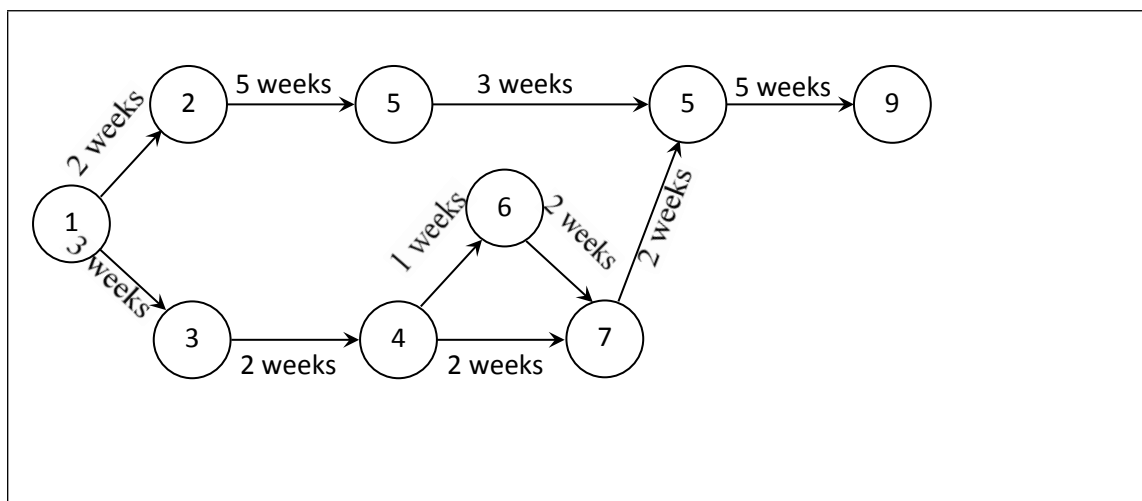
- (i). Draw the networks and obtain normal and minimum project durations.
- (ii). Find the additional cost of the project for the minimum project duration.
- (iii). Assume that overhead cost is Rs. 60 per day. What is the optimum duration?

(Adapted CA Final May 2002)

(Similar Question appeared in CA Final May 2013)

Question 34.

6.1. A net work is given below :



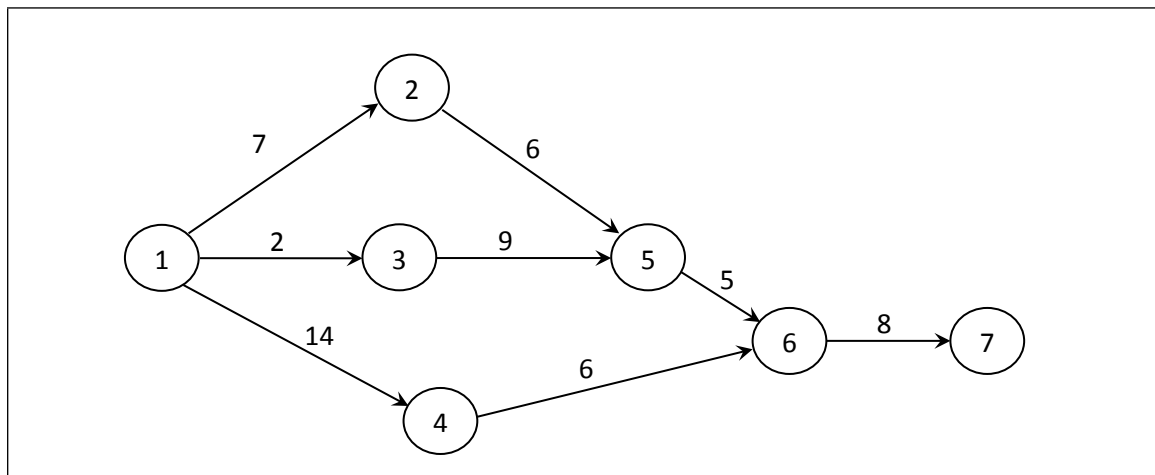
- (i). Name the paths and give their durations.

(ii). Give three different ways of reducing the duration of the project by 4 days.

(CA Final Nov. 2006)

Question 35.

At the end of activity 6-7, a project is to be launched and the date has been announced for the inaugural function, based on the normal duration of activities as given in the network below. Activities have been outsourced by the project manager to contractors A.B.C.D.E.F.G and H as indicated given below. Each contractor offers a discount on his contract price for each day given to him in addition to the normal days indicated in the network. What will be the maximum discount that the project managers may earn for the company without delaying the launch of the product?



Activity	Contractor	Discount/day	Activity	Contractor	Discount/day
1-2	A	300	3-5	E	400
1-3	B	200	4-6	F	1000
1-4	C	1200	5-6	G	600
2-5	D	500	6-7	H	500

(CA Final Nov. 2010)

Question 36.

