# Chapter-6 = PERT | CPM

DATE 28 | 11 | 2016

## # Operation Research 0 to 30 Marks

(1) Assignment
(2) Transportation
(3) Simulation
(4) Linear Program
(5) Learning Curve
(6) Network Construction. (Pert/CPM)

# Operation Research is a Mathematical observation which is based on Past experience or Industry Average. This observation provides the results which are closer to the optimum / or optimum. In other words we can say such mathematical observation is to be applied where situations are uncertain. Uncertainity means uncertain situation here means.

(a) Estimation of Demand ⇒ SIMULATION.
(b) Estimation of Consumption of Time for New Product ⇒ LEARNING CURVE.
(c) ~~tak~~ Utilization of limited resources in optimum Manner ⇒ LINEAR PROGRAMING
(d) Allocation of Work within different Workers ⇒ Assignment
(e) Distribution Channel Manage ⇒ TRANSPORTATION
(f) Project Management. ⇒ NETWORK CONSTRUCTION.

## # Network Construction ⇒ (Pert/CPM)

(1) Construction of Network
(2) Crashing
(3) PERT Calculation
(4) Resource Smoothing & Allocation (updating of Network)

(1) **Network Construction** ⇒ Network is a arrow diagram, presented to the management Consisting / Presenting the details of all activities along with its consumption of time and cost, Instead of preparing a detailed Project Report.

With the help of Network diagram management may manipulate the resources to reduce the cost / Time.

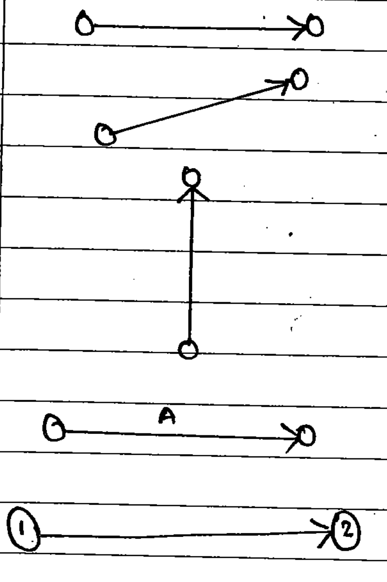
## Basic Term in Network Construction. ⇒

(1) **Activity :-** Activity means the work / task which Consumes Time & Cost. Activity always represented by an Arrow Connecting ~~to~~ TWO(2) Circle.

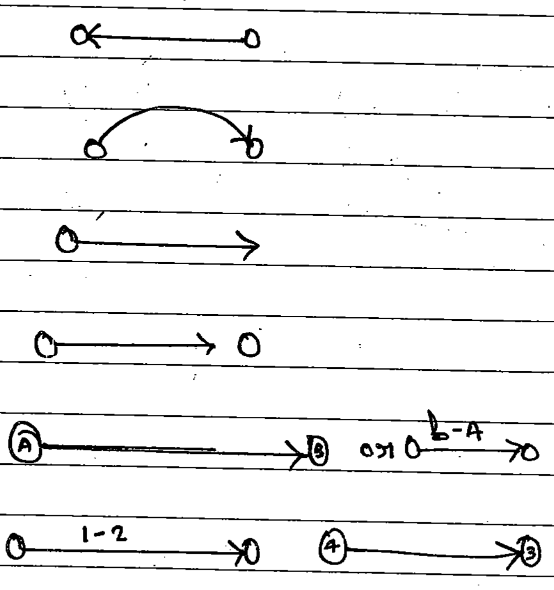
eg 

Activity should be represented by an Arrow in Forward / Upward / Downward but Not ~~Reverse~~ in Reverse direction & always presented by Straight line but not in curve.

**Correct Presentation.**



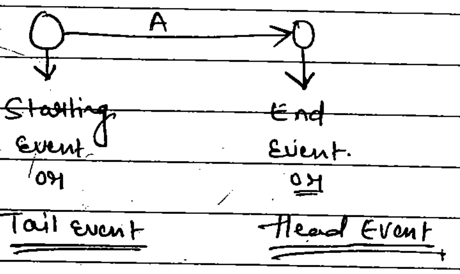
**Incorrect Presentation.**

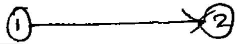


Activity should be presented ~~by~~ either by Alphabatically or Numerically & in Ascending order.

②
**#) Event** ⇒ Event means the point from we shall start any activity & complete event always represented by Circle.

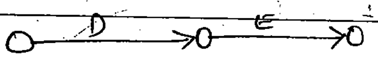
eg





③ **Preceeding & Succeeding Activity** ⇒ The activity which will start only after the completion other activity is called succeding Activity. Starting Activity will be known as succeeding Activity. Completing Activity will be known as Preceeding Activity.

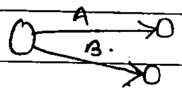
(eg.)



D= "Preceeding Activity"
E= "Succeding Activity"

④ **Simultanious Activity | Parallel.** → When two (2) Activity start with same point (event) then such Activity will be known as parallel or ~~Simult~~ simultaneous Activity.

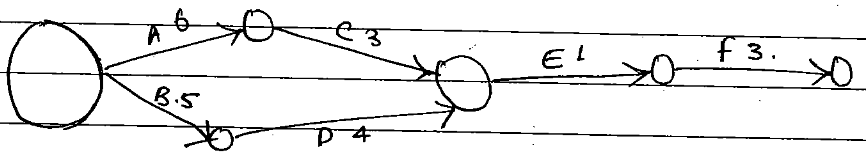
eg)



# # How to Draw the Network

## Ground Floor

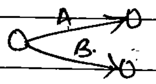
| | | Days |
|---|---|---|
| A : | Construction of front wall | 6 days |
| b : | Construction of side wall | 5 " |
| C : | Finishing of front wall | 3 days. |
| D : | finishing of side wall | 4 days |
| E : | flooring | 1 " |
| F : | Roof | 3 " |

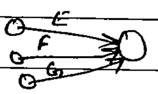


## Rule to draw the Network :-

(1) Direction of Network Should be from left to Right Instead of TOP to bottom.

(2) Starting Point & End point of the Network must be a single event.





(3) Network is not a Time scale Diagram. i.e, Just we should place the duration on the activity

(4) Presentation of Network may be in different Manner but maintain Preceeding / Succeeding relation.

(5) All activity should be presented either by Alphabatically / Numerically in ascending order.

(1)
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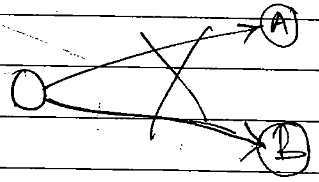

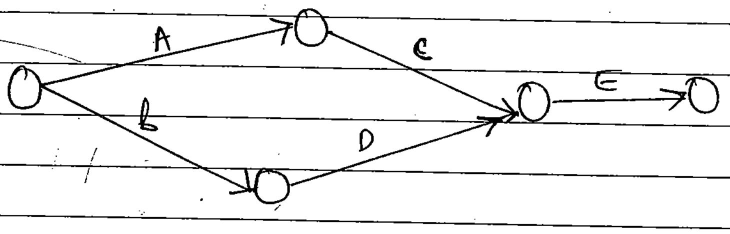

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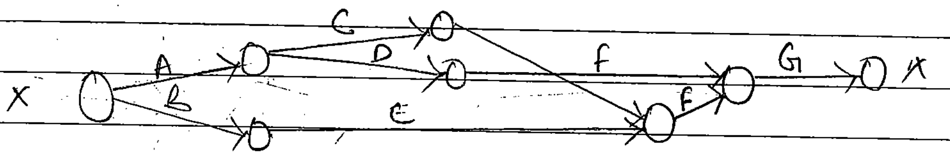

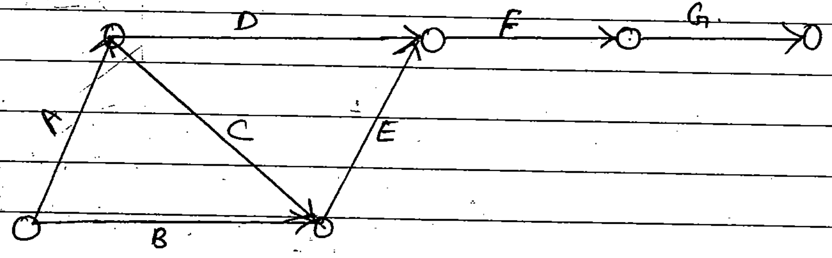

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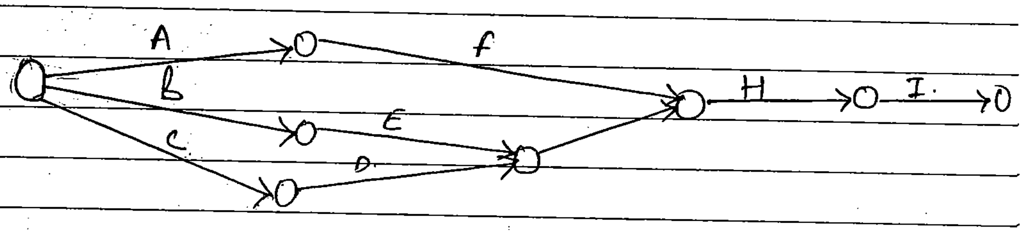

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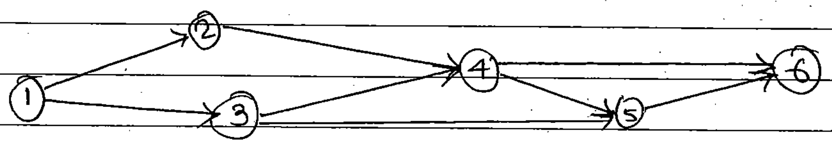

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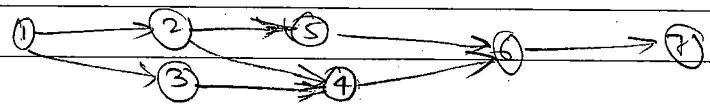
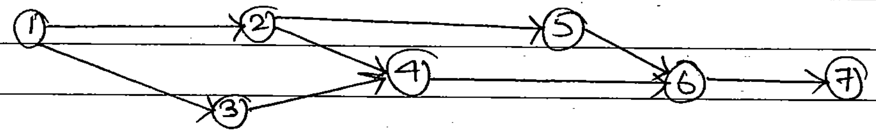
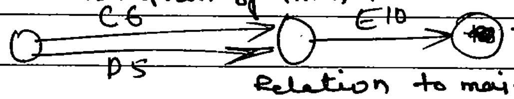




(5) Dummy Activity :- Dummy Activity means the activity which does not Consume any Time & Cost. Dummy Activity always represented by Dotted Lines & without indicating any alphabet. [Time ⇒ Productive Time]

Application ⟹ [Case-1] when Two activity ~~having~~ same ~~Start event & end event with different~~ Consumption of Time, then always we should

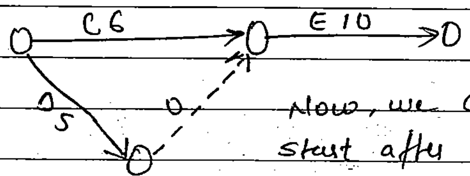
(#) Incorrect way



Introduce 'Dummy' Relation to maintain more logical.

'E' will start only after Completion of 'C' & 'D'.

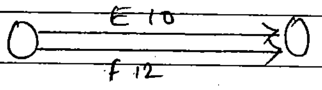
(#) Correct ways.



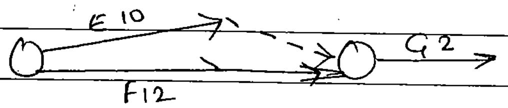
Now, we can say 'E' start after 'C' & 'D' one Particular Path incate only one direction.

Same distance having 6 days can't be represented for 5 days in order to have more logical, we should introduce a Dummy Relation.

Wrong



Correct



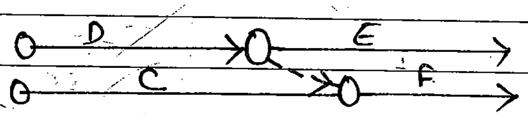
It means 'G' Start after E & F.

**Case No-2**

If any activity occur atleast two times in preceeding Coloumn i.e One time seperately and Other Time simuntaneous with Other Activity, then we should introduce a Dummy Relation to establish such relation.

(eg)

| Activity | Preceeding |
|---|---|
| E | D. |
| F | C, D. |



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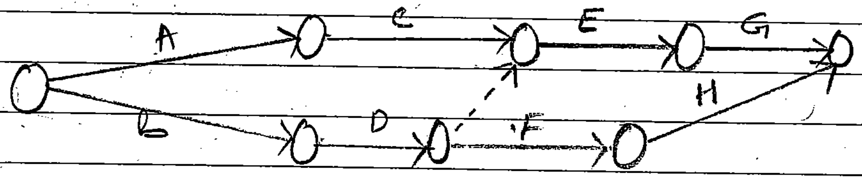

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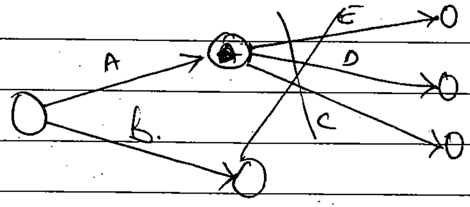

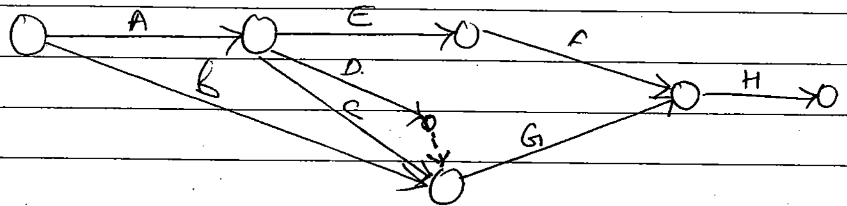

㊟ We should not Introduce dummy for 'B' & 'C' because B & C have same end ~~em~~ event but different Start event.

(7)
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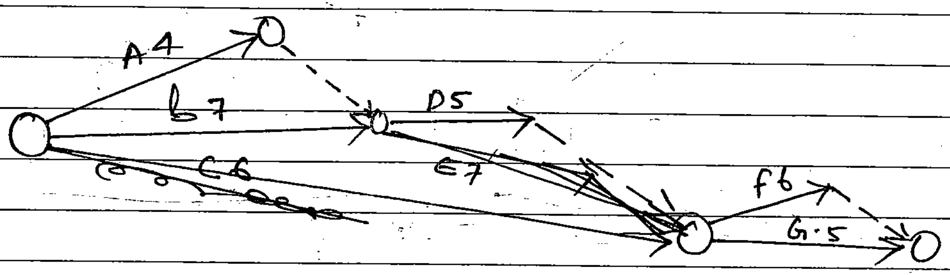

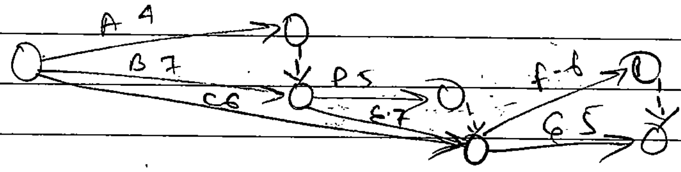

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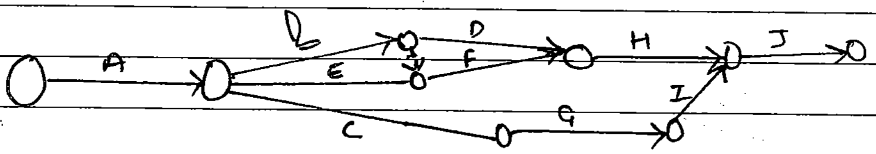

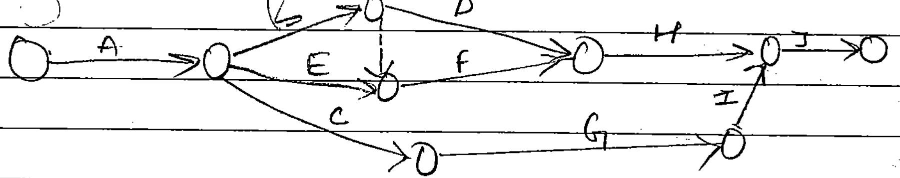

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(5)
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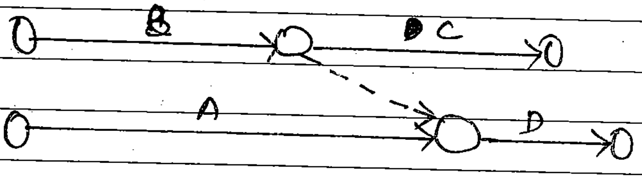
| Activity | Preceeding |
|---|---|
| C | B |
| D | A, B. |



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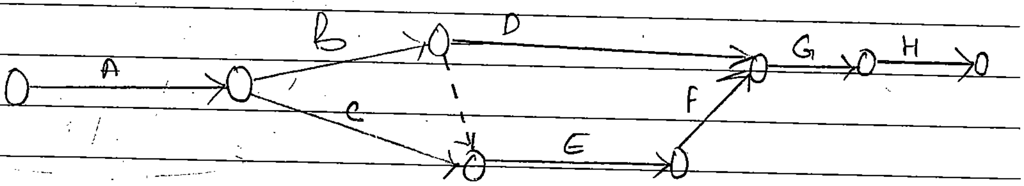

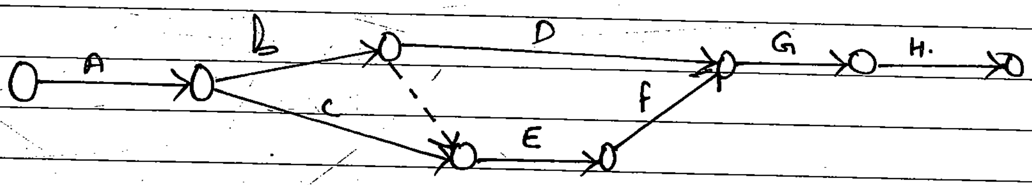

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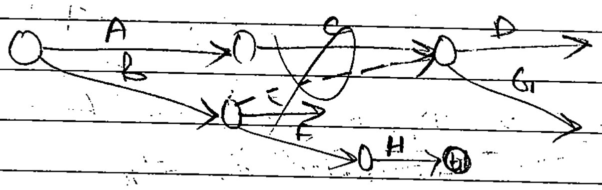

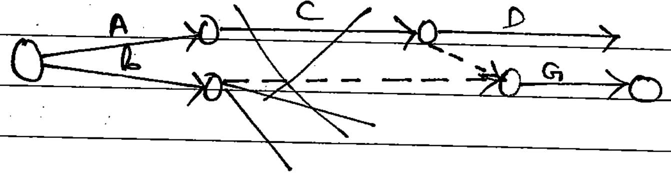

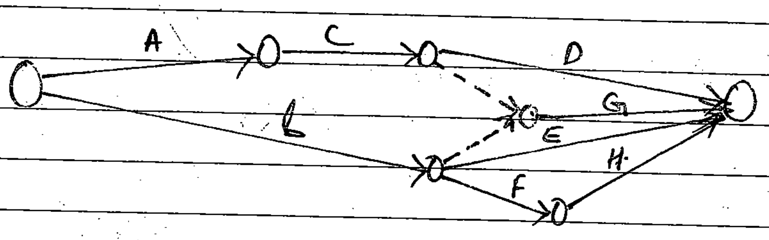

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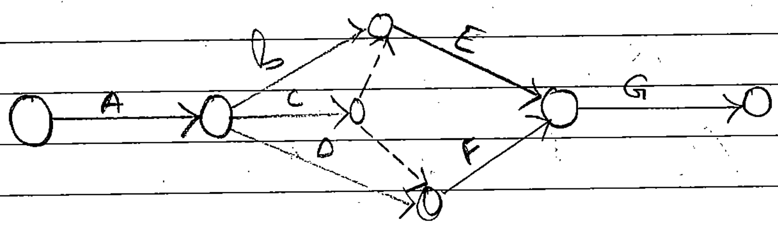

(11)
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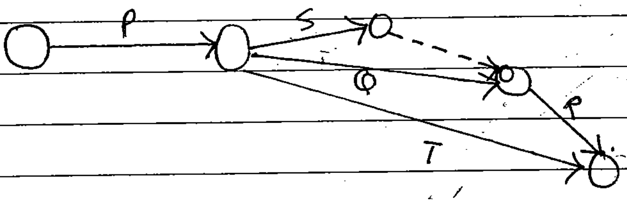
P & Q are known as : 'P' is: ~~succe~~ Preceeding for 'Q'
'Q' : succeeding for 'P'.

Correct Network. is a below



(12)
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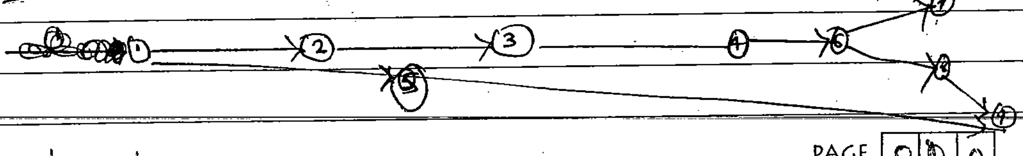
The following Errors Exists

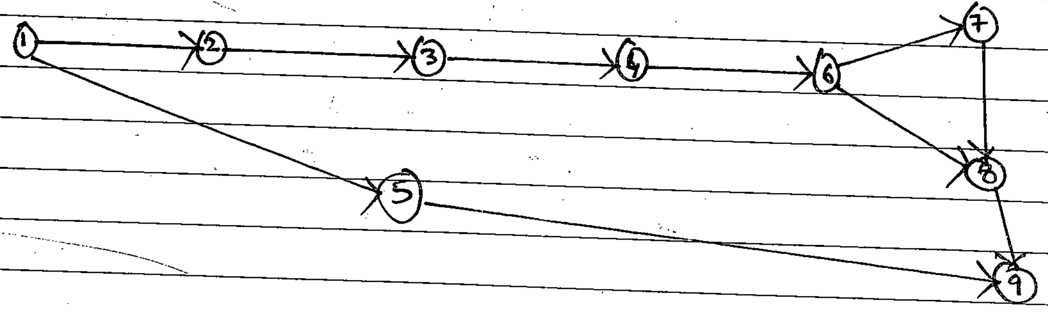
(a) Activity 2 to 3 represents only one relation. Hence it should be represented by only one Activity with straight line instead of two(2) curve shape

(b) Activity 2 to 5 should be represented by straight line

(c) 5 to 9 should be connected by an Activity because every Network end on a single event.

Correct Network is as

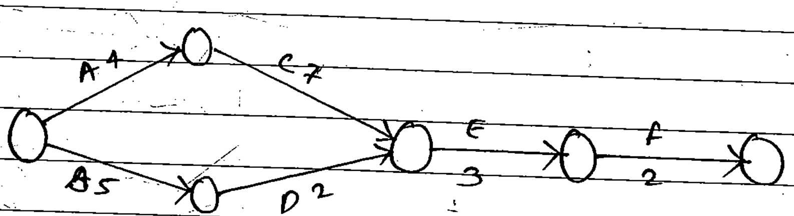




## # Basic team for Crashing.

(1). **Project Duration.** How much time the project will take to Complete i.e Project Completion Time.

(eg)

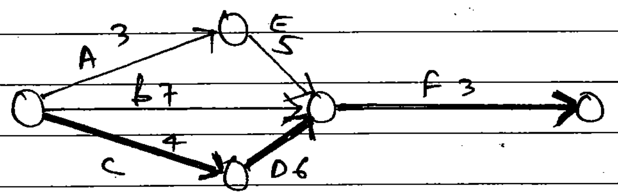


| S.No. | Possible Path. | Days. |
|---|---|---|
| 1 | A-C-E-F | 4+7+3+2 = 16 Days. |
| 2 | B-D-E-F | 5+2+3+2 = 12 Days |

∴ Project Duration= 16 Days.

# Identify all possible path from ~~bagging~~ begging to end independently and select the path having maximum Duration, such Path is known as Longest Path (Critical Path) & Such Duration is considered as Project Duration.

(eg) Identify Project Duration from the following Network

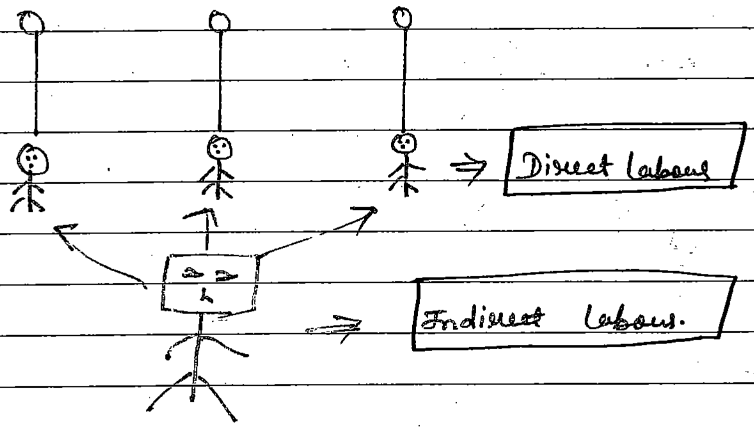


| S.No. | Possible Path | Days. |
|---|---|---|
| (1) | A-E-f | 3+5+3= 11 Days |
| (2) | b-f | 7+3= 10 Days |
| (3) | C-D-f | 4+6+3= 13 Days. |

∴ Project Duration= 13 Days. i.e CRITICAL PATH or Longest Path.

Direct cost are cost

(2) **Direct Cost :-** Which can be identified directly with a particular activity. It includes Direct Material, Direct Labour and Direct expenses.



Results Direct Labour Cost (Direct Cost) will Increase as the duration of any activity decrease

(eg) Present 4 Labour X 4 days X 100/day = ₹1600 [Labour Cost]

Now, 8 Labour ~~for~~ 2 days.

4 Labour X 2 days X ₹100/day ⇓ ₹800

4 (New Labour) X 2 days X ₹120/days ⇓ ₹960

₹800 + ₹960

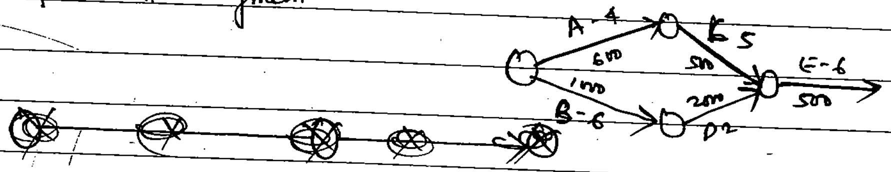
= ₹

(3) **Indirect Cost** :- Indirect Cost will reduce as the duration of activity (Project) reduces.

Indirect Cost here means

(a) Machine Rent

(b) Supervisor Payment. P.D = 15 Days.



Machine Rent = 1000/days.

∴ For 15 Day = 15000

∴ If Days Reduce = ∴ 13000.
Say 13 Day.

(4) **Total Cost of the Project**. Direct Cost of each activity xxx
+ Indirect Cost for Project Duration xxx

(5) **Cost Slope** Extra Cost per days.

(a) **Normal Time** :- Predetermined (scheduled) Time to complete an activity. 1-2 days. eg

| NT | NC | CT | CC |
|---|---|---|---|
| 6days. | 2000. | 4Days. | 3000 |

(b) **Normal Cost** :- Direct cost to be incurred to complete the activity.

(c) **Crash Time** :- Minimum Time to Complete an activity (It's not Possible to complete an activity earlier then a Particular time, such particular time is known as CRASH TIME)

(d) **Crash Cost** :- Direct Cost to be incurred to Complete an activity within Crash time.

Extra Cost = 3000 - 2000 = 1000

Days - Save = 6 - 4 = 2 days.

