Portfolio Management {Imp. Formulas}

Important Formulas

Markowitz

1. Variance ($\sigma^2 x$) $= \dfrac{\Sigma (x-\bar{x})^2}{n}$ or $\Sigma P (x-\bar{x})^2$

2. S.D. (σx) $= \sqrt{\dfrac{\Sigma (x-\bar{x})^2}{n}}$ or $\sqrt{\Sigma P (x-\bar{x})^2}$

3. Covariance $(x,y) = \dfrac{\Sigma (x-\bar{x})(y-\bar{y})}{n}$ or $\Sigma P (x-\bar{x})(y-\bar{y})$

   $\mathrm{Cov}(x,y) = (x-\bar{x})(y-\bar{y})$ i.e. net $(x-\bar{x})(y-\bar{y})$

   = net risk of stock x · net risk of stock y

4. Co-relation Coefficient (r) $= \dfrac{\mathrm{Cov}(x,y)}{\sigma x \cdot \sigma y}$ = $\dfrac{\text{net risk of stock } x}{\sigma x} \cdot \dfrac{\text{net risk of stock } y}{\sigma y}$

5. Return of a portfolio = W_x · Return of x + W_y · Return of y - - - - - -

6. Risk of a portfolio ($\sigma^2 p$) = $w^2x \cdot \sigma^2 x + w^2 y \cdot \sigma^2 y$ + Remaining weights × Cov (x,y)

   or

   $w^2x \cdot \sigma^2 x + w^2 y \cdot \sigma^2 y + 2 \cdot W_x \cdot W_y \; \mathrm{cov}(x,y)$

7. Risk of a portfolio (σp) = $\sqrt{w^2x \cdot \sigma^2 x + w^2 y \cdot \sigma^2 y + 2 \cdot W_x \cdot W_y \; \mathrm{cov}(x,y)}$

8. Weights of a minimum Risk portfolio

$$W_x = \frac{\text{Risk red}^n \text{ of } y}{\text{Risk red}^n \text{ of both } x \text{ and } y} = \frac{\text{Variance of } Y - \text{Cov}(x,y)}{\text{Variance of } x - \text{Cov}(x,y) + \text{Variance of } y - \text{Cov}(x,y)}$$

$$\downarrow$$

$$\frac{\sigma^2 y - \text{cov}(x,y)}{\sigma^2 x - \text{Cov}(x,y) + \sigma^2 y - \text{Cov}(x,y)}$$

$$W_y = \frac{\text{Risk red}^n \text{ of } x}{\text{Risk red}^n \text{ of both } x \text{ and } y} = \frac{\sigma^2 x - \text{Cov}(x,y)}{\sigma^2 x - \text{Cov}(x,y) + \sigma^2 y - \text{Cov}(x,y)}$$

$$\Downarrow$$

OR

$$W_y = 1 - W_x$$

$\text{Cov}(x,m) = B_x \cdot \sigma^2 m$

CAPM

1. $\text{Beta}_x = \frac{\text{Cov}(x,m)}{\sigma^2 m}$ * OR $\frac{\text{ret}(x-\bar{x})}{m-\bar{m}}$ OR $\frac{\text{net risk of stock}}{\text{S.D. of market}}$

OR net risk of market

i.e. $\text{Beta} = \frac{\text{Net risk of stock}}{\text{S.D. of market}}$ *

$\therefore$ Net risk of stock $= \text{Beta} \cdot \text{S.D. of market}$

$= B \cdot \sigma m$ *

Now,

$\text{Cov}(x,m) = \text{net risk of stock} \cdot \text{net risk of market}$

$= B \cdot \sigma m \times \sigma m$

$= B \cdot \sigma^2 m$

$\left\{ B = \frac{\text{Cov}(x,m)}{\sigma^2 m} \right\}$

Again,

Covariance betⁿ stock x and stock y

$$Cov(x,y) = \text{Net risk of stock } x \cdot \text{Net risk of stock } y$$
$$= \beta_x \cdot \sigma_m \times \beta_y \cdot \sigma_m$$
$$= \beta_x \cdot \beta_y \cdot \sigma^2_m$$

or

$$Cov(x,y) = r_{xm} \cdot \sigma_x \cdot r_{ym} \, \sigma_y$$
$$= r_{xm} \cdot r_{ym} \cdot \sigma_x \cdot \sigma_y$$
$$= r_{xy} \cdot \sigma_x \cdot \sigma_y$$

$$r_{xy} = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y}$$

$$Cov(x,y) = r_{xy} \cdot \sigma_x \cdot \sigma_y$$

Co-relation co-efficient

$$r_{xm} = \frac{Cov(x,m)}{\sigma_x \cdot \sigma_m} = \frac{\text{net}(x-\bar{x})\cancel{(m-\bar{m})}}{\sigma_x \cdot \cancel{\sigma_m}} = \frac{\text{net risk of stock}}{\sigma_x}$$

$$r_{xm} = \frac{\text{net risk of stock}}{\text{S.D. of stock}} \qquad \beta_{xm} = \frac{\text{net risk of stock}}{\text{S.D. of market}}$$

CAPM formulas to Remember

① $\beta = \dfrac{Cov(x,m)}{\sigma^2 m}$ or $\dfrac{\text{Net risk of stock}}{\text{S.D. of market}}$ or $\dfrac{r_{xm} \cdot \sigma_x}{\sigma_m}$

② Net risk of stock $= \beta \cdot \sigma_m$

③ Net risk of stock $= r_{xm} \cdot \sigma_x$

④ $Cov(x,y) = \text{Net risk of } x \times \text{net risk of } y$
$$= \beta_x \cdot \sigma_m \times \beta_y \cdot \sigma_m$$
$$= \beta_x \cdot \beta_y \cdot \sigma^2_m$$

To calⁿ Net risk of stock

1. Market ke S.D. me apna Beta laga do

2. Apne S.D. me apna or market ka correlation i.e. r_{xm} laga do.

Jensen's Alpha

$\alpha = E(R) - Re$,

α positive = Underpriced

α negative = Overpriced

α is zero = Correctly Priced

SML $(Re) = Rf + (Rm - Rf)$ Beta → Beta should not be filled

Characteristic line $(E(R)) = \alpha + \beta \cdot Rm$ { Rf assumed to be zero }

Rm should not be filled.

Total Risk = Systematic Risk + Unsystematic Risk

σ^2_x ... $\beta^2 \cdot \sigma^2_m$ ... σ^2_{ei}

→ Unsystematic Risk = Total Risk − Syst. Risk

$$\text{Sharpe ratio} = \frac{E(R) - Rf}{S.D.}$$

$$\text{Treynor ratio} = \frac{E(R) - Rf}{\text{Beta}}$$

Arbitrage pricing theory

E(R) = Re = Rf + { Risk Premium × Factor 1 Sensitivity } + { Risk Prem × Factor 2 Sensitivity } + { Risk Prem × Factor 3 Sensitivity } + ...

Beta ... GDP ... Inflation

$= Rf + \{RP_1 \times F_1S\} + \{RP_2 \times F_2S\} + \{RP_3 \times F_3S\} \ldots\ldots$