The Arcot Machinery Company has been offered a contract to build and deliver mine extruding presses to the Home Bottling Company. The contract price negotiated is contingent upon meeting a specified delivery time, with a bonus offered for early delivery. The marketing department has established the following cost and time information:

Activity	Normal Weeks A: $t_0$	B: $t_p$	Time M: $t_m$	Normal Cost (Rs.)	Crash Time (Weeks)	Crash Cost (Rs.)
1 - 2	1	5	3	5000	1	9000
2 - 3	1	7	4	8000	3	14000
2 - 4	1	5	3	4000	2	6000
2 - 5	5	11	8	5000	7	6000
3 - 6	2	6	4	3000	2	5000
4 - 6	5	7	6	2000	4	3600
5 - 7	4	6	5	10000	4	14000
6 - 7	1	5	3	7000	1	10600

Normal delivery time is 16 weeks for a contract price of Rs. 62,000.

On the basis of the calculated profitability for each delivery time specified in the following table, what delivery schedule do you recommend that the company may implement?

Contract Delivery Time (Weeks)	Contract Amount (Rs.)
15	62,500
14	65,000
13	70,000
12	72,500

(Here  $a = t_0$ : optimistic time,  $b = t_p$ : pessimistic time,  $m = t_m$ ; Most Likely time.)

Answer:

Last crashing – 18.700.

It can be seen that the profit is maximum when the project duration is 13 weeks. Hence the company should implement the delivery schedule of 13 weeks at a contract amount of Rs. 70,000 to gain maximum profit of Rs. 21,000.

Question 37.

A small project is composed of 7 activities whose time estimates are listed below :

Activity	Optimum duration in weeks	Most likely duration in weeks	Pessimistic duration in weeks
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

Draw project network. Find expected duration and variance of each activity. What is the expected time of completion of the project? Calculate SD of the project length. What is probability of project being completed at least three weeks before the expected date of completion? What is the probability of the project being completed not more than 3 weeks of expected completion date?

Give :

Z	0.50	1.00	1.50	2.00	2.50
Probability	0.1915	0.3431	0.4332	0.4772	0.4938

**Question 38**

If the critical path of a project is 20 months along with a standard deviation of 4 month's. What is the probability that the project will be completed within?

- (a) 20 months (b) 18 months (c) 24 months (d) 17 to 22 months (e) before 19months (f) after 23 months.

**Question 39.**

Consider the following project :

Activity	Time estimates in weeks	Predecessors
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	<b>Optimistic</b>	<b>Most likely</b>	<b>Pessimistic</b>	
A	3	6	9	NONE
B	2	5	8	NONE
C	2	4	6	A
D	2	3	10	B
E	1	3	11	B
F	4	6	8	C, D
G	1	5	15	E

Find the critical path and its SD. What is the probability that the project will be completed by 18 weeks?  
(CA Final Nov. 1992)

Give :

Z	1.21	1.26	1.41	1.46	1.51
Probability	0.3849	0.3962	0.4207	0.4279	0.4345

Question 40.

One of the activities in a PERT problem has expected duration of 12 weeks with a standard deviation of 2 weeks. The most likely time estimate of this activity is 12 weeks. Calculate the optimistic and pessimistic time estimates for this activity.

(CA Final Nov. 1992)

Question 41.

A project consists of eight activities with the following time estimates :

<b>Activity</b>	<b>Time estimates in days</b>			<b>Predecessors</b>
	<b>Optimistic</b>	<b>Most likely</b>	<b>Pessimistic</b>	
A	1	1	7	NONE
B	1	4	7	NONE

C	2	2	8	NONE
D	1	1	1	A
E	2	5	14	B
F	2	5	8	C
G	3	6	15	D, E
H	1	2	3	F, G

- (i). Find the expected time for each activity.
- (ii). Draw PERT network.
- (iii). Determine the critical path.
- (iv). Determine the earliest event times and latest allowable times.
- (v). Determine the total slack for each activity.
- (vi). What is the probability that the project will be completed in (a) 22 days (b) 18 days (c) 19 days?
- (vii). What project duration will have 95% chance of completion ( $Z_{0.95}=1.65$ )?

**Give :**

Z	0.33	0.66	0.99	1.00	2.00
Probability	0.1293	0.2456	0.3389	0.3413	0.4772

(CA Final May 1991)

Question 42.

A project consists of eight activities with the following time estimates :

Activity	Time estimates in weeks		Optimistic
	Optimistic	Most likely	
1-2	21	7.5	3
1-3	27	8	3
2-4	8	8	8

2-5	3.5	2	0.5
3-5	10	10	10
4-5	1.7	1	0.3
4-6	9	7.5	3
5-6	5	3	1

- (i). Draw PERT network and identify all paths through it.
- (ii). What is the expected project completion time?
- (iii). Find the approximate probability of completing the project no more than 4 weeks later than expected. (CA Final Nov. 1987)

Question 43.