$\therefore \frac{1000}{2} = 500$/days.

$$\boxed{\text{Extra Cost/day [Cost slope]} = \frac{\text{Change in cost}}{\text{Change in Time}}}$$

$$= \frac{3000 - 2000}{6-4} = \frac{1000}{2} = 500/\text{days}.$$

$$\boxed{\text{Extra Cost/day (Cost Slope)} = \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crash Time}}.}$$

(eg) Calculate Cost Slope from the following.

| Activity | NT | NC(₹) | CT | CC ₹ |
|---|---|---|---|---|
| 5-6 | 7 days | 5000 | 5 days | 7000 |

Solution Cost Slope $= \frac{\text{Change in Cost}}{\text{Change in time}} = \frac{2000}{2} = 1000/\text{days}.$

This means

| | |
|---|---|
| 7 | 5000 |
| 6 | 6000 |
| 5 | 7000. |

We can't go for 4 days because before 5 days (Crash Time) its not possible to complete the activity

(#) Cost Slope (Extra Cost Per days) should be calculate for each activity seperately.

⑥ **Crashing** :- Crashing means reduction in Project duration with change in cost is known as Crashing.

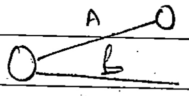
Change in Cost here means Cost may Increase or Cost may Decrease.

# Crashing Procedure How to reduce Project Duration?
(to be done by
Management Accountant)

(1) We should not reduce the activity which exist on Non Critical Path i.e, We should reduce the activity which ~~exist~~ on Critical Path. having least Cost Slope.

(2) We should Crash/Reduce activity which will reduce the project duration. so that the original Critical Path duration becomes equal to / closer to next longest Path. In other words we can say "Critical Path Should remain ~~critic~~ Critical" i.e, Critical Path will never ~~to~~ becomes non Critical but "Non Critical Path would becomes Critical."

(3) When original Critical Path duration & next longest Path duration becomes equal the Both Path Should be reduce Simultaniously. So that it becomes equal to or ~~to~~ closer to the next Longest Path.



3rd Class

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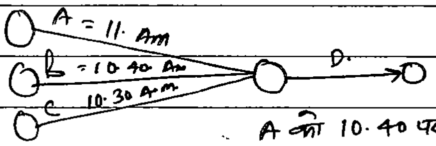
Summary Crashing Project duration reduce, Cost change

Cost slope = Extra Cost per day Reduction.

Total Cost ~~per day~~ = Direct Cost of each activity
of the project = + Indirect Cost of Project Duration

(1) least Cost slope exist on ~~ref.~~ critical Path. Critical Path should remain critical
both longest path should be reduced simuntaneously
closer to [II] i.e. (C).



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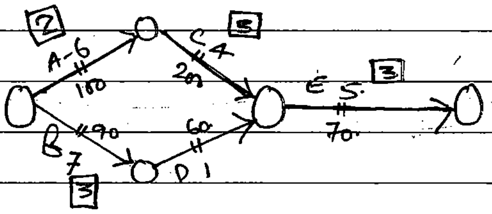
## (#) Points to be Considered in Crashing.

eg (1)



A = NT = 6
CT = 4

C = NT = 4
CT = 1

B = NT = 7
CT = 4.

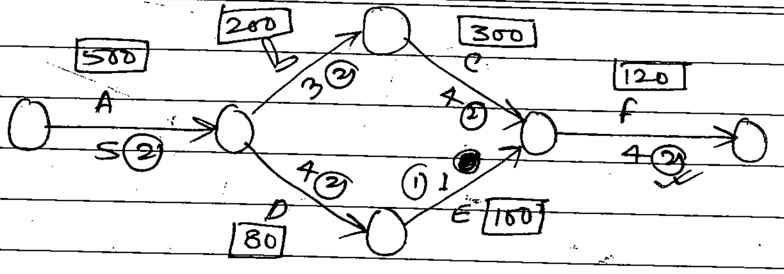
E = NT = 5
CT = 2

Critical Path = A-C-E = 15 ~~[illegible]~~ days.

Now Critical Path: A-C-E = 12 days. [6+4+2]
& Non critical Path = B-D-E = 10 days [7+1+2]

(#) If any activity exist on two (2) path or more path and we select that activity for crashing. It means other path will get automatically reduced.

(eg)
(2)



| | | |
|---|---|---|
| | Now(F) | (B) |
| Critical Path = A-B-C-F = 16 days. | 14 days | 12 day |
| Non Critical Path = A-D-E-F = 14 days. | 12 days | 12 day |

① 'F' activity should be reduced by 2 days.
② 'B' activity should be crashed by 2 days.
③ Crash 'C' on Ist Path & 'D' on IInd path.

(#) When during Crashing two (2) path becomes Critical then we should Crash both path simuntemeously by preparing Statement of Comparative Cost.

## (#) Steps for Crashing

(1) Draw Network Diagram.
(2) Identify all possible path (Critical & Non Critical, Path)
(3) Calculate Total Cost of the Project. & Calculate Possible reduction for each activity [CT-NT] along with cost slope.
(4) ~~Along with~~ Prepare statement of Crashing.
⑤ When all activity have been crashed in any Critical Path, than further Crashing couldn't be applied.

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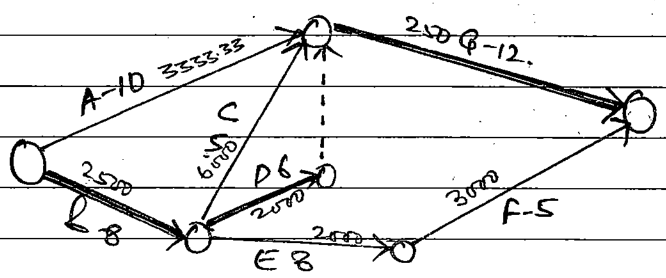

Possible Path

| | | | |
|---|---|---|---|
| 1) | A-G | 22 weeks | |
| 2) | b-C-G | 25 weeks | |
| 3) | b-D-G | 26 week | ⇒ Critical Path. |
| 4) | b-E-F | 21 week | |

Statement of Cost slope & Possible Reduction.

| Activity | Duration (weeks) | Cost Slope $\left[\frac{CC-NC}{NT-CT}\right]$ | Possible Reduction. |
|---|---|---|---|
| A | 10 | $\left[\frac{30000-20000}{10-7}\right]$ = 3333.33 | 3 weeks |
| b | 8 | $\left[\frac{20000-15000}{8-6}\right]$ = ~~10,000~~ 2500 | 2 " |
| C | 5 | $\left[\frac{14000-8000}{5-4}\right]$ = 6000 | 1 " |
| D | 6 | $\left[\frac{15000-11000}{6-4}\right]$ = 2000 | 2 " |
| E | 8 | $\left[\frac{15000-9000}{8-5}\right]$ = 2000 | 3 " |
| F | 5 | $\left[\frac{8000-5000}{5-4}\right]$ = 3000. | 1 " |
| G | 12 | $\left[\frac{4000-3000}{12-8}\right]$ 25[illegible] | 4 " |

Total Cost of Project

Direct Cost of each activity: 71000
Indirect Cost of Project Duration: 91000 [26 week × 500 ₹ × 7 day]
Total Cost = 1,62000

## Statement of Crashing

| Possible Path. | Normal ~~Original~~ Duration. | ① Duration. | ② Duration | ③ Duration | 4 Duration |
|---|---|---|---|---|---|
| B-D-G | 26 week | 22 week | 21 week | 19 week | 18 week |
| B-C-G | 25 week | 21 week | 21 week | 19 week | 18 week |
| A-G | 22 week | 18 week | 18 week | 18 week | 18 week |
| B-E-F | 21 week. | 21 week | 21 week | 19 week. | 18 week |
| TC | ₹1,62,000 | ₹ 149000 | ₹ 1,47,500 | ₹ 1,45500 | ₹1,52,000 |

Crashing I) Crashing activity 'G' by 4 weeks., TC = [162000 + (4 week × 250) − (4 week × 500 × 7)]
= 1,49,000.

(a) Crashing II ⇒ Crash activity 'D' by 1 weeks, [TC = 149000 + (2000×1) − (1×7×500)]
= ₹ 147500

Crashing III ⇒ Crash activity 'B' by 1 weeks, TC = [1,47500 + (2500×2 week) − (2×7×500)]
= ₹ 1,45500

Crashing IV ⇒ Crash activity 'D', 'C' & 'E' by 1 week on Path I, II, & III respectively, TC = [145500 + (2000+6000+2000) − (1×7×500)]
=

### Statement of Comparative Cost (18 week)

↳ तभी बनाते हैं जब दो पाथ को काम करता हो और Path common हो।

| Common Activity | least Cost slope other than common |
|---|---|
| B = 1 = 2500 | D = 1 − 2000 |
| | & C = 1 = 6000 |
| | & E = 1 = 2000 |
| 2500 | 10000 |

All activities have been crashed on Ist Critical Path, Hence Crashing Procedure could not be extended.

Optimum Duration = 19 week
Optimum Cost = ₹ 145500.

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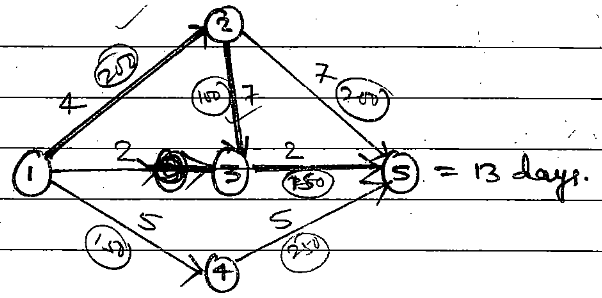

= 13 days.

150
200
150

| Possible Path | | Total Cost of the Project |
|---|---|---|
| 1-2-5 | = 11 days. | TC = 600+400+750+400+800+ |
| 1-2-3-5 | = 13 days. | 500+600 = 4050 |
| 1-3-5 | = 4 days | +Indirect Cost = [13days×500] 6500 |
| 1-4-5 | = 10 days. | Total Cost. ₹10,550 |

## Statement of cost slope & Possible Reduction

| | Cost slope | Possible Reduction |
|---|---|---|
| ✓1-2 | 200 | 1 ✓ |
| 1-3 | — | — |
| 1-4 | 150 | 1 ✓ |
| ✓2-3 | 100 | 2 ✓ |
| 2-5 | 200 | 1 ✓ |
| 3-5 | 150 | 1 ✓ |
| 4-5 | 250. | 1 |

## Statement of Crashing

| Possible Path | Normal Duration | Duration | Duration | Duration | Duration |
|---|---|---|---|---|---|
| 1-2-3-5 | 13 day | 11 days | 10 days | 9 days | |
| 1-2-5 | 11 days | 11 days | 10 days | 9 days | |
| 1-4-5 | 10 days | 10 days | 10 days | 9 days | |
| 1-3-5 | 4 days. | 4 days | 4 days. | 3 days. | |
| Total cost | ₹ 10,550 | ₹9750 | ₹9450 | ₹9450 | |

[Crashing - I] ⇒ Crash 2-3 activity for 2days. TC = 10,550 + (2days×100) - (2days×500) = 9750

[Crashing II] ⇒ Crash 1-2 activity for 1 days. TC = 9750 + (1day×200) - (1days×500) = 9450

[Crashing III] ⇒ Crash 3-5 in Path-I, 2-5 in Path II & 1-4 in Path III. for 1 day

[Crashing - IV] ⇒ TC = [9450 + (150+200+150) - (1×500)] = 9450

## Statement of Comparative Cost

| Common Activity | | other | | |
|---|---|---|---|---|
| 1-2 | 200 | 3-5 | 1 | 150 |
| ~~8~~ | | & 2-5 | 1 | 200 |
| | 200 | | | 350 |

All activities have been Crashed on Path I (Critical Path). Hence Crashing procedure could not be extended.

Optimum Duration: 9 days (having least cost)
Optimum Cost = ₹94501/—

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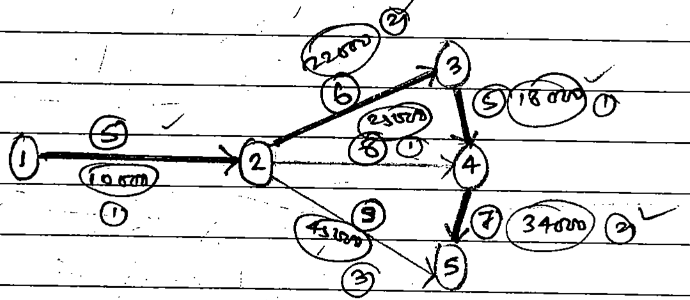

Possible Path.
1-2-5 = 14 days.
1-2-3-4-5 = 23 days. = Critical Path
1-2-4-5 = 20 days.

Statement of Cost slope & Reduction.

| | Cost slop | Reduction |
|---|---|---|
| 1-2 | 10000 | 1 |
| 2-3 | 22000 | 2 |
| 2-4 | 25000 | 1 |
| 2-5 | 45000 | 5 |
| 3-4 | 18000 | 1 |
| 4-5 | 34000 | 2. |

Statement of Crashing.

| Possible Path. | Duration. | Duration | Duration. | |
|---|---|---|---|---|
| 1-2-3-4-5 | 23 days | 22days | 21days | ~~79~~ days |
| 1-2-4-5 | 20 days | 19 days | 19 days | 19 days |
| 1-2-5 | 14 days. | 13 days. | 13 days | 13 days. |
| Total Cost. | ~~455000.~~ 570000 | 560000. | 558000. | 592000. |

[Crashing-1] Crash Activity 1-2 by 1 days.= Total Cost = ~~455000~~ 570000 + 10000 − 5000−15000 = 560000
(2) Crash Activity 3-4 by 1 days = Total Cost = 580000 + 18000 − 20000 = 558000
Crash Activity 2-3 by 2 days = Total Cost = 558000 + 44000 − 20000 = ~~580000~~ 597000.
Crash Activity 4-5 & by 2 days. Total Cost = 582000 + 68000 − 10000 = 640000.

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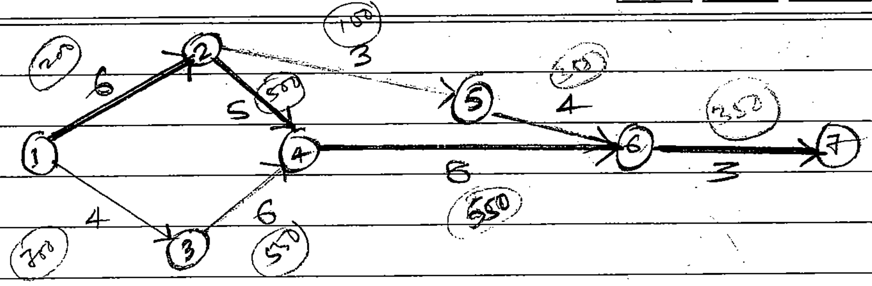

## Possible Path

1-2-5-6-7 = 16 days
1-2-4-6-7 = 22 days ⇒ Critical Path.
1-3-4-6-7 = 21 days.

Total Cost

Direct Cost = 4700
+ Indirect Cost = 8800
for 22days
(22×400)

13500

## Statement of Cost of Slope & Possible Reduction

| Activity | Cost of Slope $\left[\frac{CC-NC}{NT-CT}\right]$ | Possible Reduction |
|---|---|---|
| 1-2 | $\left[\frac{1000-600}{6-4}\right] = 200$ | 2 |
| 1-3 | $\left[\frac{2000-600}{4-2}\right] = 700$ | 2 |
| 2-4 | $\left[\frac{1500-500}{5-3}\right] = 500$ | 2 |
| 2-5 | $\left[\frac{650-450}{3-1}\right] = 100$ | 2 |
| 3-4 | $\left[\frac{2000-900}{6-4}\right] = 550$ | 2 ✓ |
| 4-6 | $\left[\frac{3000-800}{8-4}\right] = \cancel{600}\ 550$ | 4 ✓ |
| 5-6 | $\left[\frac{1000-400}{4-2}\right] = 300$ | 2 |
| 6-7 | $\left[\frac{800-450}{3-2}\right] = 350$ | 1. ✓ |

## Statement of Comparative Cost (1 days)

| Common Activity | Least other slope |
|---|---|
| 6-7 - 1days: 350 | 1-2 1 day 200<br>&<br>3-4 1 days 550 |
| 350 | 750 |

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## Statement of Crashing

| Possible Path. | Normal Duration | ~~No~~ Duration | Duration | Duration | Duration | Duration | Duration |
|---|---|---|---|---|---|---|---|
| 1-2-4-6-7 | 22 days | 21 days | 20 days | 16 days | 15 days | 14 days | 13 days |
| 1-3-4-6-7 | 21 days | 21 days | 20 days | 16 days. | 15 days | 14 day | 13 days |
| 1-2-5-6-7 | 16 days. | 15 days. | 14 days. | 14 days | 13 days. | 13 days | 13 days |
| | 13,500 | 13300 | 13,250 | 13850 | 14,200 | 14850. | 15650 |

[Crashing 'I'] = Crashing 1-2 activity for 1 days. Total Cost = [13500 + (1×200) - (1×400)]
= 13,300

[Crashing - II] = Crashing '~~1-2~~ 6-7' activity for 1 days. [Total Cost = [13300 + (1×350) - (1×400)]
= 13,250

[Crashing - III] = Crashing 4-6 activity by 4 days. Total Cost = [13,250 + (4×550) - (4×400)]
= 13850.

[Crashing - IV] = Crashing '1-2' & 3-4 activity by 1 days. Total Cost = [13850 + (1×200) + (1×550) - (1×400)] = 14200

[Crashing - V] = Crashing 2-4 & 3-4 activity by 1 days. Total Cost [14200 + (1×500) + (1×550) - (1×400)] = 14850

[Crashing VI] = Crashing 2-4 & 1-3 activity by 1 days = Total Cost [14850 + (1×500) + (1×700) - (1×400)] = 15650.

## Statement of Comparative Cost.

| Common Activity | | | Others least | | Slope. |
|---|---|---|---|---|---|
| 4-6 | 1 days. | ₹550 | 1-2 | 1 day | 200 |
| | | | & 4-6 | 1 day | 550 |
| | | 550 | | | 750 |

"Optimum Cost = 13250 with optimum Duration 20 days."

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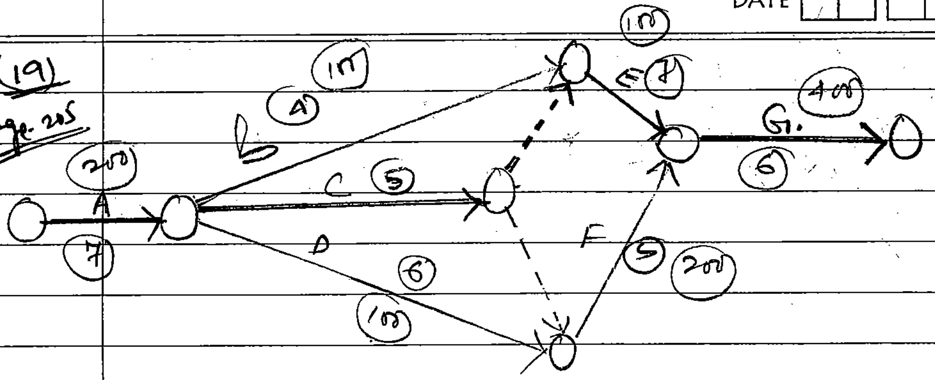

Possible Path = ~~7~~

A-b-E-G = 7+4+7+6 = 24 days.

A-C-E-G = 7+5+7+6 = 25 days → Critical Path.

A-C-F-G = 7+5+5+6 = 23 days.

A-D-F-G = 7+6+5+6 = 24 days.

Total Cost = ~~As~~ Direct Cost = 4500.

~~Costslope~~ Statement of Cost Slope & Reduction in Cost.

| | Costslope | Possible Reduction |
|---|---|---|
| A | 200 | 2day |
| b | 100 | 2 days. |
| C | 0 | 0 |
| D | 100 | 2days. |
| E | 100 | 3days. |
| F | 200 | 3 days. |
| G | 400 | 2 days. |

Statement of Crashing.

| Possible Path. | Normal Duration | Duration | Duration | Duration | Duration. | Duration |
|---|---|---|---|---|---|---|
| A-C-E-G | 25days. | 24days | 22days | 21 days | 20day | 18 day |
| A-b-E-G | 24 days. | 23days | 21 days | 20 days | 19 days | 17 days |
| A-D-F-G | 24 days | 24 days | 22 days | 21 days | 20 days | 18 days |
| A-C-F-G | 23 days | 23 days | 21 days | 21 days | 20 days | 18 days |
| Total Cost | 4500 | 4,600. | 5,000. | 5200 | 5500 | 6300 |

[Crashing - I] = Crashing 'E' activity by 1 days = Total Cost = [4500 + (1 × 100)] = 4600

[Crashing II] = Crashing 'A' activity by 2 days = Total Cost = [4600 + (2 × 200)] = 5000.

Now we should crash I & III Path
Simuntaneously hence we should
Prepare Cost of Comparative statement

| | |
|---|---|
| A - 1 days = 200. | E - 1 = 100<br>&<br>D - 1 = 100 |
| 200 | 200 |

[Crashing - III] =

Statement of Comparative Cost

| | |
|---|---|
| Common Activity | Others least Cost slope |
| G = 1 days 400 | E 1 days. 400<br>&<br>D 1 days 100 |
| ₹ 400 | ~~2~~00 |

(#) Although 'E' & 'D' can be reduced by 2 days, I, II & III will reduce but IV path could not be reduced, which result Critical Path becomes Non Critical.

∴ Crash 'E' & 'D' Activity by 1 days each

∴ Total Cost = [5000 + (1 × 100) + (1 × 100)] = 5200

[Crashing IV]

Statement of Comparative Cost

| | |
|---|---|
| Common Activity | Others least Cost Slope. |
| G = 1 days 400 | 'E' 1 days. 100. |
| | ~~'D'~~ ~~1 days~~ ~~100~~. |
| | 'F' 1 days 200. |
| 400 | Common 300 |

# we will have to reduce 'F' Activity for IV Path, which results, II Path will get automatically reduced. Hence no need to select 'D' Activity for II path.

∴ Crash 'E' activity in Path I & 'F' Activity in Path III & IV by 1 days.

Total Cost = [5200 + (1 × 100) + (1 × 200)] = 5500.

[Crashing V] = Crashing 'G' activity by 2 days.
Total cost = [5500+(2X400)] = 6300.

① Project Duration = 25 days.
& Minimum Duration = 18 days.

② change in Cost to Complete ~~a~~ the project in 21 days.

= ₹ 700 [5200 - 4500]

$\% = \frac{700}{4500} \times 100 = 15.56\%$

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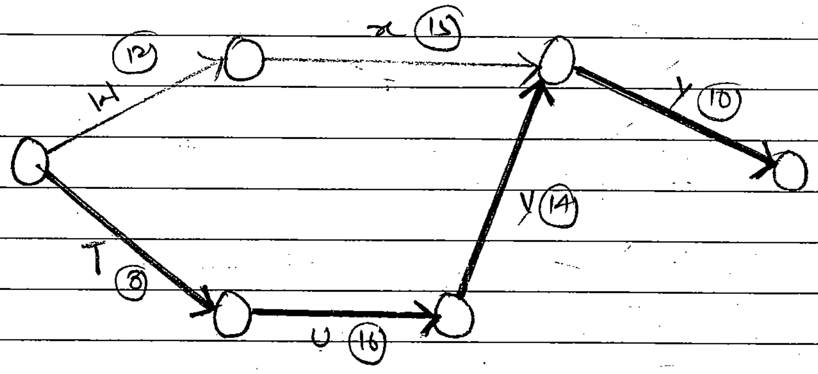


Possible Path.

T - U - V - Y = 48 day ⇒ Critical Path.
W - x - Y = 37 days

Total Cost = 12875. (2250 +1875+ 2250 +3000 +1000+2000)

Statement of Cost slope & Possible Reduction

| | Cost slope | Possible Red |
|---|---|---|
| T | 250 | 2 |
| U | 175 | 5 |
| V | 150 | 5 |
| W | 250 | 3 |
| x | 1500 | 1 |
| Y | 180 | 2 |

## Statement of Crashing.

| Possible Path | Normal Duration | Duration. | Duration | Duration. |
|---|---|---|---|---|
| $T - \overset{x}{U} - \overset{x}{V} - Y$ | 48days | 43 days | 38 days | 37days. |
| 250 175 150 180 | | | | |
| W – x – Y | 37days. | 37days. | 37days. | 37days |
| 250 1500 180. TC | ₹12875 | ₹13625 | ₹14500 | ₹14680 |

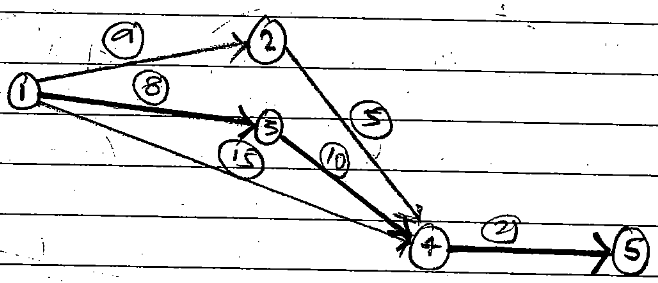
[Crashing – I] = Crash Activity 'V' by 5 days.
Total Cost = [12875 + (5 X 150)] = 13625.

[Crashing – II] = Crash Activity 'U' by 5 days.
Total Cost [13625 + (5X175)] = 14500

[Crashing – III] = Crash Activity 'Y' by 1 day =
Total Cost = [14500 + (1X180)] = 14,680

(#) Cost will be ₹ 14680 to Complete the project withing 37 days. i.e. reducing by 11 days.

⑳ ~~page-286~~



Possible Pathe.

1 – 2 – 4 – 5 = 16 day

1 – 3 – 4 – 5 = 20 days → Critical Path.

1 – 4 – 5 = 17 days.

## Statement of Possible Reduction

| | Reduction Duration |
|---|---|
| 1-2 | 3 days |
| 1-3 | 3 " |
| 1-4 | 5 " |
| 2-4 | 2 " |
| 3-4 | 4 " |
| 45 | 1 " |

## Statement of Crashing.

| Possible Path | Duration | Duration | Duration | Duration | Duration | Duration |
|---|---|---|---|---|---|---|
| 1- 3- 4- 5 | 20 days | 17 days | 16 days | 15 days | 13 days | 12 days |
| 4000 2000 6000 | | | | | | |
| 1- 4 -5 | 17 days | 17 days | 16 days | 15 days | 13 days | 12 days |
| 4500 6000. | | | | | | |
| 1 - 2 - 4- 5 | 16 days | 16 days | 15 days | 15 days | 13 days | 12 days |
| 2000 1500 6000 | Extra Cost ⟹ | 60000 | 120000 | 1,85000. | 3,85000 | 5,00,000 |

[Crashing - I] = Crash Activity '3-4' by 3 days.
Extra Cost = 3 days x 20000 = 60000

[Crashing - II] = Crash Activity '4-5' by 1 days.
Extra Cost = 1 days x 60000 = 60,000 + 60000 = 1,20,000

We should reduce Path I & II simuntaneously by Preparing

## Statement of Comparing Cost

| Common Activity | | | Other least cost slope | | |
|---|---|---|---|---|---|
| 4-5 | 1 days | 6000 | 3-4 | 1 day | 2000 |
| | | | & | | |
| | | | 1-4 | 1 day | 4500 |
| | | 6000. | | | 6500 |

[Crashing - III] = Crash Activity '3-4', 1-4, Activity in Path I & II respectively.
Extra Cost = 1,20,000 + 2000 + 4500 = 1,85000

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[Crashing-IV] = I, II & iii Path should be simuntaneouly reduced.
1-3, 1-4, & 2-4 by 2 days each.

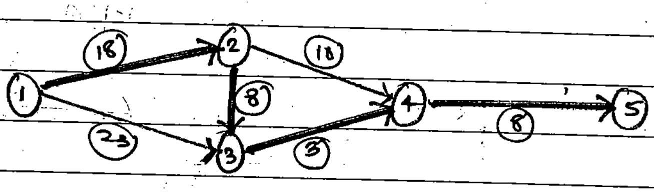
Extra Cost = 1,85000 + (2days x 40000) + (2days x 45000) + (2x15000)]
= 1,85000 + 200000
= 3,85000.

[Crashing V] = I, II & iii Path should be reduced simuntaneously.
1-3, 1-4, & 1-2, by 1 day each.

Extra Cost = 3,85000 + (1 x 40000) + (1 x 45000) + (1 x 30000)
= 3,85000 + 1,15000
= 5,00000.

① Normal Duration - 20 days
Minimum Duration = 12 days.

(ii) Additional Cost involved for Complete the work in the Shortest duration: ₹ 5,00,000.



Possible Path
1-2-4-5 = 36 days.
1-2-3-4-5 = 37 days. - [Critical Path]
1-3-4-5 = 34 days.

Statement of Crashing.

1-2-3-4-5
1-2-4-5
1-3-4-5.
Total Cost

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Exam Q Repeated

Network

| C.P.M. [Critical Path Method] | PERT. [Programme Evaluation Review Technique] |
|---|---|
| ① Regular Project | ① First (1st) Time Project |
| ② Developed in Industry since long period of Time. | ② Developing in Industry as New project. |
| ③ Consumption of Time for each activity/as well as for project always Certain/predetermined | ③ Consumption of Time for each activity as well as for project duration is uncertain. |
| ④ No Fluctuation Exist. Always Exist Certainity | ④ Fluctuation & Uncertainity always exist. |
| ⑤ Time of project will change only when predetermined activity change. | ⑤ Event Oriented Project |
| | ⑥ Innovations Projects like introduction of New Project/ Work for Us as well as Industry. eg. Mobile. |

(#) Method of Construction of Network, is same for C.P.M/PERT/ Crashing Procedure, also. Only difference is "To calculate Project Duration in PERT"

[Q.1] How to Calculate Project Duration in PERT ?

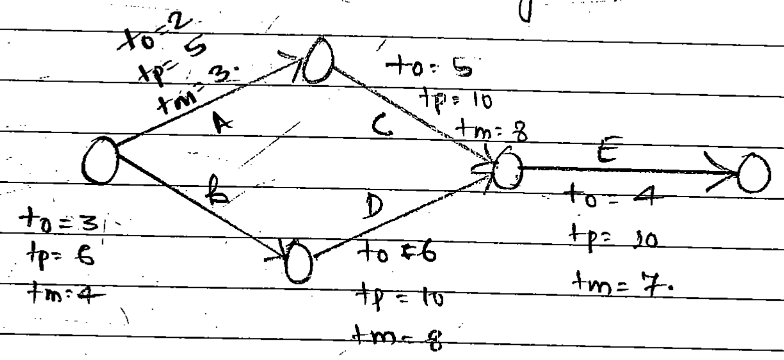
**Research Work** :- They observed Three (3) types of Timingx

(a) **Optimistic Time. ($t_o$)**: Minimum Time to Complete an Activity when all Eventx becomes favourable. i.e Resource will be managed as per Scheduled.

(b) **Pessimistic Time ($t_p$)**: Maximum Time to Complete an Activity When all eventx become Unfavourable. Resource (material/Labour/Machine) may/could not be managed as per schduled.

(c) **Mostlikely Time ($t_m$)**: When all event occur in normal Condition i.e, neither favourable nor Adverse, Consumption of Time is known as Most likely, Time.

(eg)



(#) Itx not possible to say "All Activity will occur either in itx $t_o$, $t_p$, $t_m$"

Some Activity will Occur in itx $t_o$.

" " " " " " $t_p$

" " " " " " $t_m$

(#) : **They Developed a Result.**

① Trial$_1$ (Avg) $\frac{t_o + t_p + t_m}{3} = t_e$ —— XX (Result is not Proper).

Because every time has equal possibility while there will /may be possibility of ~~tm~~ is higher as compared to $t_o$ and $t_p$.

② Trial 2 ⇒ $\frac{t_o + t_p + 2t_m}{4} \Rightarrow t_e$ ××

③ Trial 3 ⇒ $\frac{t_o + t_p + 3t_m}{5} \Rightarrow t_e$ ××

④ $\boxed{\text{Trial 4}}$ ⇒ $\boxed{\frac{t_o + t_p + 4t_m}{6}}$ ⇒ Good to some extent

(#) They observed that the relation $\boxed{t_e = \frac{t_o + t_p + 4t_m}{6}}$ provides the Result more appropriately as Compared to others, it becomes a Standard Relation to Calculate average time ($t_e$) for each activity & Consequently Calculate Project Duration.

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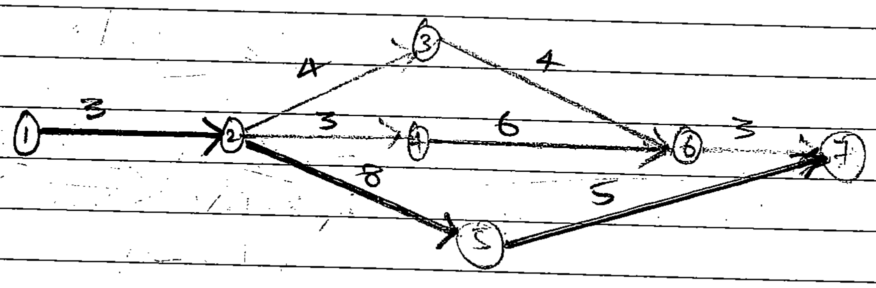
## Statement of Avg Time & Cost Reduction & Cost Slope.

| Activity | $\left[t_e = \frac{t_o + t_p + 4t_m}{6}\right]$ | Cost Slope | Possible Reduction |
|---|---|---|---|
| 1-2 | $\frac{1+5+(4\times3)}{6}$ = 3 week | $\left[\frac{9000-5000}{3-1}\right]$ = 2000 | 2 weeks. |
| 2-3 | $\left[\frac{1+7+(4\times4)}{6}\right]$ = 4 weeks | $\left[\frac{14000-8000}{4-3}\right]$ = 6000 | 1 weeks. |
| 2-4 | $\left[\frac{1+5+(4\times3)}{6}\right]$ = 3 weeks | $\left[\frac{6000-4000}{3-2}\right]$ = 2000 | 1 week |
| 2-5 | $\left[\frac{5+11+(4\times8)}{6}\right]$ = 8 week | $\left[\frac{6000-5000}{8-7}\right]$ = 1000 | 1 week |
| 3-6 | $\left[\frac{2+6+(4\times4)}{6}\right]$ = 4 weeks | $\left[\frac{5000-3000}{4-2}\right]$ = 1000 | 2 week |
| 4-6 | $\left[\frac{5+7+(4\times6)}{6}\right]$ = 6 week | $\left(\frac{3600-2000}{6-4}\right)$ = 800 | 2 week |
| 5-7 | $\left[\frac{4+6+(4\times5)}{6}\right]$ = 5 weeks | $\left[\frac{14000-10000}{5-4}\right]$ = 4000 | 1 week |
| 6-7 | $\left[\frac{1+5+(4\times3)}{6}\right]$ = 3 weeks. | $\left[\frac{10600-7000}{3-1}\right]$ = 1800 | 2 week. |



Possible Path.

①—②—③—⑥—⑦ = 14 weeks

①—②—④—⑥—⑦ = 15 weeks

①—②—⑤—⑦ = 16 weeks. ⇒ Critical Path.

## Statement of Crashing.

| Possible Path. | Duration | Duration | Duration | Duration | Duration. |
|---|---|---|---|---|---|
| ①—②—⑤—⑦<br>2000 1000 4000 | 16 week | 15 weeks | 14 week | 13 week<br>1 | 12 week |
| ①—②—④—⑥—⑦<br>2000 2000 800 1800 | 15 Week | 15 weeks | 14 week | 13 Week | 12 week |
| ①—②—③—⑥—⑦<br>2000 6000 1000 1800 | 14 weeks | 14 week. | 13 week | 12 week | 12 week |
| Total Cost | 44000. | 45000 | 47000 | 49000 | 53,800 |
| Profit: | 56000 | 65000 | ~~81000~~ 73000 | 81000 | 96200 |

[Crashing 1] = Crash Activity '2-5' by 1 week = Total Cost: 44000+1000 : 45000

[Crashing-2] = Crash Activity '1-2' by 2 week = 45000+2000 = ~~49000~~ 47000

~~[Crashing 3]~~ = ~~Crash Activity~~

[Crashing 2] = Crash Activity '1-2' by 1 week = 45000 + 2000 = 47000.

## Statement of Comparative Cost.

| Common | | | | Other least Cost slope | | |
|---|---|---|---|---|---|---|
| 1-2 | — 1 weeks | 2000 | | 5-7 | 1 week | 4000 |
| | | | | & 4-6 | 1 week | 8000 |
| | | 2000 | | | | 4800 |

[Crashing 3] = Crash 1-2 by 1 week (47000 + 2000) = 49000.

[Crashing 4] = Crash '5-7, '4-6' By 1 weeks = TC = 49000 + 4000 + 800
= 53800

∴ Its better to Complete the project within 12 weeks with higher Amt of Profit i.e.(£150000 − 53800) = 96200

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## PERT Calculation [Careerbased Calculation/Question]

(1) Concept of Standard Deviation. (Variances)

(2) Area under Normal Distribution.

## (#) Standard Deviation :- (Normal Data) (Avg of All Deviation)

Higher the SD $\Longrightarrow$ Higher the Risk & vise versa.
Standard Deviation is a measure to evaluate the Risk factore.

## (#) Standard Deviation in Project. (PERT)

**Observation[I]** :- Normally every activity has its (te) ie Normal time (Avg) just closer to its (tm) or equal to (tm).
mean Variation does not occur due to [tm] time but variation occur due to tp & to time.
Hence They observed a relation of variance ie, Difference of to & tp over $\boxed{6}$ No. of Times.
i.e, $\left[\frac{tp - to}{6}\right]^2$ is the variance time for each Activity.

**Observation (II)** :- Calculate Variance for only those activity which exist on Critical Path (Longest Path)

∴ Project Variance = Addition of all variance exist on Critical Path.

∴ Project S.D = $\sqrt{\text{Project Variance}}$

eg Project Duration = 20 days.
SD = 3days.

This means: Project may be complete in 17 day (20 day) 23 day
3day 3days.

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## (#) Steps PERT Calculation

(1) Calculate Average Time (te) for each activity

$$\left(Te = \left[\frac{to + tp + 4tm}{6}\right]\right)$$

(2) Draw Network & identify Critical Path

③ Calculate Variance of activity exist on Critical Path.

i.e, $\left(\frac{tp - to}{6}\right)^2$

④ Add Variance of all Critical Path = Project Variation.

Project S.D $= \sqrt{\text{Project Variance}}$

project S.D. means : Variation in project Duration i.e, Project may complete Early or delay with variance of its S.D.

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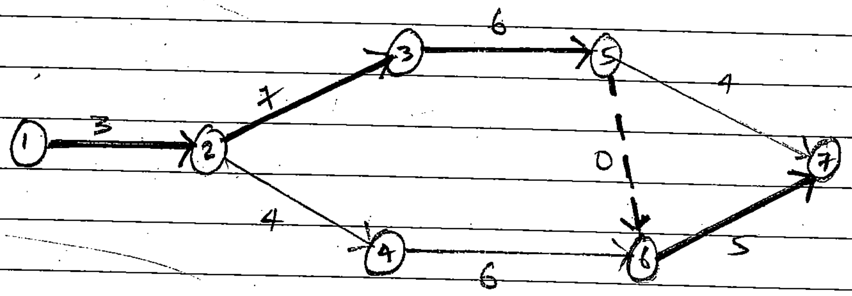
① Statement of Project Time & Variance.

| Activity | $te = \frac{to + tp + 4tm}{6}$ | Variance $= \left(\frac{tp - to}{6}\right)^2$ |
|---|---|---|
| 1-2 | 3 weeks | 0 |
| 2-3 | 7 weeks | $\left(\frac{12-6}{6}\right)^2 =$ 1 week |
| 2-4 | 4 weeks | $\left(\frac{6-2}{6}\right)^2 =$ 4/9 week |
| 3-5 | 6 week | $\left(\frac{8-4}{6}\right)^2 =$ 4/9 week |
| 4-6 | 6 week | $\left(\frac{8-4}{6}\right)^2 =$ 4/9 week |
| 5-6 | ~~4 weeks~~ | 0 |
| 5-7 | 4 week | $\left(\frac{5-3}{6}\right)^2 =$ ~~0~~ 1/3 Week |
| 6-7 | 5 week | $\left(\frac{8-2}{6}\right)^2 =$ 1 weeks. |



Possible Path.

1 - 2 - 3 - 5 - 7 = 20 days.
1 - 2 - 4 - 6 - 7 = 18 days
1 - 2 - 3 - 5 - 6 - 7 = 21 day ⟹ Critical Path.

② Project Variance.
1-2-3-5-6-7 = $0 + 1 + 4/9 + 0 + 1 = \frac{22}{9}$ weeks.

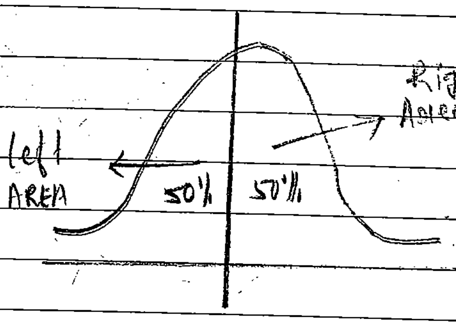
③ Project SD = $\sqrt{\frac{22}{9}}$ = 1.563 week.

i.e, $19.437 \xleftarrow[\text{1.563 weeks}]{} \underline{21 \text{ weeks}} \xrightarrow[\text{1.563 weeks}]{} 22.563$ weeks.

## # Calculation of Probability. for the Completion of Project within a particular period of Time.

Probability :- Area :- Area under Normal Distribution

Normal Distribution



$$\boxed{Z = \frac{x - \bar{x}}{SD}}$$

**Observation** Area (Prob) in all field will be almost same if we actual Data in a % forms.

% here means = Deviation of Actual data from its mean as relation of its (%) SD.

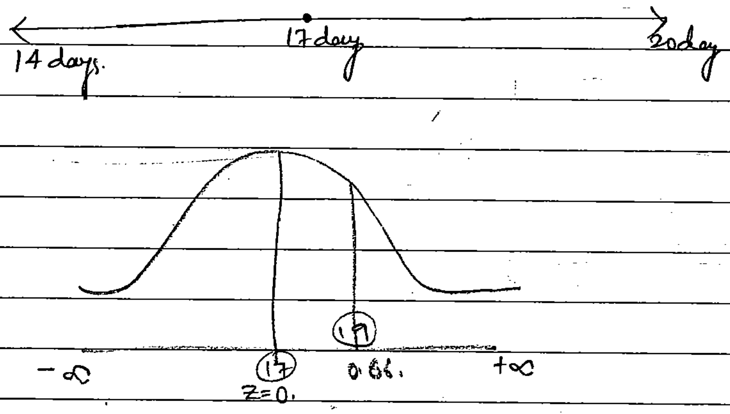
% to be ~~taken~~ known a 'Z' Standrize value.

i.e, $\boxed{Z = \frac{x - \bar{x}}{SD}}$

$x$ = Actual value
$\bar{x}$ = Project Duration Critical Path.
SD = SD of Project Duration.
Z = Standardize value.

**(eg)** What is the probability the project would to be Completed within 19 days. having Project length 17 days with standard Deviation = 3 days.

**Solution**



$= P(x \leq 19 \text{ days})$

$= P\left(Z \leq \frac{x - \bar{x}}{SD}\right)$

$= P\left(Z \leq \frac{19-17}{3}\right.$

$= P(Z \leq 0.66)$

$= P(-\infty \text{ to zero}) + P(\text{zero to } 0.66)$

$= 50\% + 0.2456$

$= 0.7456$

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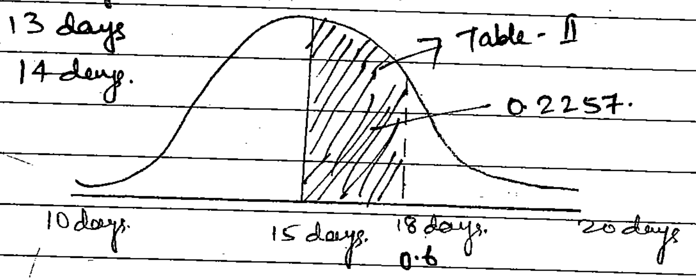
(eg) What will be the probability to complete the project within 18 days having 5 days SD, Project Duration 15 Days.

19 days?
~~17 days~~
13 days
14 days.



㊉ $= P(X \leq 18 \text{ days})$

$= P\left(Z \leq \frac{x-\bar{x}}{SD}\right)$

$= P\left(Z \leq \frac{18-15}{5}\right.$

$= P(Z \leq 0.6)$

$= -\infty$ To zero + zero To + ~~∞~~Z

$0.5 + 0.2257$

$= 0.7257$

i.e, 72.57%.

㊉ $P(x \leq 19 \text{ days})$

$P\left(Z \leq \frac{x-\bar{x}}{SD}\right)$

$P = \left(Z \leq \frac{19-15}{5}\right)$

$= P(Z \leq 0.8)$

$P(Z \leq 0.8)$

$= -\infty$ To zero + zero To + ~~∞~~ Z

$= 0.5 + 0.2881$

$= 0.7881$ or, 78.81%

㊉ ~~P~~ $P\left(Z \leq \frac{x-\bar{x}}{SD}\right)$

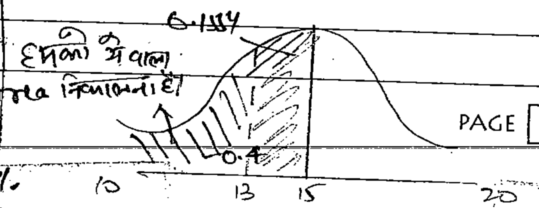
$P\left(Z \leq \frac{14-15}{5}\right)$

$= P(Z \leq -0.2)$

$= P(Z \leq 0.0793)$

$P = 0.5 - 0.0793$

$= 0.4207$ or 42.07%.



㊉ $= P(x \leq 17 \text{ days})$

$= P\left(Z \leq \frac{x-\bar{x}}{SD}\right)$

$= P\left(Z \leq \frac{17-15}{5}\right)$

$= P(Z \leq 0.4)$

$= -\infty$ To zero + zero To + ~~∞~~Z

$= 0.5 + 0.1554$

$= 0.6554$ or 65.54%.

㊉ $= P(x \leq 13 \text{ days})$

$= P\left(Z \leq \frac{x-\bar{x}}{SD}\right)$

$P\left(Z \leq \frac{13-15}{5}\right)$

$P(Z \leq -0.4)$

$=$ ~~$-\infty$ To zero − zero to~~

$= -\infty$ To zero − Z to zero

$= 0.5 - 0.1554$

$= 0.3446$ or 34.46%.

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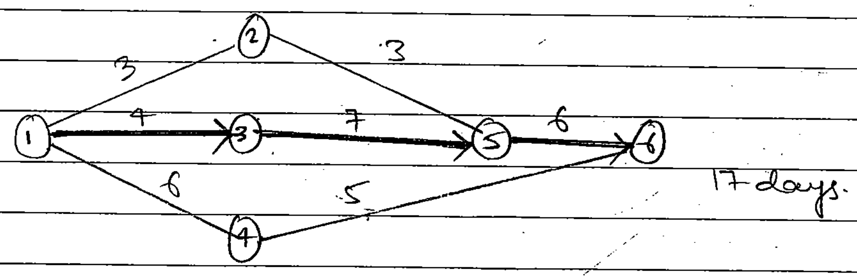
## Statement of Average Time & variance.

| | $\left[t_e = \frac{t_o + t_p + 4t_m}{6}\right]$ | Variance $\left(\frac{t_p - t_o}{6}\right)^2$ |
|---|---|---|
| 1-2 | $\frac{1+5+3\times4}{6}$ = ~~15~~ 3days | $\left(\frac{5-1}{6}\right)^2 = 4/9$ |
| 1-3 | $\frac{1+7+(4\times4)}{6}$ 4 days | $\left(\frac{7-1}{6}\right)^2 = 1$ |
| 1-4 | 6 days | 16/9 |
| 2-5 | 3 days | 1 |
| 3-5 | 7 ~~4~~ days | 4 |
| 4-6 | ~~3~~ 5 days | 1 |
| 5-6. | ~~3~~ 6 days. | 4. |



Expected Time = 17 days.

Projected Path.

1 - 2 - 5 - 6 = 12 days

1 - 3 - 5 6 = 17 days - Critical Path.

1 - 4 - 6. = 11 days

Project variance

1-3-5-6. $1+4+4 = 9.$

Project S.D = $\sqrt{\text{Project variance}}$

$= \sqrt{9} = 3$. days.

(ii) P(not meeting the target)

So we know that (P(meeting due date) + P(not meeting the due date)) = 1.

∴ P(not meeting due date) = 1 - (P(meeting the due date)

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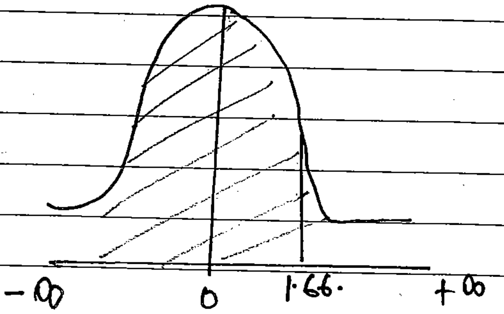
$P(x \leq 22)$

$P\left(z \leq \frac{x-\bar{x}}{sp}\right)$

$P\left(z \leq \frac{22-17}{3}\right)$

$P(z \leq 1.66)$



0%.

$-\infty$ To Zero + 0 To 1.6.

50% + 0.4515

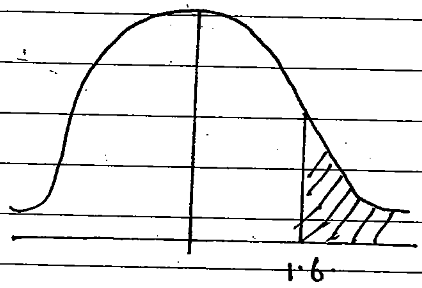
= 0.9515 [put in I]

%.

P(not meeting due date) : 1-0.9515

= 0.0485 or 4.85%.

[Alternative II]



P(not meeting due date)

$P(x > 22$ days

$P\left(Z > \frac{x-\bar{x}}{sp}\right)$

$= P\left(Z > \frac{22-17}{3}\right)$

$= P(Z > 1.66)$

0.50 less 0 To 1.66

0.50 − 0.4515

= 0.0485 or 485%.

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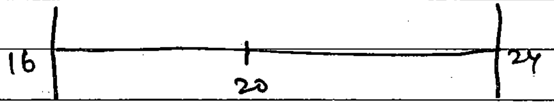


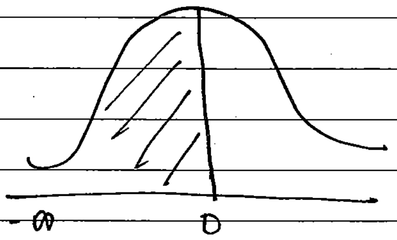
(a) $P(x \leq 20 \text{ month})$

$P(Z < \frac{x-\bar{x}}{SD})$

$P(Z < \frac{20-20}{4})$

$P(Z < 0)$

$= 50\%$



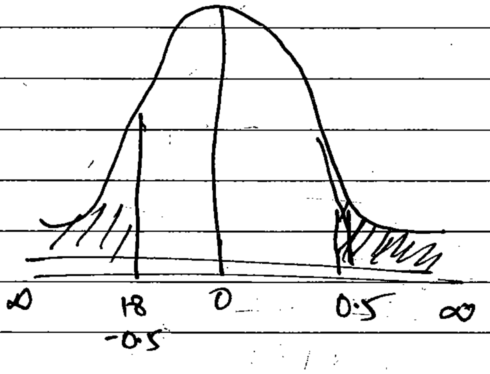
18 month

(b) $P(x \leq 18 \text{ month})$

$P(Z \leq \frac{x-\bar{x}}{SD})$

$P(Z \leq \frac{18-20}{4})$

$P(Z \leq -0.5)$



~~$Z = 0.1915$~~

$P(Z \geq 0.5)$

$P(50\% \text{ less } 0 \text{ To } 0.5)$

$= 0.50 - 0.1915$

$=$ ~~0.38~~ $0.3085$ or $30.85\%$

(c) 24 month ie $P(x \leq 24 \text{ month})$

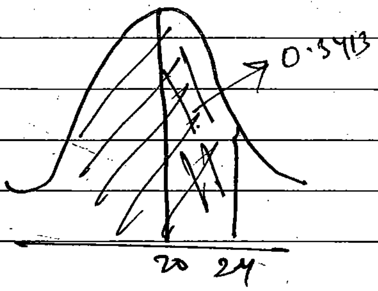
$P(Z \leq \frac{x-\bar{x}}{SD})$

$P(Z \leq \frac{24-20}{4})$

$P(Z \leq 1)$

$0.50 + 0.3413 = 84.13\%$

$=$ ~~0.1587 or 15.87%~~



0·8413.

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Z Table

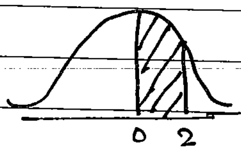
z

0·0 50·

0·1·

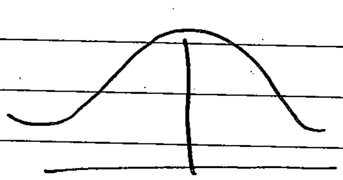
\# If ~~the~~ first value of Table is Zero & all other are <50%

Table type - II



(#) If first value of Table is 0.5· & all other values are >50%.

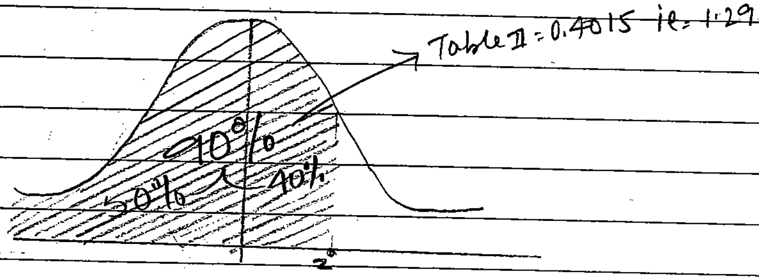
Table Type - I·



\# Managemt will provide the probability & would like to know project duration which apply to the given probability.

(eg) If a project duration is 15 days, with S.D. 5 days. When the project be completed if probability is 90%.

Solution



Z = 1.29.
i.e, 0.4015

As we have Table Type II (i.e, 0 to z), area to the left = 50%. balance Probability = 90-50 = 40% (area to the Right)

For 40% Area =

Corresponding value of Z applicable to 40% Area = 1.29

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$Z = 1.29.$

$$\frac{x-\bar{x}}{SD} = 1.29$$

$$\frac{x-15}{5} = 1.29$$

~~x~~ $1.29 \times 5 + 15 = x$

$x = 21.45$

∴ Project will be completed in 21.45 days (approx)

Project Duration will be approx 21 or 22 days
with 90% probability.

**Concept :-** If value of Probability is given
& Duration is required then we
should follow the following step.

**Step-1** Calculate balance Probability i.e given 50%.
[only in cases of Table-II]

**Step-2** Identify the Value of 'Z' Corresponding to balance probability from table (reverse order)

**Step-3** Calculate Value of 'x' by applying

$$Z = \frac{x-x}{SD} = \text{value}$$

∴ Value of 'x' is the required duration.