A project consists of eleven activities with the following time estimates :

Activity	Time estimates in weekly			Predecessors
	Optimistic	Pessimistic	Most likely	
A	2	4	3	NONE
B	8	8	8	NONE
C	7	11	9	A
D	6	6	6	B
E	9	11	10	C
F	10	18	14	C
G	11	11	11	C, D
H	6	14	10	F, G
I	4	6	5	E
J	3	5	4	I
K	1	1	1	H

- (i). Draw the PERT network.



- (ii). Compute the slack for each activity and determine the critical path.
- (iii). The contract specifies a penalty of Rs. 5000 per week for each week the completion of the project extends beyond 37 weeks. What is the probability that this company will have to pay a maximum penalty of Rs. 15000?

**Give :**

Z	1	1.09	1.18	1.48	1.58
Probability	0.1587	0.1379	0.1190	0.0694	0.0571

(CA Final May 1994)

Question 44.

A project consists of seven activities and the time estimates of the activities are furnished as under :

Activity	Optimistic Days	Most likely days	Predecessors
1-2	4	10	16
1-3	3	6	9
1-4	4	7	16
2-5	5	5	5
3-5	8	11	32
4-6	4	10	16
5-6	2	5	8

Required :

- (i). Draw the network diagram.
- (ii). Identify the critical path and its duration.
- (iii). What is the probability that the project will be completed in 5 days earlier than the critical path duration?
- (iv). What project duration will provide 95% confidence level of completion ( $Z_{0.95}=1.65$ )?

Give :

Z	1	1.09	1.18	1.25	1.33
Probability	0.1587	0.1379	0.1190	0.1056	0.0918

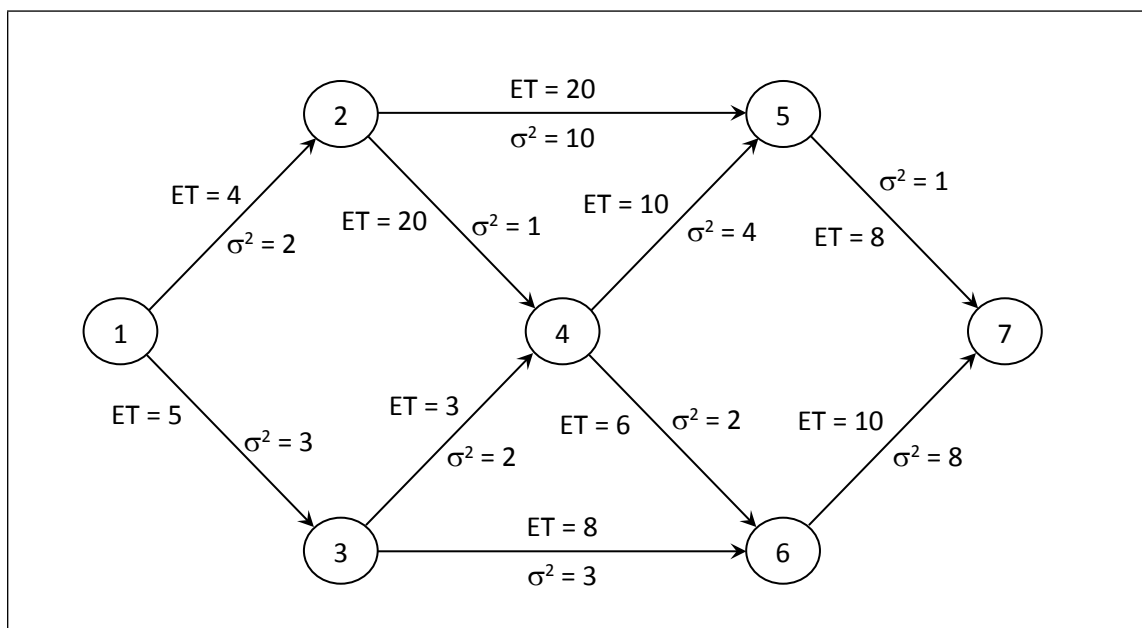
(CA Final Nov. 2008)

Question 45.

Given the following project network, determine :

- (i). Earliest expected completion time for each event.
- (ii). Latest allowable completion time for each event
- (iii). Slack time for each event
- (iv). The critical path
- (v). The probability that the project will be completed on schedule, if the scheduled completion time is 38.

**Project Diagram**



Give :

Z	1	1.11	1.18	1.25	1.41
Probability	0.1587	0.1335	0.1190	0.1056	0.0793

**Question 46.**

The following table gives the activities in a construction project and the time durations with associated probability of each activity:

Activity	Predecessors	Time (in days)	Probability
A	---	6	0.50
		8	0.50
B	--	4	0.30
		5	0.20
		6	0.50
C	A	8	0.50
		16	0.50
D	A, B	8	0.30
		10	0.70
E	C, D	2	0.20
		4	0.80

To simulate the project, use the following random numbers taking the first five random numbers digits (representing the five activities) for each trial and so on:

11, 16, 23, 72, 94; 83, 83, 02, 97, 99; 83, 10, 93, 4, 33; 53, 49, 94, 37, 7

**Required**

Determine the 'Critical Path' and the 'Project Duration' for each trial.