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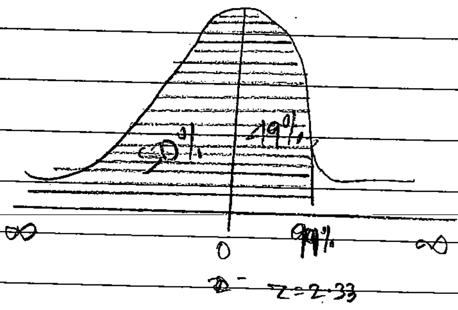
## Balance Probability.



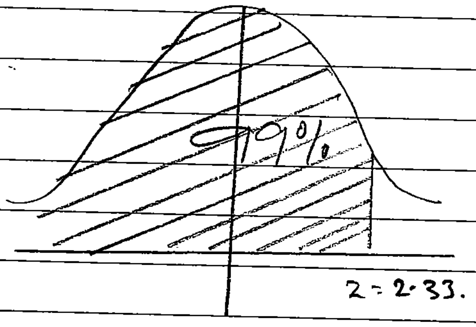
Corresponding value of Z= 2·33.

$\frac{x-\bar{x}}{SD} = 2\cdot33$

$\frac{x-15}{3} = 2\cdot33$

$x = 2\cdot33\times3+15$

$= 21\cdot99 \simeq 22$ days. Ans



Corresponding value of Z= 2·33

$\frac{x-\bar{x}}{SD} = 2\cdot33$ Variance = ~~8~~ 9

∴ SD = 3·

$x = 2\cdot33\times3+60$ week.

$=$ 67 week Answer

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Project Duration = 18 weeks.

$SD = \sqrt{\text{variance}}$

$= \sqrt{9}$

$= 3$ weeks.

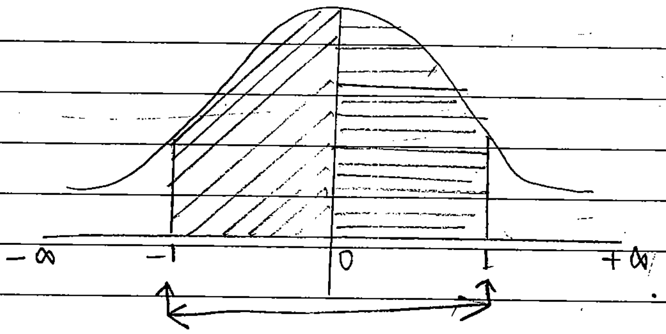
$P(x \geq 15)$ & $P(x \leq 21)$

i.e $P(15 \leq x \leq 21)$

$: P\left(\frac{x-\bar{x}}{SD} \leq Z \leq \frac{x-\bar{x}}{SD}\right)$

$P\left(\frac{15-18}{3} \leq Z \leq \frac{21-18}{3}\right)$

$P(-1 \leq Z \leq 1)$



$P(-1 \text{ T0 } 1) = P(-1 \text{ to zero} + \text{zero to } 1)$

$P(0 \text{ to } 1) + P(0 \text{ to } 1)$

$0.3413 + 0.3413.$

$= 0.6826$ or $68.26\%$.

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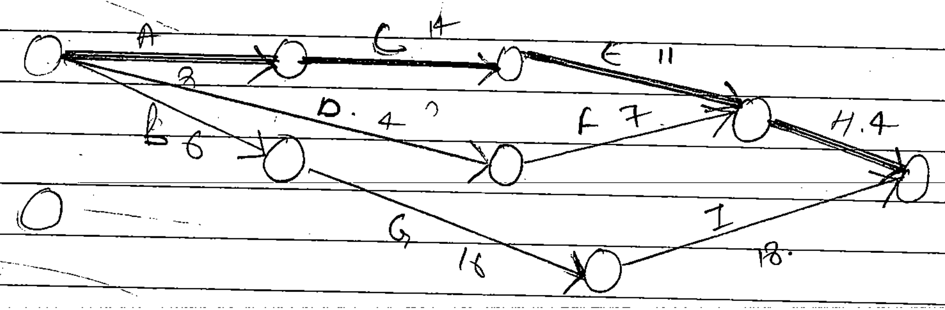
33
Page-21 (i)



## Statement of Average Time & Variance.

| (i) Activity. | $\left[t_e = \frac{t_o + t_p + 4t_m}{6}\right]$ | $Var = \left(\frac{t_p - t_o}{6}\right)^2$. |
|---|---|---|
| A | $\left[\frac{4+16+(4\times7)}{6}\right] = \frac{48}{6} = 8$ days. | $\left(\frac{16-4}{6}\right)^2 = 4$ days. |
| B | $\left[\frac{1+15+(4\times5)}{6}\right] = 6$ days. | $\left(\frac{15-1}{6}\right)^2 = 5.4$ da |
| C | $\left[\frac{6+30+(12\times4)}{6}\right] = 14$ days | $\left(\frac{30-6}{6}\right)^2 = 16$ dag. |
| D | $\left[\frac{2+8+(5\times4)}{6}\right] = 5$ days | $\left(\frac{8-2}{6}\right)^2$ 4 |
| E | $\left[\frac{\overset{5}{3}+\overset{17}{\cancel{15}}+(\overset{11}{8}\times4)}{6}\right] = 11$ days | $\left(\frac{17-5}{6}\right)^2$ 4 |
| F | $\left[\frac{3+15+(6\times4)}{6}\right] = 7$ days. | $\left(\frac{15-3}{6}\right)^2$ 4 |
| G | $\left[\frac{3+27+(9\times4)}{6}\right] = 11$ days | $\left(\frac{27-3}{6}\right)^2$ 16 |
| H | $\left[\frac{1+7+(4\times4)}{6}\right] = 4$ days | $\left(\frac{7-1}{6}\right)^2$ 1 |
| I | $\left[\frac{4+28+(19\times4)}{6}\right] = 18$ days. | $\left(\frac{28-4}{6}\right)^2$ 16. |

(ii) Projected Path.

A-C-E-H = 37 days — [Critical Path]
A-D-F-H = 24 days
A-B-G-I = 35 days

(iii) Exp. Duration = (CP) = 37 ~~week~~. days.

SD = $\sqrt{\text{Project Variance}}$

$= \sqrt{4+16+4+1}$

$= \sqrt{25} = 5$ days.

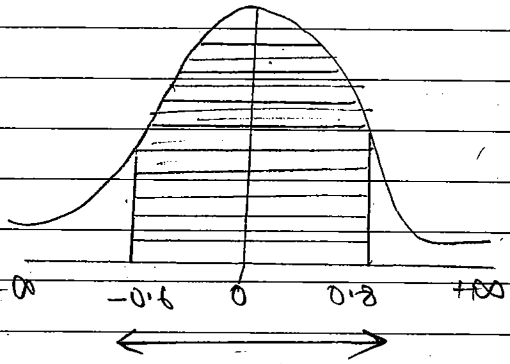
(iv) $P(34 \le x \le 41)$

$P(34 \le z \le 41)$

$P\left(\frac{x-\bar{x}}{SD} \le Z \le \frac{x-\bar{x}}{SD}\right)$

$P\left(\frac{34-37}{5}\right) \le Z \left(\frac{41-37}{5}\right)$

$P\left(-0.6 \le Z \le 0.8\right).$



$P(-0.6 \text{ To Zero}) + P(\text{zero to } 0.8)$

$0.2257 + 0.2881$

$0.5138$ i.e. $51.38\%$

(V) Corresponding value of Z at 99% = 2.33. Area.

Project Duration with 99%..
Balance Probability = 49% : [99% − 49%]

$$Z = 2.33$$

$$\frac{x - \bar{x}}{SD} = 2.33$$

$$\frac{x - 37}{5} = 2.33$$

$$\therefore x = 48.65 \text{ days.}$$

∴ Date of Start = 30 oct − 48.65 days.

11 or 12 Sept (approx)

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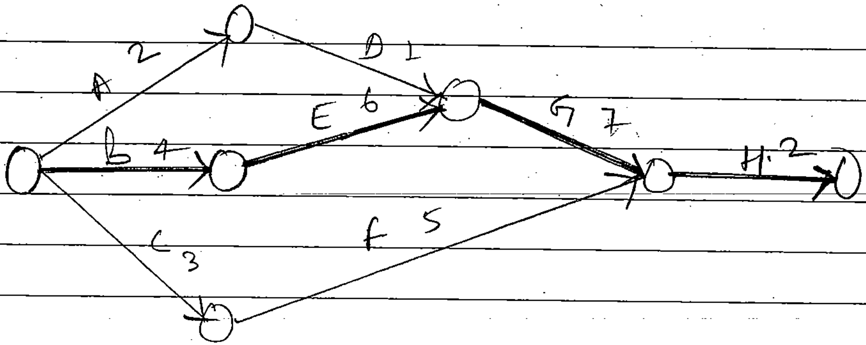

Statement of Average Time & Variance.

| Activity | Average Time $t_e = \left[\frac{t_o + t_p + 4 \times t_m}{6}\right]$ | Variance $\left(\frac{t_p - t_o}{6}\right)^2$ |
|---|---|---|
| A | $\left[\frac{1+7+(1\times4)}{6}\right] = 2$ days. | $\left(\frac{7-1}{6}\right)^2 = 1$ days |
| B | $\left[\frac{1+7+(4\times4)}{6}\right] = 4$ days | $\left(\frac{7-1}{6}\right)^2 = 1$ days |
| C | $\left[\frac{2+8+(2\times4)}{6}\right] = 3$ days. | $\left(\frac{8-2}{6}\right)^2 = 1$ days |
| D | $\left[\frac{1+1+(1\times4)^2}{6}\right] = 1$ days | $\left(\frac{1-1}{6}\right)^2 = 0$ days |
| E | $\left[\frac{2+14+(5\times4)}{6}\right] = 6$ days. | $\left(\frac{14-2}{6}\right)^2 = 4$ days |
| F | $\left[\frac{2+8+(5\times4)}{6}\right] = 5$ days | $\left(\frac{8-2}{6}\right)^2 = 1$ days |
| G | $\left[\frac{3+15+(6\times4)}{6}\right] = 7$ days. | $\left(\frac{15-3}{6}\right)^2 = 4$ days |
| H. | $\left[\frac{1+3+(2\times4)}{6}\right] = 2$ days | $\left(\frac{3-1}{6}\right)^2 = \frac{0.11 \text{ days}}{1/9}$ |

Projected Path

A-D-G-H = 12 day

B-E-G-H = 19 " ⟶ critical Path.

C-F-H = 10 "

∴ Expected Duration = 19 days.

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$S.D = \sqrt{\text{Project variance}} = \sqrt{1+4+4+1/9} = \sqrt{\frac{82}{9}}$

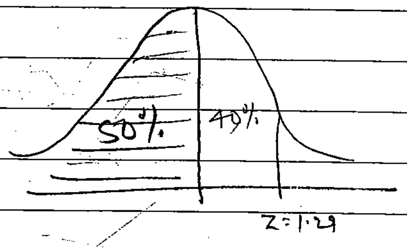
$SD = 3.01.$

(ii) Balance Probability = 40%

$Z = 1.29.$

$\frac{x-\bar{x}}{SD} = 1.29$ or $\frac{x-19}{3.01} = 1.29$

$x = 22.88.$



(iii) If Average Duration of 'f' Activity becomes = 14 days, the path C-F-H also would becomes Critical. which means project variance is the "sum of all activity variance of all exist on critical Path(s).

| Activity | Variance | Explanation |
|---|---|---|
| B | 1 days | ** As average Duration of 'f' Activity service but $t_o$, $t_p$ & $t_m$ is not given corresponding to 14 days |
| E | 4 " | |
| G | 4 " | |
| H | 1/9 " | |
| C | 1 " | |
| F ** | N.A | * 'H' occurs two (2) times. |
| H * | — | |
| | 91/9 | |

classmate

DATE

Project Duration: 19 days

SD = $\sqrt{\text{Project Variance}}$ =

$$\sqrt{\frac{91}{9}} = 3.179.$$

Balance Probability = 40%

$Z = 1.29$

$$\frac{x - \bar{x}}{SD} = 1.29$$

$$\frac{x - 19}{3.179} = 1.29$$

$x = 23.10.$ days.

㊎ If we have two (2) Critical Path then project variance is the sum of all activity exist on all "Critical Path"

7th class

DATE 14 12 2016

## Calculation of Float

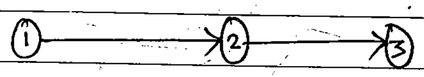
E, EST, L, LFT, LST, EFT, Float FreeFloat, Independent Float

# Purpose of These values is to

(a) Utilize the resource in Optimum manner.
(b) Manipulate the Project.
i.e, Unnesecessary we should not manage the resource at higher Cost

'E' :- Earlier start time for an Event.
'EST' :- Earlier start time for an Activity.
both have the same meaning

E → Event
EST → Activity



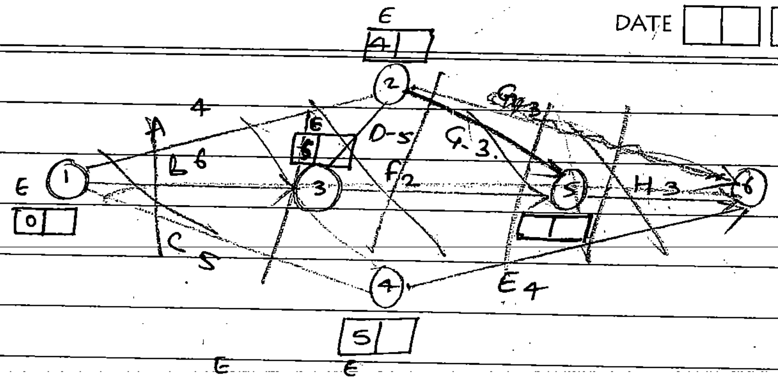
1—2 Activity
2-3 "

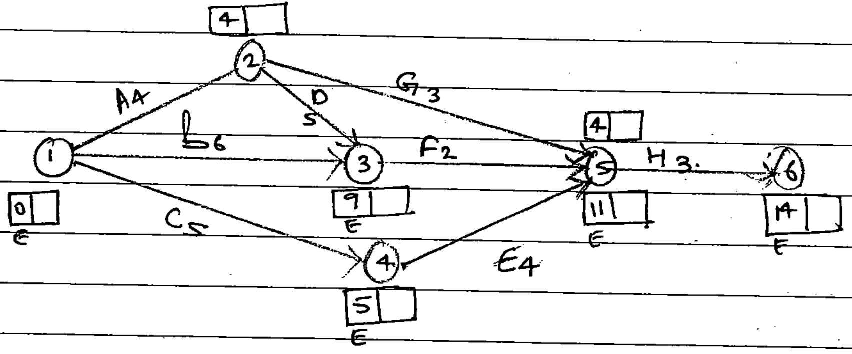
① event Tail for Activity 1-2
② event ~~Tail~~ head for Activity 1-2

② is the tail Event for Activity 2-3.

'E' :- "Its not possible to start any work from a particular Event earlier than a particular period of time"; that particular time will be known as 'E' for the Event EST for the activity.

How to Calculate? :- Identify all leading path towards the event & select the path having maximum period of Time, that maximum time period is 'E' for the Event and 'EST' for the Activity





event ⑤ envent ⑥

A-G-H = 10

A-D-F-H 14

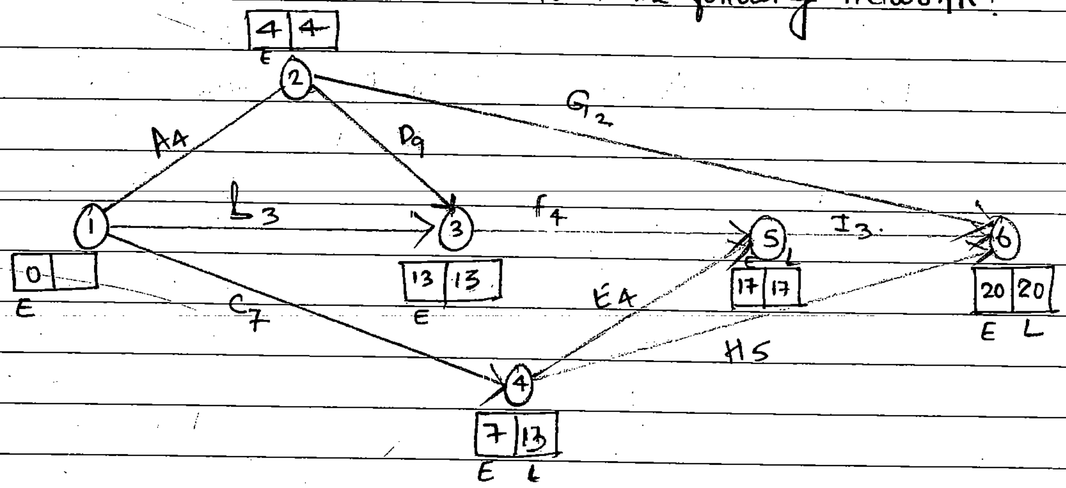
B-F-H 11

C-F-H. 12

Add duration to the preceeding E and select higher i.e, 'E' for ~~highest~~ next

| Activity | EST |
|---|---|
| A | 0 |
| B | 0 |
| C | 0 |
| D | 4 |
| E | 5 |
| F | 9 |
| G | 4 |
| H | 11. |

'E' of Tail Event is the EST for the Activity eg Calculate 'E' and 'EST' from the following Network?



| Activity | EST |
|---|---|
| A | 0 |
| B | 0 |
| C | 0 |
| D | 4 |
| E | 7 |
| F | 13 |
| G | 4 |
| H. | 7 |
| I | 17. |

## # 'L' :- Latest allowable Time for Event.

'LFT': Latest finished Time for an Activity

## How to calculate?

'L' :- All work (relating to an activity should be Completed from beginning upto a particular Event within a Maximum period of Time. (to avoid delay the Project) Such maximum period of Time is known as 'L' for the Event. & 'LFT' for the Activity

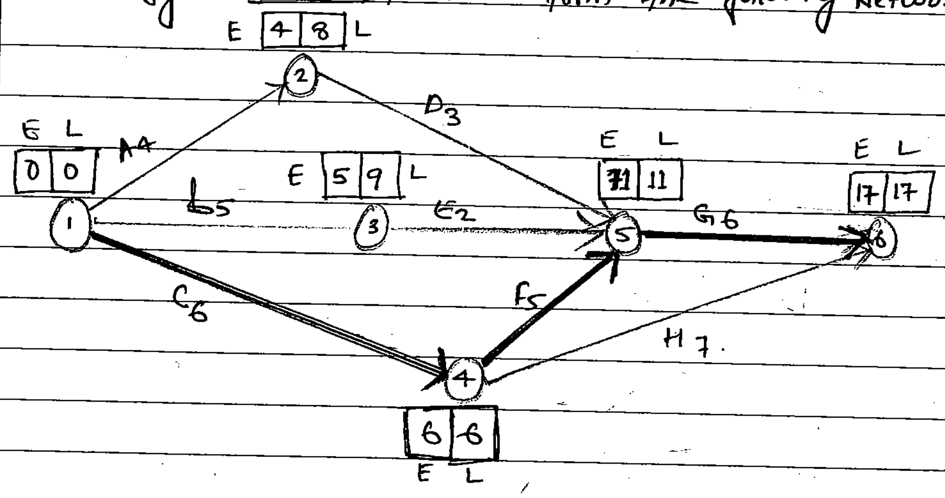
[Head Event 'L' is the 'LFT' of Activity.]

DATE

## How to Calculate ?

Identify all leading path towards the event in reverse direction & select the path having higher duration, Such higher duration should be subtracted from project duration, the resulted value is known as 'L'

(eg) Identify E, EST, L, LFT from the following Network.



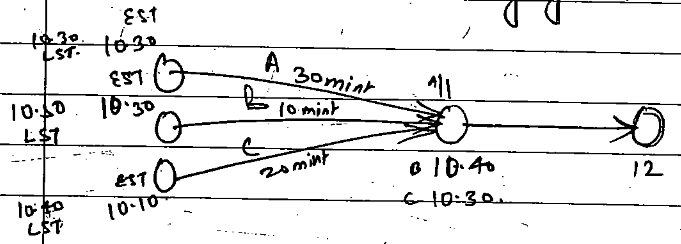
Critical Path= C-F-G.

| Activity Duration | | Tail EST | LFT. | EFT. | LST |
|---|---|---|---|---|---|
| A | 4 | 0 | 8 | 4 | 4 |
| b | 5 | 0 | 9 | 5 | 4 |
| C | 6 | 0 | 6 | 6 | 0 |
| D | 3 | 4 | ~~8~~ 11 | 7 | 8 |
| E | 2 | 5 | ~~9~~ 11 | 7 | 9 |
| F | 5 | 6 | ~~6~~ 11 | 11 | 6 |
| G | 6 | 11 | 17 | 17 | 11 |
| H | 7 | ~~10~~ 6 | 17 | 13 | 10. |

(#) **'EFT'** :- Earliest finished Time for an Activity.
Itx not possible to Complete any Activity Earlier than a particular Period of Time

EFT = Early to start + Duration
EST + Duration.

(#) LST :- Latest Start Time for an Activity. Delay in Start without delaying the project Duration.



**Explanation** If 'b' Start his Journey at 10.50 A.M & takes 10 mint. Journey Time, also he can reach latest by 11 AM. Start Time 10.50 instead of 10.30.

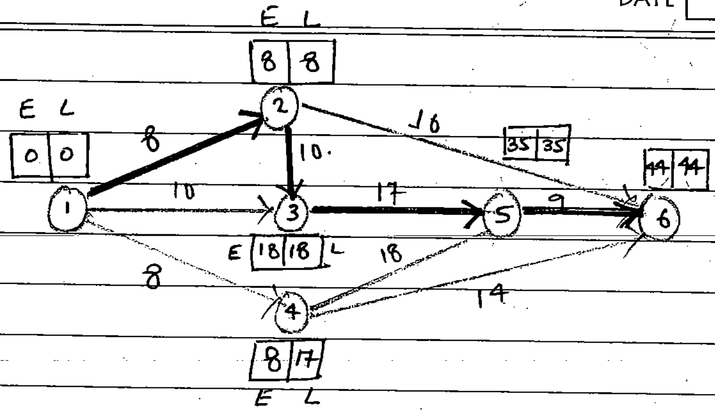
LST = LFT - Duration.
11 - 10 mint = 10.50 Am.

'C' LST ⇒ 11 - 20 mint = 10.40 A.m.

(#) **Float** :- Float is the time differences (waiting time/ Rest time) by which any activity can be delayed without delaying the project Duration.
i.e. ~~EST~~ LST - EST ⇒ (waiting in start)
LFT - EFT ⇒ (waiting at Destination).

DATE

(37)
(Page-2.14)



Critical Path= 1-2-3-5-6.

| Activity | Duration. | [Tail on E]<br>EST | [EST+Duration]<br>EFT | [Head on L]<br>LFT | [LFT-Duration]<br>LST | (F-F)or(S-S)<br>Float |
|---|---|---|---|---|---|---|
| 1-2 | 8 | 0 | 8 | 8 | 0 | 0 |
| 1-3 | 10 | 0 | 10 | 18 | 8 | 8 |
| 1-4 | 8 | 0 | 8 | 17 | 9 | 9 |
| 2-3 | 10 | 8 | 18 | 18. | 8 | 0 |
| 2-6 | 16 | 8 | 24 | 44 | 28 | 20 |
| 3-5 | 17 | 18 | 35 | 35 | 18 | 0 |
| 4-5 | 18 | 8 | 26 | 35 | 17 | 9 |
| 4-6 | 14 | 8 | 22 | 44 | 30 | 22 |
| 5-6. | 9. | 35. | 44. | 44. | 35. | 0. |

Interpretation of Float

(#) We can delay either 1-4 or 4-5 by 9 days.
or 4-6 by 22 days.

(#) if we delay 1-4 by 9 days., then 4-5 Could not be delayed
but 4-6 could be delayed by only 13 days.

(38)
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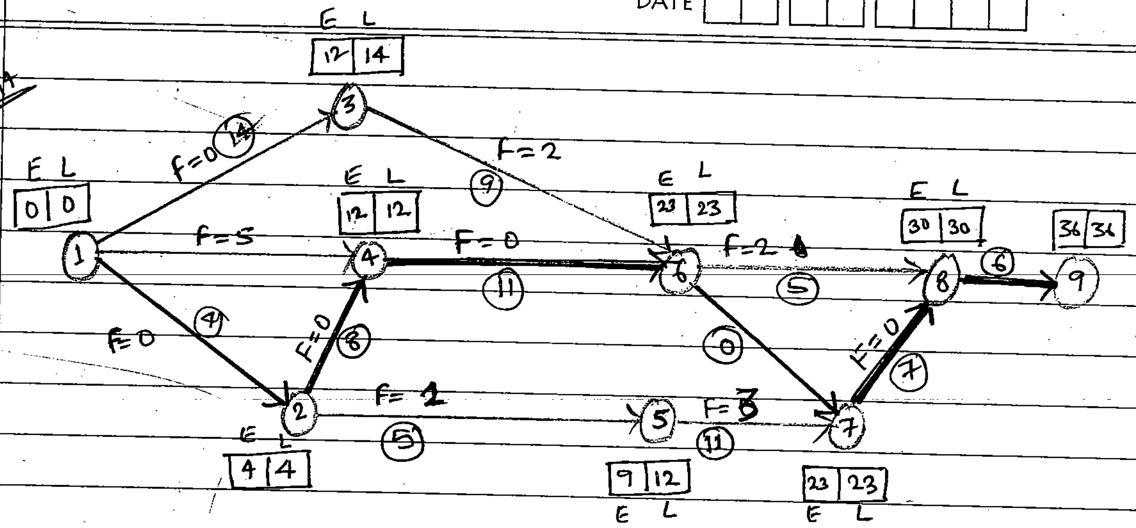


Explanation

① Activity 6-7, 7-8 have EST = 23 days means, No Duration exist between 6-7 i.e. '0' day. having also float = 0 days.

② Path 1-2-4-6-7-8 all have zero ~~path~~ float means There activity may exist on Critical path.
∴ 8-9 is linked activity.
Hence 1-2-4-6-7-8-9 is our Critical Path.

③ On critical path E=L.

④ Project Duration = 30+6 = 36 days.

Statement of Calculation of Days.

Activity. 7-8
Float = 0
Float = LFT-EFT
= LFT - (EST + Duration)
= LFT - EST - Duration.
LFT = 30
EST = 23
Duration = 30-23-0 = 7 days.

Activity = 6-8
Float = 2
Float = LFT - EFT
LFT - (EST + Duration)
LFT - EST - Duration
LFT = 30
EST. 23.
Duration = 30-23-2 = 5 days.

Activity - 4-6.
DO.
Duration = LFT-EST-Float
= 23-12-0
= 11 days.

Activity = 2-4
DO
Duration = LFT-EST-Float
= 12-4-0
= 8 days.

Activity - 5-7
DO
Duration = LFT-EST-Float
= 23-9-3
= 11 days.

Activity 1-2
DO
Duration = LFT-EST-Float
= 4-0-0
= 4 days.

Activity - 1-4
DO
Duration = LFT-EST-Float
= 12-0-5
= 7 days.

Activity - 1-3
DO
Duration = LFT-EST-Float
= 14-0-0
= 14 days.

Activity - 3-6.
Do
Duration = LFT-EST-Float
= 23-12-2
= 9 days.

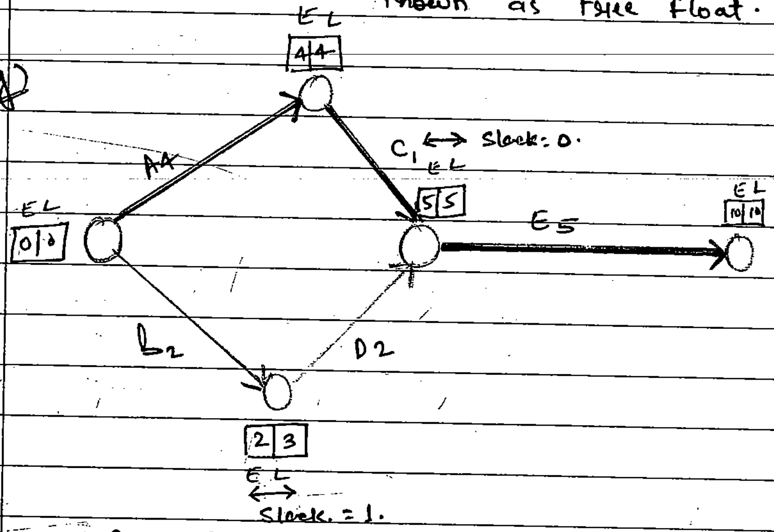
| Activity | TF | EST | Duration |
|---|---|---|---|
| 1-2 | 0 | 0 | 4 |
| 1-3 | 0 | 0 | 14 |
| 1-4 | 5 | 0 | 7 |
| 2-4 | 0 | 4 | 8 |
| 2-5 | 1 | 4 | 5 |
| 3-6 | 2 | 12 | 9 |
| 4-6 | 0 | 12 | 11 |
| 5-7 | 3 | 9 | 11 |
| 6-7 | 0 | 23 | 0 |
| 6-8 | 2 | 23 | 5 |
| 7-8 | 0 | 23 | 7 |
| 8-9 | 0 | 30 | 6 |

8th class

DATE 16 12 2016

# Free float : Any activity can be delayed beyond its 'EST' without effecting the EST, float of its Succeeding Activity, that part of Float is Known as Free float.

(eg)



LFT - EFT = Float

b = 3 - 2 = 1

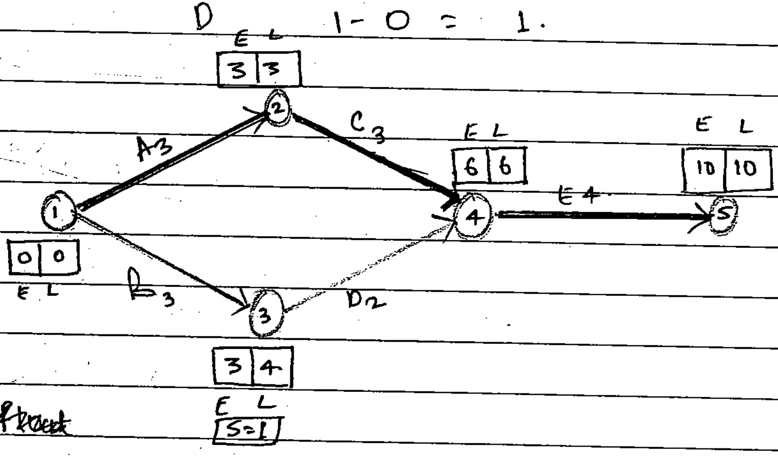
D = 5 - 4 = 1

Free float = Float - head event slack.

b 1 - ~~1~~ = 0

D 1 - 0 = 1.

(eg)



~~Float~~

b . Float = LFT - EFT = 4 - 3 = 1

D Float = LFT - EFT = 6 - 5 = 1.

Free float = Float - Head event slack

b = 1 - 1 = 0

1 - 0 = 1

## # Independent float :-

[Free float - slack of Tail Event.]

any value of float always should be positive.

## # Time scaled diagram.

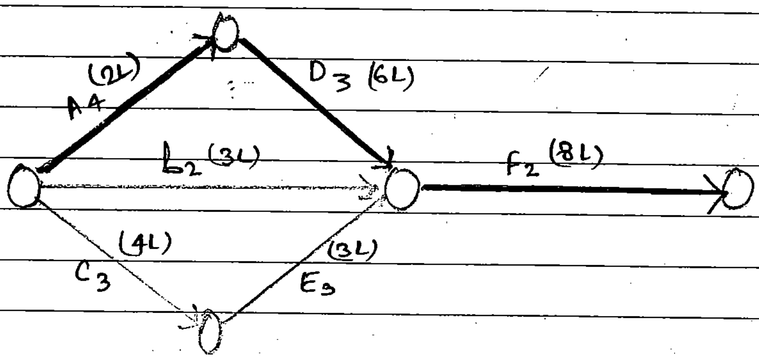
## # Resource Smothing (levelling)

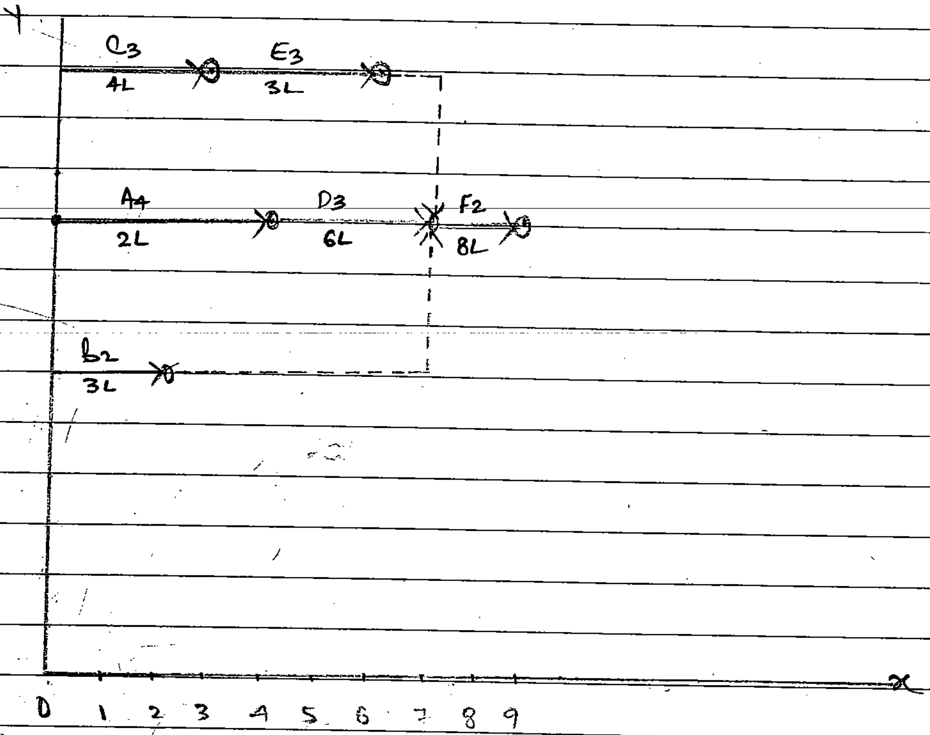
## # Resource Allocation

# Time Scaled Digram :- If management would like to know the status of Project, during the project duration (at any point of Time) then we should prepare a Diagram called Time Scaled Diagram.

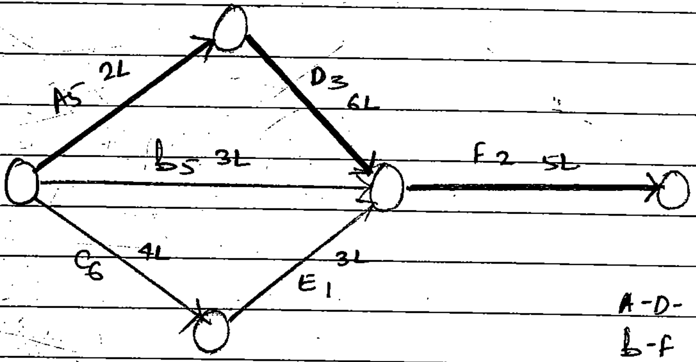
'Project status' Here means : on a particular Day (s) which activity (s) are being carried.

## How to Prepare Time scaled Diagram.

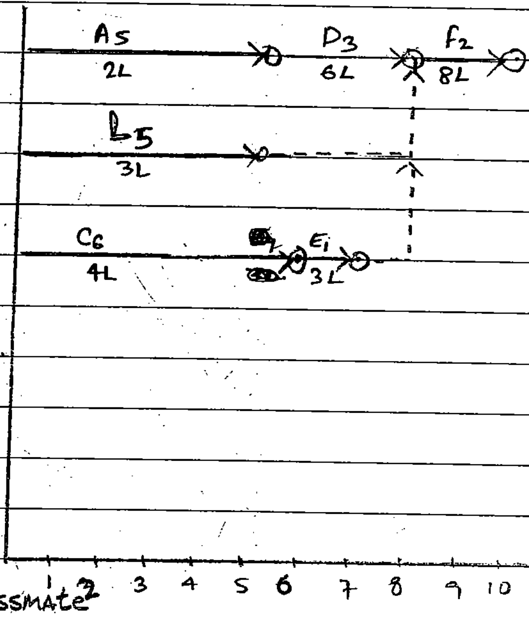




(eg)



A-D-F = 10 days - Critical Path.
b-F = 8 days
C-E-F = 9 days



| Days. | Activity. | Labour. |
|---|---|---|
| 1 | A b C | 2+3+4=9 |
| 2 | A b C | 2+3+4=9 |
| 3 | A b C | 2+3+4=9 |
| 4 | A b C | 2+3+4=9 |
| 5 | A b C | 2+3+4=9 |
| 6 | D C | 6+4=10 |
| 7 | D E | 6+3=9 |
| 8 | D | 6 = 6 |
| 9 | F | 5 = 5 |
| 10 | F. | 5 = 5 |

# **Resource Smoothing:-** To stablize/uniform the No. of Worker over the Project duration i.e, if we ~~work~~ require No of Worker/Labour uneven over the project, we should develope a system which insures that equal (almost) No of workers would be needed on every day of the project, Such System is known as **Resource Smothing (leveling)**

We Can smooth the requirement of Worker by shifting any activity (Which exist on Non Critical Path) to further by utilizing its only free float. But we should keep in mind that –

① project Duration remain Constant.
③ Peak requirement of labour should not be increased over project Duration.

(47)
(Page-217)

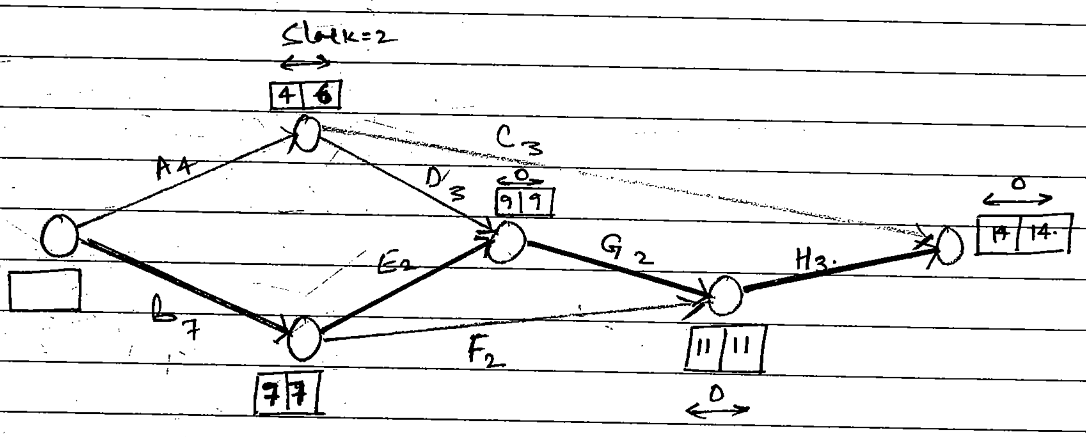
# ⑪ Steps for Resource Smoothing.

① Draw the Network, Calculate E, L, free float & identify all Possible Path

② Draw Time Scaled Digram

③ Prepare Statement of Labour requirement & Activity on each day of the project.

④ Rescheduling by utilizing free float of Non-Critical path/activity to extent maximum possible uniformity of labour requirement.



**Path**

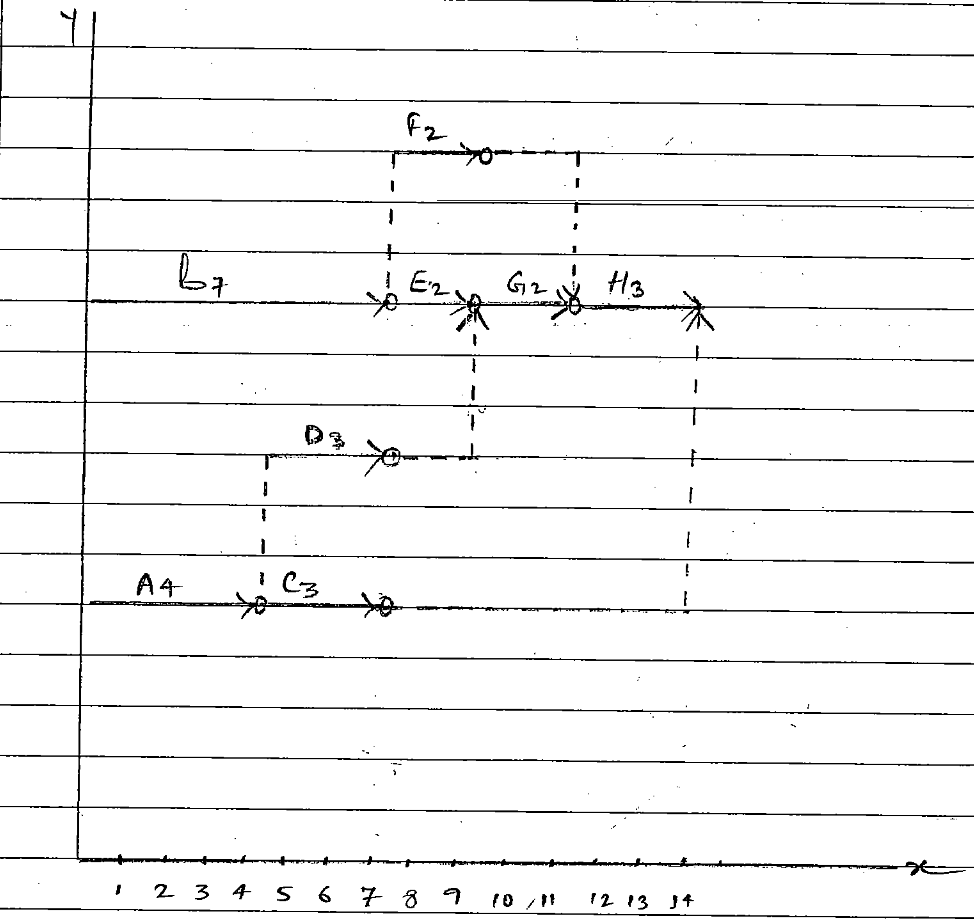
A-C = 7 day

A-D-G-H = 12 days

B-E-G-H = 14 days ⇒ Critical Path.

B-F-H = 12 days.

| Non Critical Activity. | Float | Free Float |
|---|---|---|
| A | 6-4=2 | 2-2=0 |
| C | 14-7=7 | 7-0=07 |
| D | 9-7=2 | 2-0=2 |
| F | 11-9=2. | 2-0=2. |



Statement of Resource Position.

| Days. | Activity | Labour. | Revised | Revised. | |
|---|---|---|---|---|---|
| 1 | BA | ~~2~~+4= 6 | BA 6 | | BA = 6 |
| 2 | B A | 2+4=6 | BA 6 | | BA = 6 |
| 3 | B A | 2+4= 6 | BA 6 | | BA = 6 |
| 4 | B A | 2+4=6 | BA 6 | | BA = 6 |
| 5 | B D C | 2+4+2=8 | BD= 6 | B | BD = 6 |
| 6 | B D C | 2+4+2=8 | BD=6 | B | BD = 6 |
| 7 | B D. C | 2+4+2=8 | BD=6 | BD | BD = 6 |
| 8 | E F | 6+3=9 | 9 | EFD | E~~6~~ = 6 |
| 9 | E F | 6+3=9 | 9 | EFD | E = 6 |
| 10 | G | 3 | 3 | | GF = 3+3=6 |
| 11 | G | 3 | 3 | | GF = 3+3=6 |
| 12 | H | 4 | HC: ~~1+3~~=6 | | HC = 6 |
| 13 | H | 4 | HC =6 | | HC = 6 |
| 14 | H | 4 | HC =6 | | HC = 6 |

## Reschduling No. 1

If we appoint 9 Labours for project then most of the days labours remain idle

· For this Purpose,
We can shift 'C' by 7days.
at Present C : 5-6-7 days.
Shift by 7 days.
∴ Now C = 12-13-14.

## Reschduling No. 2

D Activity Can be shifted by 2 Days.
At present D= 5-6-7 days.
Now 2 days shift
D. 7-8-9.
It will increase our peak requirement hence we should not follow it.

## Reechduling No. 3

'F' Activity can be shifted by 2 days.
At Present F= 8-9.
Now 2 days shift
F 10-11.

# ⋕ Resource ~~Levelling~~ Allocation :- If we have shortage of labour on any day(s) of Project then

project may be delayed but in order to avoid such situation, we should smooth our labour requirement i.e, Resource smoothing process should be applied. (first) After smoothing we can say the exact duration of the project

(A2)
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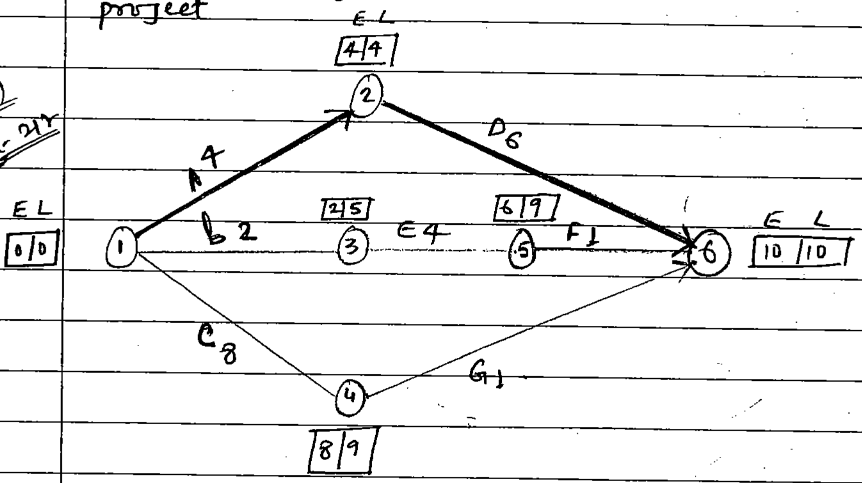
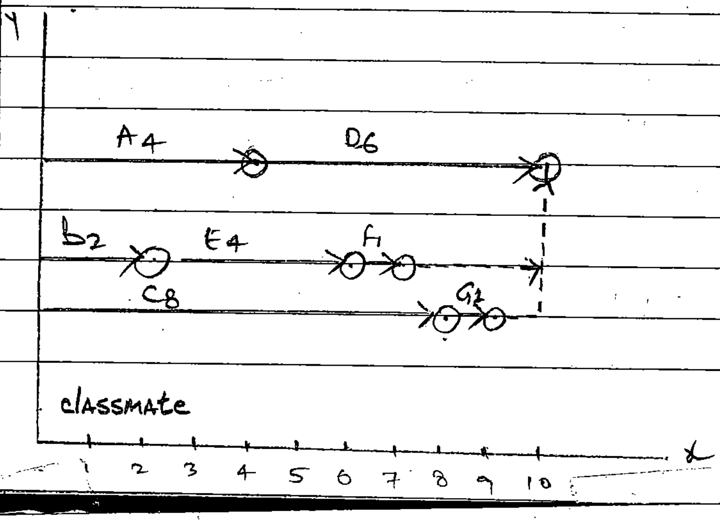


$\overset{4}{A}-\overset{6}{D}$ = 10 days. – Critical Path
$\overset{2}{b}-\overset{4}{E}-\overset{1}{F}$ = 7 days
$\overset{8}{C}-\overset{1}{G}$ = 9 Days.

| Non Critical Activity | Float [LFT-EFT] | Free (Float-slack) Float. |
|---|---|---|
| b | 5-2=3. | 3-3=0 |
| E | 9-6=3 | 3-3=0 |
| F | 10-7=3. | 3- 0=3 |
| C | 9-8=1 | 1-1=0 |
| G. | 10-9=1 | 1-0=1. |



## Statement of Resource Allocation.

| Days. | Activity | Labour. | No-1 Labour | No-2 Labour. |
|---|---|---|---|---|
| 1 | ABC | 2+3+5 = 10 | ABC=10 | ABC=10 |
| 2 | ABC | 2+3+5 = 10 | ABC=10 | ABC=10 |
| 3 | AEC | 2+2+5 = 9 | AEC=9 | AEC=9 |
| 4 | AEC | 2+2+5 = 9 | AEC=9 | AEC=9 |
| 5 | DEC | 3+2+5 = 10 | DEC=10 | DEC= 10 |
| 6 | DEC | 3+2+5 = 10 | DEC=10 | DEC= 10. |
| 7 | DFC | 3+3+5 = 11 | DFC= 11. | DC = 3+5=8 |
| 8 | DC | 3+5 = 8 | DC= 8 | DC= 8 |
| 9 | DG. | 3+8 = 11 | D = 3 | DF= 3+3=6. |
| 10 | D | 3 = 3 | DG= 3+8= 11. | DG= 11. |

**Resource smoothing No. 1**

If we appoint 11 labour for the project, most of days labour will remain idle. Although we have only 10 labour.

G.

Labour. (Free float)

| | |
|---|---|
| F | 3 |
| G | 1 |

If we utilize f for 3 days, F=7 ~~days~~

Now F= 10th days

7days = 8 labour., 10th Day 6 labour.

but we could not utilize G. in Next step. (Peak requirement would increase) hence we can delay 'G' First by 1 day

Present G = 9 day

Delay = 1 day

Now G = 10 days.

**Resource Smoothing. ②.**

Now we can delay 'F' by 2 days instead of 3 day at Present

F= 7th days.

delayed by 2days.

Now F= 9th days.

Conclusion ⇒ If we have 10 labour, First 9 days we have no Problem but 10th days we require 11 Labour. Hence Project will be delayed by 1 days.

(22) 570 560 551 547 554 561 529 (24) = 3rd page - 130

## Steps for Time Scale Diagram.

**Step 1** Draw Network & identify Critical & Non Critical Path.

**Step-2** Plot duration of Critical Path on x-Axis by taking equal distance, starting from zero.

**Step-3** Draw Critical Path activity as Parallel to X-Axis and from middle of Y-Axis as straight line.

**Step-4** Other Non Critical Path should be drawn either LHS or RHS of Critical Path line. Common Activity should be drawn only once. Non Critical Path should be linked (Joined) with Critical path by Horizontal/ Vertical. dotted line.

9th class

DATE 19 12 2016

# (#) Updating of Network

If during the project Duration,

(1) Any Addition of new activity / any activity may be deleted.

(2) Any activity to be revised

(3) Material / Labour will be in short
Supply / then

(4) Duration of the project will be revised.

(43)
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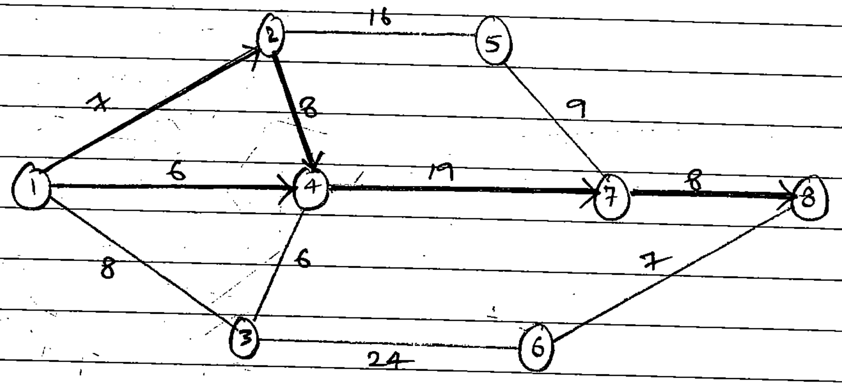


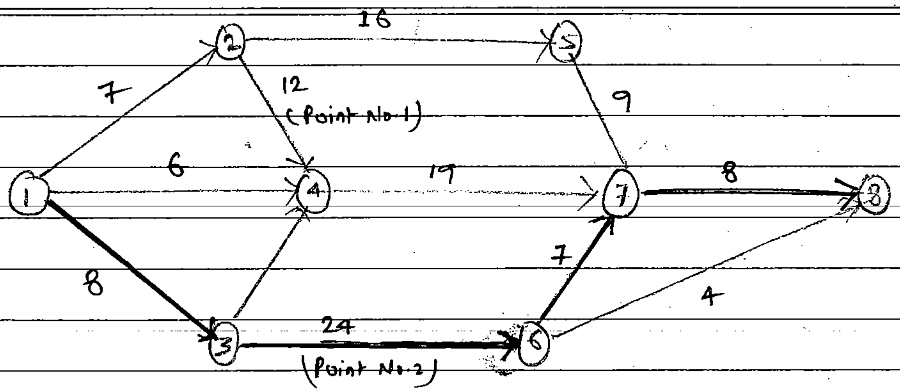
**Possible Path**

1-2-5-7-8 = 40 days = Non Critical Path
1-4-7-8 = 33 days = Non Critical Path.
1-2-4-7-8 = 42 days ⇒ Critical Path
1-3-6-8 = 39 days = Non Critical Path
1-3-4-7-8 = 41 days Non Critical Path.

**Point No1**

After 15 days we can say that at Event No 4 total Consumption of days will be 19 days (i.e 19th day). while - 1-2 consumed 7 days & means 2-4 will take (19-7) = 12 days.

P-1-0



**Point No 2**

We will be at event No 6.

on 15+17 = 32th day

1-3 already consumed 8 days means 3-6 now

take (32-8) = 24 days.

**Point No. 3**

6-7 a new Activity

**Revised Path**

1-2-5-7-8 = 40

1-2-4-7-8 = 46

1-4-7-8 = 33

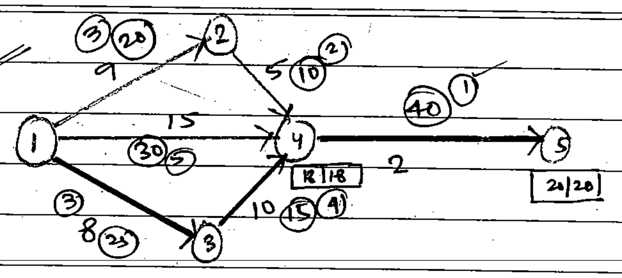
1-3-6-7-8 = 47 = Revised Critical Path.

1-3-6-8 = 36

1-3-4-7-8 = 41.

27, 24, 28, 26, 35, 36, 39, 40,

(24) Page-208 (i)



Possible Path

1-2-4-5 = 16 days
1-4-5 = 17 days.
1-3-4-5 = 20 days [Critical Path]

(iii)

[I] We should delays 1-4 by 3 days More because LFT for 1-4 is 18 days, EFT = 15 days, Float = 3 days.

Saving = 3 days X 30 = 90.
(as we should have to reach at Event '4' on 18 days)

II 1-2 Can be relaxed by 4 days, but Total relaxation should be 5 days, 3 days already done, 1-2 could be relaxed for another 2 days
Saving = 2 X 20 = 40

∴ Total Saving = 90+40 = 130

(ii)

Statement of Crashing.

| Possible Path. | Duration. | Duration | Duration | Duration |
|---|---|---|---|---|
| 1-3-4-5 | 20 days | 17 days | 16 days | 15 days |
| 1-4-5 | 17 days | 17 days | 16 days | 15 days |
| 1-2-4-5. | 16 days. | 16 days. | 15 days. | 15 days |
| Total Cost. | 1940. | 1985 | 2025 | 2070. |

∴ Cost incurred for the optimal Crashing of 5 days
= (2070 - 1940)
= £130 Ans

[Crashing I] ⇒ Crashing Activity 3-4 by 3 days. Total Cost = 1940 + 45 (15x3) = 1985
[Crashing - II] = Crashing Activity 4-5 by 1 days. Total Cost = 1985+40 = 2025
[Crashing - III] = Crashing Activity '3-4' & '1-4' by 1 days. Total Cost = 2025+15+30
= 2070

Statement of Comparative Cost.

| Common Activity | | | least Cost other slope | | |
|---|---|---|---|---|---|
| 4-5 | 1day | 40. | 4-5' & | 1day | ~~30~~ 30 |
| | | 40 | 3-4 | 1day | 15 |
| | | | | | 45 |

# Chapter- 4 - Learning Curve

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(#) **Learning Curve** As production increases - Consumption of Time decrease. Due to Experience / Learning, Technology, upto a Certain stage that stage will be known as Stable stage & upto the Stable Stage called Learning

Costing- Stage-I 10 days
Stg II 8 days
Stage III 5 days.

(#) Learning Curve theory applicable only when new work start and same work is going to repeat.

(#) Learning Curve Theory is applicable for New Worker also.

(#) The same Work should be repeated for Next Time.

(#) Learning Curve Theory should not be applied on Department where we have automatic Machine.

(#) Learning Curve Theory apply on Labour Cost only.

(#) Learning Curve Theory should ~~be~~ not be applied on Material, Fixed overhead.

(#) Learning Curve Theory helps in decision Making.

(#) How much is the Consumption of time for subsquent unit (if same type of Product repeats) so that Labour Cost Could be analysed & Minimum price refered to management accordingly this Could be possible only from Learning Curve Theory.

(#) Learning Curve Theory developed by researcher on the basis of Actual data of the company with in the same Industry

(#) Learning Curve Theory estimate the Consumption of Time on subsquent unit.

## (#) Researcher

### Actual Data (Company 1) [Unskilled Labour]

| Cumm Production | | Avg Per unit (Hr) | Total Time (Hr) | Incremental Time |
|---|---|---|---|---|
| 1 | | 10 | 10 | 10 |
| 2 | 90% | 9 | 18 | 8 |
| 3 | 90% | 8.5 | 25.5 | 75 |
| 4 | | 8.1 | 32.4 | 6.9 |
| 5 | | ✓ | ✓ | — |
| 6 | 90% | ✓ | ✓ | — |
| 7 | | ✓ | ✓ | — |
| 8 | | 7.29. | 572 | 6 |
| 9 | | ✓ | ✓ | ✓ |
| 10. | 90% | ✓ | ✓ | ✓ |
| ⋮ | | ⋮ | ⋮ | ⋮ |
| 16 | | 7 | 112 | 5. |

### Learning Curve Observation

If production increases (on same type) & becomes equal to double of preceding then only Average time per unit will be 90% of its Previous average. This is known as Trend (Tendency) developed on the basis of actual data of other Company within same Industry. This trend predict only Average Time.

Average Time/unit should be multiplied with cummulative production to have total time. & after deducting preceding one, balance is Incremental Time.

eg. Statement of Learning Curve.

| Cumm. Production | AT/PU(Hr) | TT | Incr. Time |
|---|---|---|---|
| 1 | 20 | 20 | 20 |
| 2 (LH) | 18 | 36 | 16 |
| 3 | — | — | — |
| 4 | 162. | 64.8. | 28.8 |

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## Statement of learning curve.

Production : Unit wise 80%.

| Cumm Prod. | Avg Ti/unit | TT | Increment Time |
|---|---|---|---|
| 1 | 1000 | 1000 | 1000 |
| 2 | 800 | 1600 | 600 (2nd unit) |
| 3 | – | – | |
| 4 | 640 | 2560 | |
| 5 | – | – | |
| 6 | – | – | |
| 7 | – | – | |
| 8 | 512 | 4096 | |
| 9 | – | – | |
| 10 | – | – | |
| 11 | – | – | |
| 12 | – | – | |
| 13 | – | – | |
| 14 | – | – | |
| 15 | – | – | |
| 16 | 409.6. | 6553.6. | |

## Statement of Minimum Price.

| | 1st 8 Machine | 1st 16 Machine |
|---|---|---|
| Material | 1600 000 | 32,00,000 |
| Labour | ₹100/hr X 4096<br>= 4,09,600 | ₹100/hr X6553.6hr<br>6,55,360. |
| Fixed Cost | 100 000 | 100 000 |
| | | |
| Relevant Cost (a) | 21,09,600 | 39,55,360. |
| Machine (b) | 8 | 16. |
| Cost Per Machine (a/b) | 2,63,700 | 247,210. |

# (#) (II) Production ⇒ Lot wise / batch / Group wise / Production Run / First ⸻ units.

**Lot wise :-** All units Consist in a lot to be produced simultaneously (due to nature of process).
Hence we should say that " Learning of 1 Lot should be applied on Next Lot." Consumption of Time on each unit in a Lot will be same.

If Size of lot increases & becomes double of Preceeding, Average Time per unit will be 90% of Average Time Per unit of its preceeding Lot.

(3) [Page-150]

Statement of learning Curve
Production : Lot wise : 80%
[10 unit]

| Cumm Prod. | ATPU | TT | IT |
|---|---|---|---|
| 10 units (1 Lot) | 50 | 500 | 500 |
| 20 units (2 Lot) | 40 | 800 | 300 |
| 40 units (3 Lot) | 32 | 1280 | 780 · [Next 30 units] |

Statement of Price Next 30 units.

| | |
|---|---|
| Material · $\left[\frac{3000}{10} \times 30\right]$ | 9000 |
| Labour $[₹12 \times 780 \text{ Hr}]$ | 9360 |
| Variable overhead $\left[\frac{1000}{10} \times 30\right]$ | 3000 |
| Machine Tool Cost *# | — |
| Minimum Price. | 21,360 |

Note

(#) ① Concept of Tools :- The Cost of Tools always to be charged from Initial order as life of Tools are very short although practically same Tools may be used for subsequent order.

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② **Variable overhead** : Variable overhead may be charged Qty based/ Hrs based.

Indirect Material ⇒ Qty [eg: Coal, oil]
Indirect Labour ⇒ Qty [eg: Supervisor/day]
Indirect Expenses ⇒ Hrs. [eg: Power]

(4) Page-150

Statement of learning curve. [80% : 4000 unit (1st)

| Cumm level. | ATPU | TT | IT. |
|---|---|---|---|
| 4000 (1 lot) | 1·5 hr | 6000 hr | — |
| 8000 (2 lot) | 1·2 hr | 9600 hr | 3600 Hr |
| 12000 (3 lot) | ~~0·96 hr~~ | 11 | |
| 16000 (4 lot) | 0·96 hr | 15,360 | 9360 Hr. |

[Next 12000 units over 4000 unit]

Statement of Minimum Price.
[Next 12000 units].

| | | |
|---|---|---|
| Material. | [₹35 × 12000 unit] | 4,20,000 |
| Labour | [$\frac{₹24}{1.5}$ × 9360 hr] | 1,49,760 |
| Variable overhead. | [₹15 × 12000] | 1,80,000 |
| | Minimum Price ⇒ | 7,49,760 |

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## Statement of learning Curve.

Production : Unitwise

| CP – | ATPU | TT | IT. |
|---|---|---|---|
| 30 (lot) | 200 | 6000 | |
| 60 (lot) | 180 | 10800 | |
| 90 (lot) | | | |
| 120 (lot) | 162 | 19440 | 13,440. (Next 90 units] |

## Statement of Minimum Price.

| | | |
|---|---|---|
| Material | [$\frac{10\,000}{3000}$ x 90] | 30000 |
| Direct Labour | 13,440 x 3 | 40,320 |
| Tooling Cost | | — |
| Variable overhead | [13,440 x 1] | 13,440. |

Minimum Price ⇒ 83,760

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## Statement of learning Curve.

Production : lot wise.

| CP | ATPU | TT | IT. |
|---|---|---|---|
| 10 (1 lot) | 100 | 1000 | |
| 20 (2 lot) | 90 | 1800 | |
| 40 (4 lot) | 81 | 3240 [1st 40 unit] | |
| 80 (8 lot) | 72.9. | 5832 [2nd 40 unit] | 2592 |
| 100 (10 lot)<br>160 (16 lot) | 65.61 | 10,497.6 [Next 40]. | |
| 320 (32 lot) | 59.049 | 18,895.68. | |

## Statement of Cost [For 40 units]

(i) Labour Cost & Variable O/H. [$\overset{₹/hr}{(8+2)}$ X $\overset{hr.}{3240}$] ₹ 32,400

32400

Average Cost = $\frac{32400}{40}$ = 810.

(ii) Labour for first 80 unit = 5832 hr × (8+2) ₹ = 58320

$\therefore$ Average Cost = $\frac{58320}{80}$ = 729.

(2) Incremental Cost = ₹(8+2) × 2592 hr = ₹ 25920
[for 41-80]

II If production could not be acheived as double of its preceeding. In order to calculate/estimate the Consumption of Time on subsequent production. We should apply a Mathematical formula instead of Direct observation.

Formula ⇒ $Y = \boxed{ax^b}$
↳ [When Period = unit wise]

Where = ⊕ a = "Total Time for 1st unit"
⊕ x = "Cummulative Production" for which we require to Calculate Average time.
⊕ Y = Average Time Per unit. applicable to 'x' Cummulative unit
⊕ b = "Learning Index"

the value of 'b' i.e, learning Index is although given but we can also calculate

⇒ $b_{(90\%)} = \frac{\log 0.9}{\log 2}$ How to calculate value of log

**Property of log**

⇒ $\frac{b}{80\%} = \frac{\log 0.8}{\log 2}$ ① $\log a^b = b \log a$

② $\log \frac{a}{b} = \log a - \log b$.

⇒ $\frac{b}{70\%} = \frac{\log 0.7}{\log 2}$ ③ $\log a \times b = \log a + \log b$.

(OR)

Special Case

$\log_{(95\%)} = \frac{\log 0.95}{\log 2}$

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## log Table

Say log 54326·1261

(i) Ignore value after Decimal.

(ii) Count the Integer & less 1. digit. (5-1) = 4.

(iii) first two digit in column & (iii) digit for RHS in Table. 54 ⇒ ~~0·7354~~. 0·7348

(iv) Add the value of Mean difference.

~~4·7438~~      4·7348
~~+ 2~~      2
~~4·7340~~      4·7350

eg log 1234567·152 (Ignore)

(7-1) = 6·0899
14
6·0913

(eg) log 527687·156

5·7218    [(6-1)· & 52 ka 7 = 0·7218 & ~~Right~~ left se 6 (ie 1st 3 digit छोड़ के)
5    ka 52 वां = 5 Add]
5·7223

Now we would like to know that what should be the powers of 10 which results 100.

We know that = $\log_{10}^{2}$ = 100.

log 100 = 2 ⇒ This is the power of 10 i.e all values placed in Table represents power of 10 (In order to get almost accurate, add last value (mean differences).

# $= \frac{\log 0.9}{\log 2}$

$\frac{\log \frac{9}{10}}{\log 2} = \frac{\log 9 - \log 10}{\log 2} = \frac{0.9542 - 1}{0.3010} = -0.1521$ [at 90%]

# $b = \frac{\log 0.8}{\log 2}$

$\frac{\log \frac{8}{10}}{\log 2} = \frac{\log 8 - \log 10}{\log 2} = \frac{0.9031 - 1}{0.3010} = -0.3219$ [at 80%]

(b) $\frac{\log 0.7}{\log 2} = \frac{\log 7 - \log 10}{\log 2} = \frac{0.8451 - 1}{0.3010} = -0.5146$ [at 70%]

# Antilog :-

Reverse Table (Value) Procedure log

eg A log 2.4325.

(i) First 2(two) digit upto 3rd digit after decimal.

(ii) Add '1' ~~too~~ to the left hand value placed in decimal. 2+1=3.

eg: ⇒ 2704

+ 3

2707 270.7.

Alog 1.46781

⇒ 2931

5

29.36

eg A log. 4.32567

⇒ 2113

+ 3

21160.

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(6)
[Page-161]

## Statement of learning Curve.

Production : unitwise = 90%

| Cumm Prodⁿ | A.T.P.U | T.T | I.T. |
|---|---|---|---|
| 1 | 100 | 100 | 100 |
| 2 | 90 | 180 | 80 [2nd unit] |
| 3 | 84·61 | 253·83 | 73·83·[on 3rd unit] |
| 4 | 81 | 324 | 70·17·[on 4th unit] |
| Suppose 5 | | | |
| ? | | | |

$y = ax^{b}$.

$a = 100$

$x = 3$

$$\left[\begin{array}{ll} b = & \frac{\log 0.9}{\log 2} = -0.1521 \\ 90\% & \end{array}\right]$$

$y = (100 \times 3)^{-0.1521}$

$= (100 \times 5)^{-0.1521}$

$= \log 100 + \log 5^{-0.1521}$

$= 2 - 0.1521 \log 5$

$= 2 - 0.1521$

Taking log both side.

~~$\log y = \log 300$~~

$\Rightarrow \log y = \log 100 \times 3^{-0.1521}$

$\Rightarrow \log y = \log 100 + \log 3^{-0.1521}$

$\Rightarrow \log y = \log 100 + (-0.1521 \log 3)$

$\log y = \log 100 - 0.1521 \log 3$

$\Rightarrow \log y = 2 - 0.1521 \times 0.4771$

$\log y = 1.9274$

$\Rightarrow A \log(\log y) = A \log 1.9274$

$y = 8453 + 8 = 84.61$

$y = 84.61$. [This is Avg Time / unit applicable for 1st 3 unit]

classmate

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Ⓣ If Production is as lot | Batch | Group | Production Run.

Statement of Learning Curve.

| C.P. | ATPO | TT | IT. |
|---|---|---|---|
| 10 (1lot) | 10 | 100 [a] | |
| 20 (2 lot) | 9 | 180 | |
| 30 (3 lot) $x$ | ? | | |

$y = ax^b$

Where, $a$ = Total time for first lot.

$x$ = Cummulative No. of lot. for which we should Calculate average time/unit. i.e. 3.

$y$ = Average Time Per lot. applicable corresponding to '$x$' Cummulative No. of lot.

$$\left[\text{Avg Time per Unit} = \frac{\text{Avg Time Per Lot}}{\text{No. of unit in lot}}\right]$$

$y = (100 \times 3)^{-0.1521}$

$\log y = \log 100 + \log 3^{-0.1521}$

$\log y =$

(7) [Page-152]

[L1) c] Statement of Learning Curve.

Production: lot wise 90%.

| CP | ATPU | TT | IT. |
|---|---|---|---|
| 10 (1 lot) | 100 Hr | 1,000 Hr | 1000 Hr |
| 20 (2 lot) | 90 Hr | 1800 | 800 Hr |
| 40 | 81 | 3240 | |
| 80 | | | |
| 100 (10 lot) | 70.46 Hr | 7046. | |

$y = ax^b$

$1000 \times (10)^{-0.1521}$

taking log both side.

$\log y = \log 1000 + \log 10^{-0.1521.}$

$\log y = 3 + (-0.1524) \times 1.$

$\log y = 3 - 0.1524$

$\log y = 2.8479$

$A \log y = A \log 2.8479.$

$y = 7031 + 15 = 7046$

$y = 704.6.$

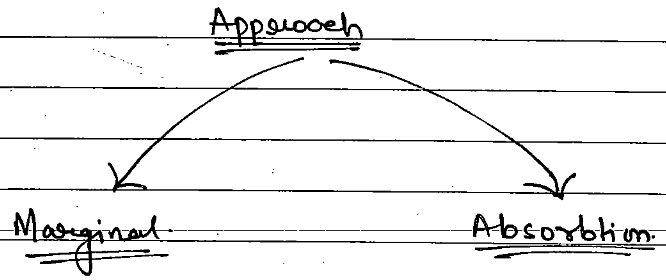
[i.e.] Average time/lot 704.6 Hr.

$\therefore$ Average time per unit = $\frac{704.6}{10} = 70.46$ Hr

Labour & Variable overhead ₹ [for 1st 100 unit]

$= \left[[70.46 \times 100] \times (8+2)₹\right]$

$=$ ₹ 70460



(1) Marginal Approach :- (Decision Making): This approach should be applied when manage-ment would like to know

(a) The Minimum Price of order/offer

(b) Decision :- Either to accept the or Reject order always.
we should calculate Relevant Cost i.e, Unavoidable Fixed Cost should not be Considered
Only Avoidable Fixed Cost should be Considered.

(2) Absorbtion Approach :- ~~Mag~~ Management would like to know the Total Cost of the product (Regular)/offer, Just before its production.

eg April 2nd, First production is going to produce.

| | €/unit | | |
|---|---|---|---|
| Material | 100 | Fixed Cost | |
| Labour. | 200 | Rent | 200000 |
| Variable O/H. | 100 | Salary | 100 000 |
| Variable Cost | 400 | Depⁿ | 100 000 |
| + Fixed Cost | ? | | 400 000 |
| Total Cost. | XX | Qty | 100 |
| | | FC/unit | 400. |

but management would like to ~~know~~ know TC Just before Production & we are calculating at end of Month.

This can be resolved by estimating Monthly Production.

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| | |
|---|---|
| Budgeted Fixed Cost | 500000 |
| Budgted Peroduction | 1000 unit |
| Recovery Rate / Standard Rate | 500 / Units |

if at the end of Month Actual Production becomes less than / greater than budgeted production, Adjustment will be Carried at the end of Month.

(eg)

## Cost Sheet

| | $FG_1$ | $FG_2$ | $FG_3$ | FG.8n. | | |
|---|---|---|---|---|---|---|
| Material | 100 | 100 | 100 | 100 | | XX |
| Labour | 200 | 200 | 200 | 200 | | XX |
| Variable O/H | 100 | 100 | 100 | 100 | | XX |
| Fixed O/H. | 500 | 500 | 500. | 500 | | 400000 * Recovered. |
| Total Cost | 900 | 900 | 910 | 900 | Factory Cost | XXX |
| | | | | | +O/A | XX |
| | | | | | COGS | XXX |
| | | | | | COGS | XX |
| | | | | | SD. | XX |
| | | | | | Cost | XXX |
| | | | | | Sales | XXp |
| | | | | | Profit | XXp |
| | | | | | Less- Under Recovery | ** 100000 |
| | | | | | | XXXV |

Budgeted Fixed overhead = 500 000

Let Actual Fixed overhead. 500 000

Budgeted Production 1000 unit.

Actual Production 800 unit

**Result:-** If we adopt Marginal Costing, Existing burden of Fixed Cost Cost should not be charged from offer as Calculation of Minimum Price.

If we adopt Absorbtion, every Product (Regular/offer) will absorb its burden of Fixed Cost through recovery rate i.e, Recovery Rate = $\frac{\text{budgeted Fixed O/H}}{\text{budgeted output}}$.

classmate

| Marginal | Absorption |
|---|---|
| # Minimum Price | # Total Cost |
| # Decision is required. | # Recovery Rate / standard Rate is given. |
| | # Company adopt Cost + Mark up. policy |

(10) [Page-153]

Recovery Rate = $\frac{\text{budgeted fixed overhead}}{\text{budgeted Production / Hours.}}$

budgeted fixed overhead = 28,800

budgeted Hours = Hrs available = 20 Labours X 24 days X 8 hrs.
= 3840 Hrs.
(-) Down Time (25%) 960 (Normal Loss)
2880 Hr

∴ Recovery Rate = $\frac{28800}{2880}$ = ₹10 / Hrs

Statement of learning Curve.
Production = unit wise = 80%

| Cumm Prod. | ATPU | TT | IT. |
|---|---|---|---|
| 1 | 40 | 40 | 40 |
| 2 | 32 | 64. | 24. |
| 4 | 25.6 | | |
| 8 | | | |
| 16 | | | |
| 30 | 13.38 | 401.4 | |
| 32 | | | |

$y = ax^b$

$\log y = \log 40 + \log (30)^{-0.322}$

$\log y = \log 40 - 0.322 \log 30.$

$\log y = \cancel{1} - \cancel{0.322} \times 1$ $\quad 1.6021 - 0.322 \times 1.4771.$

$\log y = \cancel{0.678}$ $= 1.1265.$

Taking Anti Log both side

$Y = 1337+1$

$= 1338$

$Y = 13.38.$

Statement of Total Cost

| | | |
|---|---|---|
| Material | [30 unit x ₹100] | 3000 |
| Labour | [~~30unit~~x 401.4 Hr x 4) | ~~48168~~.1605.6 |
| Variable over head. | [401.4 Hr x 2] | 802.8. |
| Fixed overhead | [401.4 x 10 Hr ] | 4014 |
| Total cost ⇒ | | 9422.4 |

(2) Statement of learning Curve

| C.P. | ATPU | TT | I.T. |
|---|---|---|---|
| 1 | 40 | 40 | |
| 2 | | | |
| 4 | | | |
| 16 | | | |
| 30 | 13.38 | 401.40 | |
| [30+20] 50 | 11.35 | 567.5 | 166.1 (next 20 units) |

$Y = ax^b$

$Y = 40(50)^{-0.322}$

$\log y = \log 40 + (\log 50)^{-0.322}$

$\log y = \log 40 - 0.322 \log 50.$

$\log y = 1.6021 - 0.322 \times 1.6990$

$\log y = 1.6021 - 0.547078$

$\log y = 1.055022$

Antilog taken both side

$y = A\log 1.055022$

$y = 11.35$

Average time / unit for 1st 50 unit.

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## Statement of Price.

| | | |
|---|---|---|
| Material | [20unit X 100] | 2000 |
| Labour | [166.1 Hr X ₹4] | 664.4 |
| Variable o/H | [166.1 X ₹2] | 332.2 |
| fixed overhead | [166.1 X ₹10] | 1661 |
| Total Cost | | 4657.6 |
| Add:- Profit Mark up 25%$\overset{\rightarrow 1/4}{}$ on sale i.e 1/3 of cost | | 1552.5 |
| Sale Price | | 6210.1 |
| Qty | | 20 unit |
| Sale Price/unit = | | ₹ 310.51 |

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## Working Hours Available

| Department | A | B. |
|---|---|---|
| Total Hours available | 6000 | 6000 |
| Utilized for Regular Work | 3840 | 2100 |
| Balance Hours for special order. | 2160 | 3900 |
| (a) for 100 unit | 1800 | 900. |
| (b) for 200 unit (First 200 unit) | 2880 | 1260 |
| Overtime Hrs Required [2880-2160] | 720 | — |

(i) Statement of Total Cost [Absorbtion]

| | | [First 100 unit] | | [First 200 unit] |
|---|---|---|---|---|
| **Department 'A'** | | | | |
| Material | [₹60 X 100 Unit] | 6000 | [50 X 100 unit] | 10,000 |
| Labour | [₹20 X 1800 Hr] | 36000 | [₹20X2160Hr] | 43200 |
| Overtime premium | | — | [₹30X720 Hr] | 21600 |
| Variable O/H. | [₹30X 1800 Hr] | 54000 | [₹30X 2880] | 86400. |
| Fixed O/H | [₹40X 1800 Hr] | 72000 | [₹40X2880] | 1,15,200 |
| **Department 'B'** | | | Department 'B' | |
| Labour | [₹30X900Hr] | 27000 | [₹30X 1260 Hr] | 37800 |
| Overtime premium | | — | | — |
| Variable o/H | [₹60X900 Hr] | 54000 | [₹60 X1260] | 75600 |
| Fixed O/H | [₹50X900 Hr] | 45000 | [₹50 X 1260] | 63000 |
| Tooling cost | | 1200 | Tooling | 1200 |
| Cost/unit = 2952 | | 245200 | Cost/unit = 2270 | 454000 |

Working Statement of learning Curve. [80%]

| C.P. | ATPU | TT | IT. |
|---|---|---|---|
| [Department A] | ~~18~~ | | |
| 100 [1 lot] | 18 | 1800 | |
| 200 [2 lot] | 14.4 | 2880. | |
| 400 [4 lot] | 11.52 | 4608 | |
| 800 [8 lot] | 9.216 | 7372.8 | 4492.8 [for 600 units over ~~1st~~ 200 units] |
| [Department B] | [70%] | | |
| 100 [1 lot] | 9 | 900 | |
| 200 [2 lot] | 6.3 | 1260. | |
| 400 [4 lot] | 4.41 | 1764 | |
| 800 [8 lot] | 3.087 | 2469.6. | 1209.6 [For 600 unit over first 200 unit |

(2) Statement of Minimum. Price for Next 600 Unit.

Department A

| | |
|---|---|
| Material ~~[illegible]~~ [W. N# 2] | 22000 |
| Labour [₹20 x 4492.8] | 89856 |
| Variable overhead. [₹30 x 4492.8] | 1,34784. |

Department B

| | |
|---|---|
| Labour [₹30 x 1209.6] | 36288 |
| Variable O/H [₹60 x 1209.6] | 72576 |
| Tooling Cost [∵ sunk] | — |

Minimum Price ⇒ 355,504.

∴ Minimum Price/unit = 592.51.

Working Note Relevant Cost of Material.

| Unit | Cost/unit | Total Cost. | Relevant Cost |
|---|---|---|---|
| 100 | 60 | 6000 | |
| 200 | 50 | 10000 | 4000 [For Next 100 unit] |
| 800 | 40 | 32000 | 22000 [For Next 600 unit] |

| | | |
|---|---|---|
| | Cost to be Incurred due to offer [600x40] | 24000 |
| (−) | Benefit to be achieved due to offer [200x10] | 2000 |
| | Material. Cost for 600 unit of | 22000 |

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## Pointx to be Remember

1) Overtime Premium ⇒ No Capacity Constraint
2) Fixed overhead ⟹ Marginal
3) Tooling cost → Repeat Order.

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## Working Note [#1]

Learning Effected Cost => Labour & Variable O/H.
Non learning effected Cost = Material.

Production = Lot wise ⇒ 1000 unit
Labour Rate = ?

Variable manufacture Cost = 4400
(for 1st Batch)

Material (Non learning effected Cost) 50% = 2200.

Labour & Variable O/H (learning effective Cost) = 2200
4400

### Statement of learning Curve. [Let the Rate be x]

| Cumm Prodⁿ | ACPU | TC | IC |
|---|---|---|---|
| 1000 (1 lot) | 2200. | 22,00,000 | |
| 2000 (2 lot) | 2200x | | |
| 4000 (4 lot) | [2200x X x] | | |

According to Question

Learning effected Cost + Non learning effected cost = Unit Avg Cost

$2200x^2 + 2200 = 4120$

$x^2 = \frac{4120-2200}{2200}$

$x^2 = 0.872\dot{7}$ ∴ Learning Rate = 93.41%

$x = 0.9342$

## Statement of Comparative Cost.

| Price (S.P) (a) | Qty. (d) | Material. (b) | Labour & Var OH (c) | Cost $[a-[b+c]]\times d$. |
|---|---|---|---|---|
| 11,100 | 1000 | 2200 | 2200 | 6700000 |
| 10,700 | 2000 | 2200 | 2055.02<br>[2200×93.41%] | 1,28,89,960 |
| 9,600 | 3000 | 2200 | 1961 | 1,63,17000. |
| 8,700. | 4000. | 2200 | [2055.02×93.41%]<br>1919.59. | 1,83,21,640. |

**Rough**

$y = ax^{b}.$

$y = 2200000 \times (3)^{b} - 1.$

b

$b = \frac{\log 0.9341}{\log 2}$

$b = \log 0.9341 - \log 2$

$b = \log \frac{9341}{1000} - \log 2$

$b > \log 9341 - \log 1000 - \log 2$

$b = \frac{-0.0315}{0.3010}$

$b = -0.10465.$

taking log both side.

$\log y = \log 220000 + \log (3)^{-0.10465}$

$= \log 2200000 - 0.10465 \log 3.$

$=$ ~~log 6.3424 − 0.10465~~.

$= 6.3424 - 0.8913$

$y = A\log 6.29247$

$\frac{1959}{2}$

$1961000.$

$y = 1961000$ Avg Cost Per Lot

$ACPU = \frac{1961000}{1000.}$

$y = ax^{b}$

$y = 2200000 \times (3)^{b}$

$\log y = \log 2200000 + \log 3^{b}$

Now $b = \frac{\log 0.9341}{\log 2.}$

$b = \log \frac{-0.0315}{0.3010} = -0.10465$

$b = -0.10465$

$\log y = \log 2200000 + \log 3^{-0.10465}$

$\log y = 6.3424 - 0.10465 \times \log 3.$

$\log y = 6.3424 - 0.10465 \times 0.4771$

$\log y = 6.3424 - 0.04992$

$\log y = A\log 6.29247$

$y = \frac{1959}{2}$

$1961000.$

$y = 19,61,000$ Avg Cost/Lot.

$ACPU = \frac{₹1961000}{1000/lot} = 1961.$

∴ Better to Select ₹8700/unit as price.

(#) If extract table is given, It means.

x: 1. ⇒ Always Initial order. (What ever Qty exist in Initial order represented by '1')

1·3 ⇒ Multiple of Initial order i.e, If Initial order = 100, 1·3 Mean 130 units.

Y = 100% ⇒ Avg Time (cost) Per unit on every unit for first Initial order.

(16) Page-15

## Statement of learnings Curve.

| x. | Y. | TT | IT. |
|---|---|---|---|
| 1 (100 unit) | 100% : 40 Hr | 4000 Hrs. | |
| 1·3 (130 unit) | 91·7% = 36·68 Hr. | | |
| 1·4 (140 Unit) | 89·5% = 35·8 Hr | 5012 Hrs. | 1012 [Next 40 unit] |
| 1·5 (150 unit) | 87·6% = 35·04 Hr | | |
| 1·6 (160 unit) | 86·1% = 34·44 Hr | | |
| 1·7 (170 unit) | 84·4% = 33·76 Hr. | | |
| 1·8 (180 unit) | | | |
| 1·9 (190 unit) | | | |
| 1·2 (200 unit) | 80% = 32 Hr | 6400 Hrs. | 1388 [Next. 60 unit] |

## Statement of Cost & Price.

| Order | | First 100 unit (₹) | Next 40 unit | Next 60 unit. |
|---|---|---|---|---|
| Material | | 10 000 | 4000 | 6000 |
| | | [₹100 X 100] | [₹ 100 X 40] | [₹100 X 40] |
| Labour. | A | 20,000 | 8 000 | 12000 |
| | ~~b~~. | [20 hrs X 100 X ₹10] | [20 hr X 40 X ₹10] | [20 Hr X 60 X ₹10] |
| | b | 80 000 | 2,02,400 | 27,760 |
| | | [40 Hr X 100 unit X ₹20]. | [1012 Hr X ₹20] | [1388 Hr X ₹20]. |
| Variable O/H. | | | | |
| | A | 2000 | 1800 | 1200 |
| | | [2000 X 1] | [1800 X 1] | [1200 X 1]. |
| | b | 8000 | 2024 | 2776 |
| | | [4000 X 2] | [1012 X 2] | [1388 X 2]. |
| Fixed O/H. | A | 20 000 | 8000 | 12000 |
| | | ~~[2000 hr X 10]~~ | ~~[800 X 10]~~ | ~~[1200 X 10]~~ |
| | b | 80 000 | 20240 | 27760 |
| | | ~~[4000 hr X 20]~~ | ~~[1012 X 20]~~ | ~~[1388 X 20]~~ |
| Total Cost | | 2,20,000 | 63304 | 89496 |
| Mark up @ 20% | | 44000 | 12661 | ~~17899~~ |
| Sales | | 2,64000 | 75965 | 107,395 |
| SP/unit | | 2640/- | 1899·12/- | 1789·92. |

12th class

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## Extract Table

x:1: Qty Exist in Initial order.

Y: 100%: AUTP for quantity exist in Initial order

(17) [Page-15]

### Statement of learning Curve (Dep-II)

| CP(x) | (ATPU)(Y) | TT(xy) | IT |
|---|---|---|---|
| 1: 160 | 2 | 320 | |
| 1.2: 192 | 93.3% X2 = 1.866 | 358.272 | 38.272 (Next 32 unit) |
| 1.7: 272 | 84.4% X2 = 1.688 | 459.136 | 100.864 (Next 80 unit) |
| 2.7: 432 | 73.2% X 2 = 1.464 | 632.448 | 173.31 (Next 160 unit) |

Dept ⇒ III.

| | | | |
|---|---|---|---|
| 1: 160 | 1 | 160 | |
| 1.2: 192 | 0.933 | 179.136 | 19.136 (Next 32 unit) |
| 1.7: 272 | 0.844 | 229.588 | 50.432 (Next 80) |
| 2.7: 432 | 0.732 | 316.224 | 86.658 (Next 160) |

## Working Note:-

Dept I (Automatic Machine)

learning curve could not applied for Dept-I

learning Curve could be applied for Dept-II/III

### Statement of Cost (Absorbtion)

| Qty. | 160 unit | 32 | 80 | 160 |
|---|---|---|---|---|
| Material A: 32 | | | | |
| B 18 | 50 | 10 | 25 | 50 |
| Labour Dept-I | 160 × L | $\frac{100}{160}$ × 32 × 1 = 20 | 50 Hr × L = 50<br>100. | 100 Hr × L : 100 |
| Dept II | 320 Hr × 2 | ~~$\frac{380}{160}$ × 2 × 32 = 124~~<br>76.544<br>38.272 Hr × 2 | [100.864 × 2]<br>201.728 | 173.31 × 2 = 346.62<br>hr ₹ |
| Dept-III | 160 Hr × 3 | ~~$\frac{160}{160}$ × 32 × 3 = 96.~~<br>19.136 Hr × 3<br>57.408 | [50.432 × 3]<br>151.296 | 86.658 × 3 = 259.974.<br>hr ₹ |

classmate

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Statement of Cost.

| | | | | |
|---|---|---|---|---|
| Qty | 160 unit | 32 | 80 | 160 |
| Material A 32<br>B 18 | 50 | 10 | 25 | 50. |
| **Labour** | | | | |
| Dept-I | 100X1=100 | $\frac{100}{160}$X32X1= 20 | $\frac{100}{160}$ ~~X25~~ 50X1=50<br>$[\frac{100}{160}X80X1]$ | 100HrX1=100<br>$[\frac{100}{160}X160X1]$ = ~~100~~. |
| Dept-II | [320hrX2]=640 | [38·272X2] = 76·544 | [100·864X2] = 201·728 | [173·31X2] = 346·62 |
| Dept-III | [160HrX3] = 480 | [19·136X3] = 57·408 | [50·432X3] = 151·296 | [86·658X3] = 259·974. |
| **Variable O/H** | | | | |
| Dept I | 100X0·2=20 | 20·~~X~~0·2= 4 | 50X0·2=10 | 100X0·2·= ~~5~~20 |
| Dep-II | 320X0·4=128 | 38·272~~X~~ 0·4=15·3088 | 100·864X0·4=40·3456 | 173·31X0·4=69·324 |
| Dep-III | 160X0·6=96 | 19·136X 0·6=11·4816 | 50·432X0·6=30·2592 | 86·6X0·6 = 51·96 |
| **fixed O/H** | | | | |
| Dept-I | 100X3=300 | 20X3=60 | 50X3=150 | 100X3=300 |
| Dept II | 320X4= 1280 | 38·272X4=153·088 | 100·864X4=403·456 | 173·31X4=693·24 |
| Dept-III. | 160X5= 800 | 19·136X5=95·68 | 50·432X5=252·16 | 86·6X5 = 433 |
| Total Cost= | 3894 | 503·5104· | 1314·2448 | 2274·118 |

(20)
~~Page-139~~

statement of learning Curve. (80%, unitwise)

| | Cumm. Production | ATPU. | TT | IT. |
|---|---|---|---|---|
| Learning stage | 1 | 30 | 30 | |
| | 2 | 24 | 48. | |
| | ⋮ | | | |
| | 249. | 5.079 | 1264.671 | |
| | 250 | 5.073. | 1268.25 | 3.579 [IT for 250th unit] |
| Stable stage | 257 | [illegible] | [illegible] | 3.579 |
| | 252 | | | 3.579 |
| | 253 | | | 3.579. |
| | ⋮ | | | |
| | 1000 | | | 3.579 |

Total time for 1000 units

Total time for 1st 249 unit= i.e. 5.073 x 250. = 1268.25
Total time for Next 751 Unit= i.e. 3.579 x 751 = 2684.25
Total Labour Hour ⇒ 3952.5

Statement of Profit. [1000 units]

| | |
|---|---|
| Revenue [1000 unit x ₹5000] | 50,00,000 |
| Material [1000 unit x ₹1850] | (18,50,000) |
| Labour [~~1000 unit~~ x 3952.5 Hr x 80] | (3,16,200) |
| Variable overhead Cost [1000 unit x ₹1000] | (10,00,000) |
| Packing Material. | (500,000) |
| Profit ⇒ | 13,33,800 |

Let $x$ be Labour Cost (Total)

50,00,000 - 18,50000, - $x$ - 10,00,000 - 500000 = 1000 unit x 800.

1650000 - $x$ = 800000

$x$ = 1650,000 - 800000

$x$ = ₹ 8,50,000.

∴ Average Target Labour Cost/unit = 8,50000 ÷ 1000 = ₹850/unit.

~~Working~~
Now we require Average Time per unit.

| On 1st 250 unit | On 1st 249 unit |
|---|---|
| $Y_{250} = ax^b$ | $Y_{249} = ax^b.$ |
| $= 30(250)^{-0.3219}$ | $Y = 30(249)^{-0.3219.}$ |
| $Y = [0.1691 \times 30]$ | $Y = 30(0.1693)$ |
| $Y = 5.073$ | $Y = 5.079.$ |

(9) [152] If We select to utilize our own Labour (₹8 per Hour), all 8 unit (4 unit for 'P' & 4 unit for 'Q') will be produced in our factory One by one i.e, learning of 1 unit on 2 unit and on Next upto ~~8~~ 8 units

~~Khosh~~

Statement of learning Curve.

| C.P. | ATPP | ~~IT~~ | IT. |
|---|---|---|---|
| 1 | 2000 Hr | 2000 Hr | – |
| 2 | 1600 Hr | 3200 Hr | 1200 Hr |
| 4 | 1280 Hr | 5120 Hr | 1920 Hr |
| 8 | 1024 Hr | 8192 Hr | 3072 Hr. [Next 4 unit]. |

If we accept 'P' ltd labour (@2₹/Hr), ~~P~~ 4 unit for P ltd. will be produced seperately from another 4 units for Q ltd.

Working

Statement of Learning Curve [own labour]

| | | |
|---|---|---|
| 1 | 2000 | 2000 |
| 2 | 1600 | 3200 |
| 4 | 1280 | 5120 |

Statement of learning Curve ['P' ltd labour]

| | | |
|---|---|---|
| 1 | 2000 | 2000 |
| 2 | 1600 | 3200 |
| 4 | 1280 | 5120. |

P.T.O

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## Statement of Profit.

[OPtion-I: own labour.]

| | |
|---|---|
| Revenue = P [33000 X 4 unit) = 152000 | |
| Q [₹34400 X 4 unit) = 137600 | 2,69,600 |
| Less | |
| Material [₹20000 X 8unit] | (160000) |
| Labour. [₹8 X 8192 Hr] | (65536) |
| Variable overhead. [₹4000 X 8] | (32000) |
| Contribution. [overall]. | 12064 |

## Statement of Profit

[option-II - 'P' ltd Labour].

| | |
|---|---|
| Revenue .. P = [₹28000 X 4Unit] = 1,12,000 | |
| Q = [₹34400 X 4 unit] = 137600 | 249600. |
| Less | |
| Material [₹20000 X 8 unit] | (160000) |
| Labour. 1st 4 unit = [5120 Hr X ₹8] = 40960 | |
| 2nd 4 Unit [5120 X ₹ 2] = 10240 | (51,200) |
| Variable over [₹4000 X 8 unit] | (32000) |
| Contribution ⟹ | 6400 |

We should not utilize 'P' ltd's Labour.

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(12)
~~Page-154~~

## Statement of Learning Curve.

| Cumm Prod^n | ATPU | TC | IC |
|---|---|---|---|
| 50 (1 lot) | 100 [100Hr x ₹1] | 5000 | |
| 100 (2 lot) | 90 [90Hr x ₹1] | 9000 | |
| 200 (4 lot) | 81 | 16200 | 7200 |
| 400 (8 lot) | 72.9 | 29160 | 12,960 |
| 800 (16 lot) | 65.61 | 52,488. | 23,328. |
| 1000 (20 lot) | 63.41 | 63410 | 10,922. |

~~$y = ax^b$~~

~~$100(100)^{-}$~~

$1~~00(1000)^{-0.1521}$~~

$y = ax^b$

$5000(20)^{-0.1521}$

$\log y = \log 5000 \cancel{+} \log 20^{-0.1521}$

$\log y = \log 5000 + -0.1251 \log 20.$

$\log y \quad 3.6990 + -0.1251 \times 0.3010$

$\log y = 3.6613449$

Anti log.

$y = \log 0.6613449.$

4581

## Statement of Price.

| Order. | 400 Unit | 800 unit | 1000 unit |
|---|---|---|---|
| Material | [150 x 400]<br>60,000 | [150 x 800]<br>1,20,000 | [150 x 1000]<br>150,000 |
| Labour | 29160 | 52,488 | 63410 |
| Overhead. | [50 x 400]<br>20000 | [50 x 800]<br>40,000 | [50 x 1000]<br>50000 |
| Total Cost | 1,09,160 | 2,12,488 | 2,63,410 |
| + Profit | 27,290 | 53,122 | 65,852.5 |
| Sales | 1,36,450 | 265,610 | 3,29,262.5 |
| Sale Price/unit | 341.125 | 332.0125 | 329.2625 |

DATE

Statement of learning Curve.
[Production unitwise, 90%].

| Cumm Prod" | ATPU | TT | IT. |
|---|---|---|---|
| 1 | 20 | 20 | |
| 2 | 18 | 36 | |
| 4 | 16·2 | 64·8 | |

(i) Statement of Price.

| | |
|---|---|
| Direct variable cost (excluding labour) [₹2000 X 4 unit] | 8 000 |
| Labour [64·8 Hrs × ₹50] | 3240. |
| Total cost | 11 240. |
| + Contribution [1500 X 4 unit] | 6000 |
| Sale Value | 17,240 |
| Qty | 4 |
| Sale Value/unit | ₹4310/unit. |

(ii) Statement of Price

| | |
|---|---|
| Direct Variable Cost | 8000 |
| Labour cost [₹50/Hr X 20 Hrs X 4 unit] | 4000 |
| Total Cost | 12000 |
| + Contribution [₹1500 X 4 unit] | 6 000 |
| Sales | 18,000 |
| SP/unit | 4,500 |

If 4 different labour made 4 different product simultaneously, learning could not be applied. Hence price would be different i.e. ₹4500.

DATE

(#) **Sensitivity of Profit** :- Profit should be recovered from Customer by adding to the Labour Cost & Calculate Revised to Labour Cost which includes Profit [Existing Labour Cost + Profit] & then Calculate [Present learning = Revised Labour Cost]

{Profit includes in Labour & identify Revised learning curve}

Revised learning Rate which provide such Revised Labour Cost, such Revised Labour Rate should be Quoted to the Customer.

**Step-1** Present Labour Cost as per Present Labour Rate

**Step-2** Revised Labour Cost = [Existing Labour Cost + Desired Profit]

**Step-3** Calculate New learning Rate applicable for Revised Labour Cost

(19) ~~Page-151~~

Statement of learning Curve.

| Cumm Unit | ACPU | TL |
|---|---|---|
| 1 | 112.5 | 112.5 |
| 2 | 101.25 | |
| 4 | 91.125 | |
| 8 | 82.0125 | |
| 16 | 73.81125 | |
| 32 | 66.430125 | |
| 64 | 59.7871125 | |
| 128 | 53.80840125 | |
| 256. | 48.436561125. | 12399.759648. |

$Y = ax^b$

$= 112.5(256)^{-0.1521}$

$= \log 112.5 - 0.1521 \log 256$

$= 2.0492 - 0.1521 \times 2.4082$

$= AL\ 1.68291$

$= 4808$

10

~~48.98~~

Labour Cost for First 256 units as per existing Learning Rate

= 12399.759648 Hr X 20₹

= 2,47,995.19296.

Revised Labour Cost = ₹2,47,995.19296 + 75000

= 3,22,995.19296.

∴ Revised Labour Hr = $\frac{3,22,995.19296}{₹20/Hr}$ = 16149.759648. Hr.

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Average time Per unit = $\frac{16149.759648 \text{ Hr}}{256 \text{ unit}}$ = 63.084998625 Hr/unit.

What should be Rate of Learning to have ATPU = 63.084998625 on first 256 unit.

As $y = ax^b$

$63.084998652 = 112.5(256)^b$.

X $\left\{ \frac{63.084998652}{112.5} = (256)^b \right.$

$= 0.5607555 4357 = (256)^b$

Log 0.5607555 X

$\log 63.084998652 = \log 112.5 (256)^b$.

~~log~~:

$1.7993 = \log 112.5 + b \log 256$.

$= 2.0492 + b \log_2 8$.

$= 2.0492 + \frac{\log r}{\log 2} \times 8 \times \log 2$

$1.7993 = 2.0492 + 8 \log r$

$1.7993 - 2.0492 = 8 \log r$

$= \frac{-0.2499}{8} = \log r$

$= \log r = (-0.0312375 + 1 - 1) AL$

$r = AL(0.9687625 - 1)$

~~Antilogs~~ ~~Alog~~ AL($\bar{1}$.9687625)

| | |
|---|---|
| 1 | 9290 |
| | 15 |
| 10 | 9305 = r |
| | 0.9305 |
| | i.e 93.05% |

classmate

(18)
[Page-158]

Statement of learning Curve.

| Cummulative Production | Avg Material per unit | Total Material | Initial Material. |
|---|---|---|---|
| 1 lot | 100 kg | 100 kg | |
| 2 lot | | | |
| ⋮ | | | |
| 319 | 65·27 | 20821.13 | |
| 320 lot | 65·26. | 20883·2. | 62·07 (320th lot) |

$Y = ax^b$

$= 100\ kg\ Y(320)^{-0.074}.$

$\log y = \log 100 - 0.074 \log 320.$

$= 2.0 - 0.074 \times 2.5051$

$=$ ~~[illegible]~~ $1.8146226$

$Y =$ Anti log $1.8146226.$

$= 65.26.$

Average Material to be used per lot upto 320 lot.

Total material for 320 lot $= 65.26 \times 320$

$= 20883.2$kg

$Y = ax^b$

$100\ (319)^{-0.074.}$

$\log y = \log 100 - 0.074 \log 319.$

$= 2.0 - 0.74 \times 2.50379.$

$Y =$ Antilog $1.8147$

$= 65.27.$

Total Material to be used for 319 lot. $= 319 \times 65.27$ kg

$=$ 20821.13 kg.

Material to be Consumed for 320 lot =

Total Material for 320 lot = 20833.20 kg

Total Material for 319 lot = 20821.13 kg

62.07 kg

Learning curve

Statement for Labour Cost (Labour)

| Cumm Production | ATPL | T.T | I.T. |
|---|---|---|---|
| 1 lot | 100 | 100 | |
| 2 lot | | | |
| ⋮ | | | |
| 319 lot | 15.62 | 4982.78 | |
| 320 lot | 15.61 | 4995.2 | 12.42 (320th lot) |

Average time for 320 lot (First)

$Y = ax^b$

$100\text{ Hr} \times (320)^{-0.322}$

$\log 100 - 0.322 \times \log 320$

$2.0 - 0.322 \times 2.5051$

Antilog 1.1933578

= 15.61

Total time for First 320 lot

= 15.61 X 320

4995.2

Average Time for 319 lot

$Y = ax^b$

$= 100\text{ Hr} \times (319)^{-0.322}$

$\log 100 - 0.322 \times \log 319$

$2.0 - 0.322 \times 2.50379$

$2 - 0.80822038$

Anti log 1.19377962

= 15.62

Total time for first 319 lot

= 15.62 × 319

= 4982.78

Data for Resource Variance

| | Budget (1 lot) | | | Revised Budget (320th lot) | | | Actual (320th lot) | | |
|---|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rate | Amt |
| Material | 100 kg | 55 | 5500 | 62.07 | 55 | 3413.85 | 80 | 50 | 4000 |
| Labour. | 100 Hr | 40 | 4000 | 12.42 | 40 | 496.8 | 20 | 50 | 1000 |
| Var. O/H. | 100 | 75 | 7500 | 12.42 | 75 | 931.5 | 20 | 70 | 1800 |

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Calculate all variances

Material Price Variance = (SR - AR) X AQ

= (55-50) X 80 = 400 F

Material Usage Variance = (SQ - AQ) X SR

= (62.07 - 80) X 55 986.15 A.

=

Labour Rate variance = (SR - AR) X AH

= (40 - 50) X 20 = 200 A.

Labour Efficiency Variance = (SH - AH) X SR

= (12.42 - 20) X 40 = 303.2 A.

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# Chapter- '5'

13th class

## SIMULATION

DATE 28 12 2016

### # Simulation

- # Estimation of Demand on the basis of past
- # Simulation Technique should be applied where situation are uncertain like. Estimation of Demand, Estimation of City weather, Estimation of winning in lottery.
- # Simulation Technique should be applied in short term decision. if we predict Estimation of Demand for Next year on the basis of past 10 years. We apply Fixed Trend in Time Series. However if we predict demand for next 10 days on the basis of past 100 days, then we should apply Simulation.

# Estimation of Future on the basis of Past.

$$\left.\begin{array}{l}\text{Past}\\ \text{Actual Data}\end{array}\right] \xrightarrow{\text{system : simulation}} \left[\begin{array}{l}\text{Future}\\ \text{Estimation}\end{array}\right.$$

# Events which occurred more times in past that Event will Occur more time in Near Future.

Past : Actual Data

$\Downarrow$

$$\text{Probability} = \frac{\text{No. of Times event occurred}}{\text{Total Possible outcomes.}} \quad [\text{Two Digit}]$$

[eg] 40 Red ball

60 Green ball.

$P(\text{Red ball}) = \frac{40}{100} = 40\%$

Future Estimation

$\Downarrow$

Random Numbers

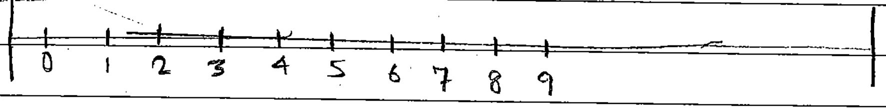
$\Downarrow$

Arbitrary No

(Two Digit)

# Simulation means to study a system in replicate manner, not in actual manner & ~~demand~~ derived a result based on imagination, so that remedial action could be taken on time.

Number



$P(3) = \frac{1}{10} =$ Every No. has equal chance of being selected,
$P(6) = \frac{1}{10}$ any No. can be selected for the system.

Arbitrary Two No.
67 Random No. i.e, Randomly selected
72 to link with probability (Past Data)
12
00.

# Simulation is a way to study the system.

There are 2 ways in simulation to study the system

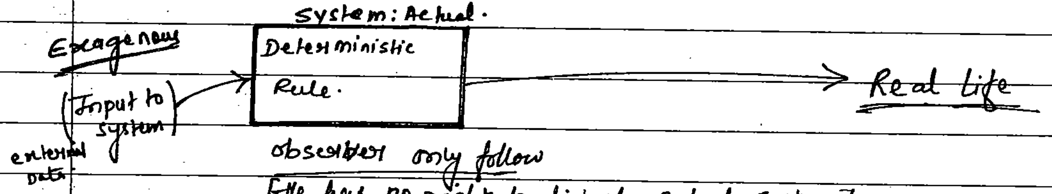
Actual system (Experiment)

Model system (Copy/Replicate) ⇐ [Simulation Model]
⇓
Experiment
⇓
Result Draw

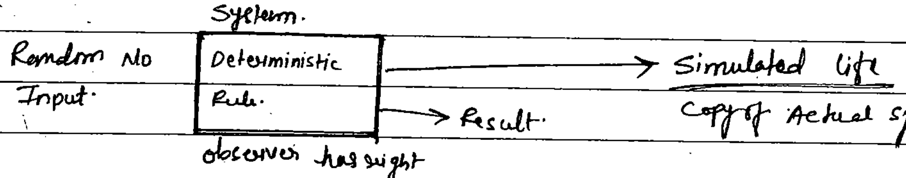
(eg) [For Develop a Colony]

⇒ [1st draw the Model of Colony and read the Model and on the basis of drawn Model take action]

[negative, remedial action will be taken immidiately to Prevent such event]



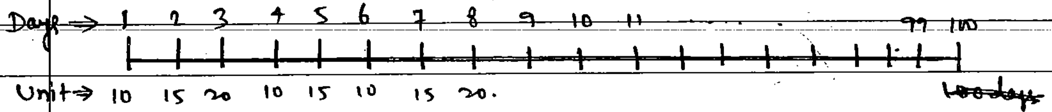
observer only follow
[He has no right to disturb Actual System].



observer has right to imagine all possible combination.

(eg) **Mathematical eg**

Past Data [Actual]. last 100 day



Manage unit: Ascending order. (System: Replicate)

| Demand (unit) | Days. | Probability | Commulative Prob. | Range. |
|---|---|---|---|---|
| 10 unit | 20 (1st 20 days) | 20% | 20% | 00-19 |
| 15 unit | 50 (Next 50 days) | 50% | 70% | 20-69 |
| 20 unit | 30 (last days) | 30% | 100% | 70-99. |

TRIAL-I

| Days | Random No | Demand (unit) |
|---|---|---|
| 1 | 15# | 10 unit |
| 2 | 72 | 20 unit |
| 3 | 25 | 15 unit |
| 4 | 19 | 10 unit |
| 5 | 92 | 20 unit. |
| | | 75 unit |

Average Demand $= \frac{75}{5} = 15$ unit

~~# on~~

# On 15th days in past data, what was out actual Demand, that Demand will occur on 1 Days.

(#) To Simulation Technique we should prepare following 2 Statement

(1) Statement of Random No. Range.

(2) Statement of Simulated Data

DATE

(2)
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## Statement of Random No. Range.

| Demand | Probability | Cummulative Prob | Range. |
|---|---|---|---|
| 0 | 10% | 10% | 0-9 |
| 15 | 20% | 30% | 10-29 |
| 25 | 30% | 60% | 30-59 |
| 35 | 20% | 80% | 60-79 |
| 45 | 5% | 85% | 80-84 |
| 50 | 15% | 100% | 85-99 |

## Statement of Simulated Data.

| Days | Random No. | Demand (unit) | Manufacture | stock |
|---|---|---|---|---|
| 1 | 15 | 15 | 35 | 20 |
| 2 | 14 | 15 | 35 | 0+20=20. |
| 3 | 68 | 35 | 35 | 0 |
| 4 | 09 | 0 | 35 | 0+35=55 |
| 5 | 24 | 15 | 35 | 20 |
| 6 | 75 | 35 | 35 | 0 |
| 7 | 98 | 50 | 35 | — |
| 8 | 34 | 25 | 35 | 10 |
| 9 | 56 | 25 | 35 | 10 |
| 10 | 77. | 35 | 35 | 0. |
|  |  | 250 |  |  |

$$\text{Average Demand} = \frac{\text{Total Demand}}{\text{No of Days}} = \frac{250}{10} = 25 \text{ Cakes / days.}$$

## # Point to be Remember

(1) In case of Perishable goods (branded Cake), Vegetable, etc), Inventory of last day should not be carried to next day.

(2) In case of Non perishable goods (Books, Car, etc) Inventory of the day should be C/f to Next day.

classmate

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(13) [Page-175]

| | | | Unsold | |
|---|---|---|---|---|
| Working | Cost Price = | 40 | Cost price = | 40 |
| | Profit | 10 | loss | 15 |
| | S.P | 50 | S.P. | 25 |

Demand > Manufacture

Penalty = 5 ₹

㊟ [Benefit to be Lost due to not being able to meet the demand. = Penalty] ⇒ opportunity cost.

(OR)

㊟ Excess Purchase cost over selling Price = Penalty

Statement of Random No. Range.

| Demad (Units) | Probability | Cummulative Probability | Range. |
|---|---|---|---|
| 27 | 15% | 15% | 0-14 |
| 28 | 10% | 25% | 15-24 |
| 29 | 20% | 45% | 25-44 |
| 30 | 35% | 80% | 45-79 |
| 31 | 20% | 100% | 80-99. |

Statement of Simulated Data / Profit & loss.

| Days. | Random No. | Demand | Manufacture | Profit. | Profit (29 unit Mfg) |
|---|---|---|---|---|---|
| 1 | 79 | 30 | 28 | [(28X10)-(2X5)] = 270 | [(29X10)-(1X5)] = 285 |
| 2 | 11 | 27 | 28 | [(27X10)-(1X15)] = 255 | [(27X10)-(2X15)] = 240 |
| 3 | 16 | 28 | 28 | [(28X10)] = 280 | [(28X10)-(1X15)] = 265 |
| 4 | 20 | 28 | 28 | [28X10] 280 | [(28X10)-(1X15)] = 265 |
| 5 | 10 | 27 | 28 | [(27X10)-(1X15)] = 255 | [(27X10)-(2X15)] = 240 |
| 6 | 99 | 31 | 28 | [(28X10)-(3X5)] = 265 | [(29X10)-(2X5)] = 280 |
| 7 | 65 | 30 | 28. | [(28X10)-(2X5)] = 270 | [(29X10)-(1X5)] = 285 |
| | | | | Profit ⇒ 1875 | Profit ⇒ 1860 |

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Statement of Random No. Range.

| Events (Time) | Probability | Cumm. Prob | Range |
|---|---|---|---|
| 45 | 10% | 10% | 0-9 |
| 60 | 20% | 30% | 10-29 |
| 15 | 30% | 60% | 30-59 |
| 45 | 25% | 85% | 60-84 |
| 15 | 15% | 100% | 85-99. |

Statement of simulated Data.

| Trial of Random No. | Random No. | Arrival Schedule Time | Service start | Consumption of Time | Departure | Waiting Time Patient | Waiting Time Doctor |
|---|---|---|---|---|---|---|---|
| 1 | 66 | 9.00 A.M | 9.00 A.M. | 45 | 9.45 | — | — |
| 2 | 17 | 9.30 | 9.45 | 60 | 10.45 | 15 mint | — |
| 3 | 79 | 10.00 | 10.45 | 45 | 11.30 | 45 | — |
| 4 | 40 | 10.30 | 11.30 | 15 | 11.45 | 60 | — |
| 5 | 82 | 11.00 | 11.45 | 45 | 12.30 | 45 | — |
| 6 | 11 | 11.30 | 12.30 | 60 | 1.30 | 60 | — |
| 7 | 34 | 12.00 | 1.30 | 15 | 1.45 | 90. | — |
| 8. | 25 | 12.30. | 1.45 | 60. | 2.45 | 75. | — |
|  |  |  |  |  |  | 390 Mint |  |

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Statement of Random No. Range.

ARRIVAL

| Event Minutes Early | Prob. | Cumm Prob | Range |
|---|---|---|---|
| 5 | 10% | 10% | 00-09 |
| 3 | 20% | 30% | 10-29 |
| 2 | 40% | 70% | 30-69 |
| On time | 20% | 90% | 70-89 |
| Late 1 | 5% | 95% | 90-94 |
| 3 | 5% | 100% | 95-99. |

Consumption of Time.

| Event (Mint) Consumption of Time | Probability | Cummulative Probability | Range. |
|---|---|---|---|
| 14 | 18% | 18% | 00-09 |
| 15 | 15% | 25% | 10-24 |
| 16 | 28% | 53% | 25-52 |
| 17 | 34% | 87% | 53-86 |
| 18. | 13% | 100% | 87-99. |

## Statement of Simulated Data. [Let schedule Time be 9.00 AM].

| Trial | Random No | Schedule Time | Arrival Time | Service start | Consumption of Time | Departure | WAITING TIME Patient | WAITING TIME Doctor |
|---|---|---|---|---|---|---|---|---|
| 1 | 94,08 | 9.00 A.M. | 9.00 A.M. | 9.01 | 14 mint | 9.15. | — | 01 mint |
| 2 | 49,98 | 9.15 | 9.13 A.M | 9.15 | 18 mint | 9.33. | 2 mint | — |
| 3 | 79,57 | 9.30 | 9.30 AM | 9.33 | 16 mint | 9.49 | 3 mint | — |
| 4 | 43,74. | 9.45 | 9.43 AM | 9.49 | 17 mint | 10.06. | 6 mint | — |
| 5 | 92,24. | 10.00 | 10.01 AM | 10.06 | 15 mint | 10.21 | 5 mint | — |
| 6. | 17,14. | 10.15. | 10.12 AM. | 10.21 | 15 mint | 10.36 | 9 mint | — |
| | | | | | | | 25 mint | 1 mint |

Average waiting Time $= \frac{25}{6} = 4.16$

If we have 2 uncertain event in any Trial, then ~~Ramd~~ Random Number should be considered pair wise. i.e,

First Random No. for First Event }
II Random No. for second event } In one Trial of 50 m.

14th class

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## Statement of Random No. Range.

### Table I = [Previous day : Rain]

| Event | Probability. | Cumm. Prob | Range |
|---|---|---|---|
| NO Rain | 25% | 25% | 0 – 24 |
| 1 CM | 50% | 75% | 25 – ~~6~~74 |
| 2 CM | 15~~0~~% | 90% | 75 - 89 |
| 3 CM | 05% | 95% | 90 - 94 |
| 4 CM. | 05% | 100% | 95 - 99. |

### Table-II [Previous Day : No Rain]

| Event | Probability | Cumm. Prob | Range |
|---|---|---|---|
| NO Rain | 75% | 75% | 0 - 74 |
| 1 CM | 15% | 90% | 75-89 |
| 2 CM | 6% | 96% | 90 - 95 |
| 3 CM | 4% | 100% | 96 - 99 |

## Statement of Simulated Data.

| TRIAL | Random. No. | Basis. | Event. |
|---|---|---|---|
| 1 | 06 | T-2 | No. Rain |
| 2 | 78 | T-2 | 1 CM |
| 3 | 76 | T-1 | 2 CM. |
| 4 | 67 | T-1 | 1 CM. |
| 5 | 63 | T-1 | 1 CM |
| 6 | 39 | T-1 | 1 CM. |
| 7. | 70. | T-1 | 1 CM. |

Total Rainfall = 7CM

Day Without Rainfall = 1 Day.

1st Day becomes the previous day for 2nd day of simulation
2nd Day becomes the previous days for 3rd day of simulation.

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## Statement of Random No. Range.

### T-I For 'x'.

| Time (Second) | Probability | Cumm. Prob | Range |
|---|---|---|---|
| 10 | 4/100 = 4% | 4% | 00-03 |
| 20 | 6/100 = 6% | 10% | 04-09 |
| 30 | 10/100 = 10% | 20% | 10-19 |
| 40 | 20/100 = 20% | 40% | 20-39 |
| 50 | 40/100 = 40% | 80% | 40-79 |
| 60 | 11/100 = 11% | 91% | 80-90 |
| 70 | 5/100 = 5% | 96% | 91-95 |
| 80 | 4/100 = 4% | 100% | 96-99. |

### For 'y'

~~Time~~

| Time (second) | Probability | Cumm. Prob. | Range |
|---|---|---|---|
| 10 | 5/50 = 10% | 10% | 00-09 |
| 20 | 4/50 = 8% | 18% | 10-17 |
| 30 | 7/50 = 14% | 32% | 18-31 |
| 40 | 6/50 = 12% | 44% | 32-43 |
| 50 | 10/50 = 20% | 64% | 44-63 |
| 60 | 8/50 = 16% | 80% | 64-79 |
| 70 | 4/50 = 8% | 88% | 80-87 |
| 80 | 6/50 = 12% | 100% | 88-99. |

## Statement of Simulated Data.

| TRIAL | (x) Random No. | Start [Sec] | Consumption of Time [Sec] | Departure [Second] | Random No | (y) Start [Sec] | Consumption of Time [Sec] | Departure [Sec] | Waiting Time [Second] |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 24 | 00. | 40 | 40 | 25 | 40 | 30 | 70 | 40 |
| 2 | 10 | 40 | 30 | 70 | 76 | 70 | 60 | 130 | 00 |
| 3 | 25 | 70 | 40 | 110 | 09 | 130 | 10 | 140 | 0 |
| 4 | 90 | 110 | 60 | 170 | 20 | 170 | 30 | 200 | 30 |
| 5 | 61 | 170 | 50 | 220 | 97 | 220 | 80 | 300 | 20. |
| 6 | 82 | 220 | 60 | 280 | 41 | 300 | 40 | 340 | 00 |
| 7 | 00 | 280 | 10 | 290 | 56 | 340 | 50 | 390 | 00 |
| 8 | 03. | 290 | 10. | 300. | 34. | 390 | 40 | 430 | 00 |

Total Waiting Time ⇒ 90 Second for 'y'.

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## Statement of Remdom No. Range

Table-1 [Time between two Consucutive Customers]. Table-2

| Event | Probability | Cumm Probability | Range |
|---|---|---|---|
| 4 | 25% | 25% | 00-24 |
| 5 | 10 % | 35% | 25-34 |
| 6 | 25% | 60% | 35-59 |
| 7 | 20% | 80% | 60-79 |
| 8 | 20% | 100% | 80-99 |

| Event | Probability | Cummulative Probability | Range |
|---|---|---|---|
| 5 | 10% | 10% | 00-09 |
| 6 | 20% | 30% | 10-29 |
| 7 | 30% | 60% | 30-59 |
| 8 | 10% | 70% | 60-69 |
| 9 | 30% | 100% | 70-99. |

## Statement of Simulated Data.

| TRIAL | Random No | Schdule Time | Arrival | Start | Consumption | Departure | Waiting Time Customer | Waiting Time Teller |
|---|---|---|---|---|---|---|---|---|
| 1 | 83, 10 | 10. A.M. | 10.08. | 10.08. | 6 | 10. 14 | — | 8 mint |
| 2 | Table I 93, Table-2 34 | | 10.16 | 10.16. | 7 | 10.23 | — | 2 |
| 3 | 33, 53 | | 10.21 | 10.23 | 7 | 10.30 | 2 | – |
| 4 | 49, 94 | | 10.27 | 10.30 | 9 | 10.39 | 3 | – |
| 5 | 37, 97. | | 10.33. | 10.33. | 9. | 10.48 | 6 | – |

Explanation :- Random No 83 means T-1 i.e, first Customer will arrive 8 mint late than its preceeding customer.
As we know that, No Preceeding Customer exist for 1st means schdule time for 1st Customer is the Arrival Time for Preceeding Customer for 1st one.

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## Statement of Random No. Range.

| Table-1 [Selling Price] | | | | Table-2 [Variable Cost] | | | | Table-3 [Sales-Volume]. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Event | Prob | Cumm Prob | Range | Event | Prob. | Cumm Prob | Range | Event | Prob | Commu Prob | Range |
| 3 | 20% | 20% | 00-19 | 1 | 30% | 30% | 00-29 | 2000 | 30% | 30% | 00-29 |
| 4 | 50% | 70% | 20-69 | 2 | 60% | 90% | 30-89 | 3000 | 30% | 60% | 30-59 |
| 5 | 30% | 100% | 70-99 | 3 | 10% | 100% | 90-99 | 5000 | 40% | 40% | 60-99. |

## Statement of Profit (Simulated.)

| TRIAL | Random. No | [(S.P-V.C)Qty sold - F.C] |
|---|---|---|
| 1 | 81, 32, 60 | [(5-2) 5000 - 4000] = 11000 |
| 2 | 04, 46, 31 | [(3-2) 3000 - 4000] = (1000) |
| 3 | 67, 25, 24 | [(4-1) 2000 - 4000] = 2000 |
| 4 | 10, 40, 02 | [(3-2) 2000 - 4000] = (2000) |
| 5 | 39, 68, 08. | [(4-2) 2000 - 4000] = Nill |
| | | - 10,000 |

Average Profit = $\frac{10000}{5}$ = 2000 Answer

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## Statement of Random No. Range.
(Type of Service)

| Event | Probability | Cummulative Probability | Range |
|---|---|---|---|
| Self Service | 60% | 60% | 00-59 |
| Attended Service. | 40% | 100% | 60-99. |

## ARRIVAL Rate

| Event (No. of Arrival) | Probability | Cummulative Probability | Range |
|---|---|---|---|
| 1 | 10% | 10% | 00-09 |
| 3 | 30% | 40% | 10-39 |
| 4 | 05 | 45% | 40-44 |
| 0 | 20% | 65% | 45-64 |
| 2 | 35% | 100% | 65-99. |

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(i) Statement of Random No. Range

Demand

| Event (Demand) | Probability | Cummulative Probability | Range- |
|---|---|---|---|
| 4 | 4/50 = 8% | 8% | 00-07 |
| 5 | 10/50 = 20% | 28% | 08-27 |
| 6 | 16/50 = 32% | 60% | 28-59 |
| 7 | 14/50 = 28% | 88% | 60-87 |
| 8 | 6/50 = 12% | 100% | 88-99. |

(ii) & (iii) Statement of Simulated Data.

| Days | Random No. | Demand | Car owned | Rental Car | Rental loss [Demand > Car owned] |
|---|---|---|---|---|---|
| 1 | 15 | 5 | 6 | 5 | – |
| 2 | 48 | 6 | 6 | 6 | – |
| 3 | 71 | 7 | 6 | 6 | 1 |
| 4 | 56 | 6 | 6 | 6 | – |
| 5 | 90 | 8 | 6 | 6 | 2 |

Total Rental for 5 days = 29        3

(ii) ∴ Average No. of Car Rented / Day = $\frac{29}{5}$ = 5.8 Car.

(iii) 3 Rental will be lost over 5 days.

(12) [Page-174]

**Replenishment** :- Time Gap between Placing the order & receiving the order. It does not include the day of placing the order & the day of receiving the order.

| eg | 1st March | 2nd&3rd March | 4th March |
|---|---|---|---|
| | Date of Placing oRder | 2 days<br>⇓<br>Replenishment | Date of Receipt |

**Lead Period** :- It includes the day of Placing / Receiving the order.

### Statement of Random No. Range.

| Event (Demand) | Probability | Cumm. Probability | Range. |
|---|---|---|---|
| 0 | 5% | 5% | 00-04 |
| 1 | 10% | 15% | 05-14 |
| 2 | 30% | 45% | 15-44 |
| 3 | 45% | 90% | 45-89 |
| 4 | 10% | 100% | 90-99. |

### Statement of Simulated Data

Option-1 order-5 books

| Cycle-No | Random. No. | Op. stock | Received | Demand | Closing stock | order o/s at begining | Order. Placed | Order o/s at the end |
|---|---|---|---|---|---|---|---|---|
| 1. | 89 | 8 | — | 3 | 5 | 6 | X | 6 |
| 2. | 34 | 5 | 6 | 2 | 9 | 0 | X | ~~0~~ |
| 3. | 78 | 9 | ~~0~~ | 3 | 6 | [illegible] | $5^{*}$ | $5^{*}$ |
| 4 | 63 | 6 | — | 3 | 3 | $5^{*}$ | * | $5^{*}$ |
| 5. | 61 | 3 | — | 3 | 0 | $5^{*}$ | $5^{**}$ | 10.$[5^{*}+5^{**}]$ |
| 6. | 81 | 0 | $5^{*}$ | 3 | 2 | $5^{**}$ | $5^{***}$ | 10.$[5^{**}+5^{***}]$ |
| 7. | 39 | 2 | — | 2 | 0 | $10^{[**+***]}$ | X | 10 $[5^{**}+5^{***}]$ |
| 8. | 16 | 0 | $5^{**}$ | 2 | 3 | $5^{***}$ | X | $5^{***}$ |
| 9. | 13 | 3 | $5^{***}$ | 1 | 7 | — | $5^{****}$ | $5^{****}$ |
| 10. | 73. | 7 | — | 3 | 4 | $5^{****}$ | X | $5^{****}$. |
| | | | | | 39 | | 4 orders | |

Relevant Cost for 1st option = Ordering Cost + Storage Cost

= [(4 orders X ₹10/order) + (39 books X ₹0.5/Book)]

= ₹40 + ₹19.5 = ₹59.5.

∴ Relevant Cost = ₹59.5.

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Option - II     Order - 8 book

| Cycle No. | Random No | op. stock | Received | Demand | closing stock | Order o/s at begging | Order Place | Order o/s at the end |
|---|---|---|---|---|---|---|---|---|
| 1 | 89 | 8 | — | 3 | 5 | 6 | X | 6 |
| 2 | 34 | 5 | 6 | 2 | 9 | — | X | — |
| 3 | 78 | 9 | — | 3 | 6 | — | $8^{I}$ | $8^{I}$ |
| 4 | 63 | 6 | — | 3 | 3 | $8^{I}$ | X | $8^{I}$ |
| 5 | 61 | 3 | — | 3 | 0 | $8^{I}$ | X | $8^{I}$ |
| 6 | 81 | 0 | $8^{I}$ | 3 | 5 | — | $8^{II}$ | $8^{II}$ |
| 7 | 39 | 5 | — | 2 | 3 | $8^{II}$ | X | $8^{II}$ |
| 8 | 16 | 3 | — | 2 | 1 | $8^{II}$ | X | $8^{II}$ |
| 9 | 13 | 1 | $8^{II}$ | 1 | 8 | 8— | X | — |
| 10 | 17 | 8 | — | 3 | 5 | — | $8^{III}$ | $8^{III}$ |
|  |  |  |  |  | 45 books |  | 3 order |  |

∴ Relevant Cost for IInd Option = ordering Cost + Storage Cost

= [(3 order × ₹10/order) + (45 books × ₹0.5/books)]

= ₹30 + ₹22.5

= ₹52.5

⇒ Second option is better.

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Statement of Random No. Range.

| Event (Production) | Probability | Cumm. Probability | Range |
|---|---|---|---|
| 196 | 15% | 15% | 00-14 |
| 197 | 10% | 25% | 15-24 |
| 198 | 20% | 45% | 25-44 |
| 199 | 35% | 80% | 45-79 |
| 200 | 20% | 100% | 80-99. |

Statement of Simulated Data.

| Days | Random No | Production Demand | Capacity of lorry | Empty | Waiting |
|---|---|---|---|---|---|
| 1 | 04 | 196 | 200 | 4 | — |
| 2 | 23 | 197 | 200 | 3 | — |
| 3 | 50 | 199 | 200 | 1 | — |
| 4 | 77 | 199 | 200 | 1 | — |
| 5 | 27 | 198 | 200 | 2 | — |
| 6 | 54 | 199 | 200 | 1 | — |
| 7. | 10 | 196 | 200 | 4 | — |
|  |  |  |  | 16 |  |

(I) No waiting.

(II) ∴ Average Empty = $\frac{16}{10}$ = 1.6 Mopeds / Day

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## Working Notes

defects 'A' = 15% — out of 100 unit, 15 unit may be defects due to A
85 unit free from defects A.

defects 'b' = 20% — out of 100 units, 20 units may be defects due to b.
80 units may be free from b.

defects 'C' = 10% — out of 100 units, 10 units may be defects due to 'C'
90 units may free from defects 'C'.

We Can say 45% Probability for defects.
We Can't add probability of defects because those defects could occur simuntaniously. However if these defects could not occur simuntaniouly i.e, independently; we could add.

### Statement of Random No. Range.

Table-1

| | Probability | Cummulative Probability | Range- |
|---|---|---|---|
| Defects A occur | 15% | 15% | 00- 14. |
| Defects A not occur | 85% | 100% | 15 - 99. |

Table-2

| | | | |
|---|---|---|---|
| Defects b Occur | 20% | 20% | 00-19 |
| Defects b not occur | 100% | 100% | 20-99. |

Table-3

| | | | |
|---|---|---|---|
| Defect C occur | 10% | 10% | 00-09 |
| Defects C not occur | 90% | 100% | 10 - 99. |

DATE

Statement of Simulated Data.

| TRIAL | Random No. | Defects | | | Rework Time |
|---|---|---|---|---|---|
| | | A | b | c | |
| 1 | 55,79,20 | No | No | NO | — |
| * 2 | 09, 55, 84 | Yes | NA | NA | ~~15 mint~~ |
| 3 | 40, 10, 56 | No | Yes | NO | 15 mint. |
| 4 | 93, 13, 11. | No | Yes | No. | 15 mint |
| * 5 | 01, 57, 52 | Yes | NA | NA | ~~15+30 mint~~ |
| 6 | 83, 09, 03 | No | Yes | Yes | 15+30 mint |
| 7 | 63, 47, 82 | No | No | No. | — |
| | | | | | 75 mint |

* N.A = Not applicable because as per Question if product is defects due to 'A' then no further processed and therefore no chance for defects of b & c occuring. that Why not applicable for defects b & c's probability of defects.

(21) [Page-180] Working

Previous Day: No stock = Order on Current Day = 110 cakes

Previous Day: stock exist = then Order should be placed on sale basis

| Sale | | order of cakes |
|---|---|---|
| 100 | → | 100 |
| 101 | → | 100 |
| 102 | → | 100 |
| 103 | → | 105 |
| 104 | → | 105 |
| 105 | → | 105 |
| 106 | → | 105 |

## Statement of Random No. Range.

### Fresh Cakes

| Events | Prob | Cumm Prob | Range |
|---|---|---|---|
| 100 | 01% | 1% | 00 - 00 |
| 101 | 03% | 4% | 01 - 03 |
| 102 | 04% | 8% | 04 - 07 |
| 103 | 07% | 15% | 08 - 14 |
| 104 | 09% | 24% | 15 - 23 |
| 105 | 11% | 35% | 24 - 34 |
| 106 | 15% | 50% | 35 - 49 |
| 107 | 21% | 71% | 50 - 70 |
| 108 | 18% | 89% | 71 - 88 |
| 109 | 09% | 98% | 89 - 97 |
| 110. | 02% | 100% | 98 - 99. |

### One day old cake.

| Events | Prob. | Cumm Probability | Range. |
|---|---|---|---|
| 0 | 70% | 70% | 00-69 |
| 1 | 20% | 90% | 70-89 |
| 2 | 08% | 98% | 90-97 |
| 3. | 02% | 100% | 98-98. |

## Statement of Simulated Data. [fresh Cake]

| Days | Random No. | order Received | Demand | Sales | Closing Stock | order Placed |
|---|---|---|---|---|---|---|
| 1 | 37 | ~~105~~ | 106. | 105 | 00 | 110 |
| 2 | 73 | 110 | 108 | 108 | 2 | 110 |
| 3 | 14 | 110 | 103 | 103 | 07 | 105 |
| 4 | 17 | 105 | 104 | 104 | 01 | 105 |
| 5 | 24 | 105 | 105 | 105 | 00 | 105 |
| 6 | 35 | 105 | 106 | 105 | 00 | 110 |
| 7 | 29 | 110 | 105 | 105 | 05 | 110. |
| 8 | 37 | 110 | 106 | 106 | 04 | 105 |
| 9 | 33 | 105 | 105 | 105 | 00 | 105 |
| 10 | 68. | 105 | 107. | 105 | 00 | 110 |
|  |  |  |  | 1051 |  |  |

## Statement of Simulated Data

### One day old Cake

| Days | Random No | opening. Stock | Demand | Sales | Dead stock |
|---|---|---|---|---|---|
| 1 | 17 | ~~00~~ | 0 | ~~00~~ | ~~00~~ |
| 2 | 28 | ~~00~~ | 0 | ~~00~~ | ~~00~~ |
| 3 | 69 | 02 | 0 | 00 | 02 |
| 4 | 38 | 07 | 0 | 00 | 07 |
| 5 | 50 | 01 | 0 | 00 | 01. |
| 6 | 57 | 00 | 0 | 00 | 00 |
| 7 | 82 | 00 | 1 | 00 | 00 |
| 8 | 44 | 05 | 0 | 00 | 05. |
| 9 | 89 | 04 | 1 | 01 | 03. |
| 10 | 60 | 00 | 0 | 00 | 00 |
| | | | | 01 | 18 |

### Statement of Profit

| | | | |
|---|---|---|---|
| Sale of Fresh Cake | (1051 X 7) | 7357 | |
| Sale of Old Cake | (1 X 2) | 2 | 7359 |
| Less Cost of ~~Fresh~~ Cakes. | [1051 + 1 + 18] X 4.5 | | (4815) |
| | Profit ⟹ | | 2544 |

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## Statement of Random No Range.

Table-1

| Events [Cash Receipts] | Prob | Cumm Probability | Range |
|---|---|---|---|
| 30 | 20% | 20% | 00-19 |
| 42 | 40% | 60% | 20-59 |
| 36 | 25% | 85% | 60-84 |
| 99. | 15% | 100% | 85-99 |

Table-2

| Events [Disbursment] | Prob | Cumm Prob | Range |
|---|---|---|---|
| 33 | 15% | 15% | 00-14 |
| 60 | 20% | 35% | 15-34 |
| 39 | 40% | 75% | 35-74 |
| 57 | 25% | 100% | 75-99 |

## Statement of Simulated Data

| Month | Random No | Opening cash (₹in crore) | Receipts | Disbursed | closing cash | |
|---|---|---|---|---|---|---|
| 1 | 17,78 | 15 | 30 | 57 | (12) | |
| 2 | 43,16 | (12) | 42 | 60 | (30) | |
| 3 | 7435 | (30) | 36 | 39 | (33) | |
| 4 | 31,23 | (33) | 42 | 60 | (51)# | |
| 5 | 72,44 | (51) | 36 | 39 | (54)# | |
| 6 | 46,92 | (54) | 42 | 57 | (69)# | |
| 7 | 51.58 | (69) | 42 | 39 | (66)# | |
| 8 | 68,08 | (66) | 36 | 33 | (63)# | |
| 9 | 93,58 | (63) | 99 | 39 | (03) | |
| 10 | 5478 | (03) | 42 | 57 | (18) → | 399 |
| 11 | 96,54 | (18) | 99 | 39 | 42 | |
| 12 | 09,77 | 42 | 30. | 57. | 15 | |

(i) Probability of Cash shortfall $= \frac{10 \text{ times}}{12 \text{ times}} \times 100 = 83.\dot{3}\dot{3}\%$

(ii) Average Monthly shortfall $= \frac{\text{Total Cash Short}}{\text{No. of Month falling shortfall}} = \frac{399}{10}$ =

39.9 Crores

(iii) Event after taking overdraft faciity of 45 crores = 5 times out of 12 times. $= \frac{5}{12} \times 100 = 41.\dot{6}\%$

(#) shortfall even after taking overdraft facility of ₹ 45 crore

DATE

(Q20)
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## Statement of Random No. Range.

| Time between Arrival | | | | ~~&~~ Service Time | | | |
|---|---|---|---|---|---|---|---|
| Time | Prob. | Cumm. Prob | Range. | Service Time (min) | Prob | Cumm Prob | Range. |
| 1 | 5% | 5% | 00-04 | 1 | 1/10 = 10% | 10% | 00-09 |
| 2 | 20% | 25% | 05-24 | 2 | 2/10 = 20% | 30% | 10-29 |
| 3 | 35% | 60% | 25-59 | 3 | 4/10 = 40% | 70% | 30-69 |
| 4 | 25% | 85% | 60-84 | 4 | 2/10 = 20% | 90% | 70-89 |
| 5 | 10% | 95% | 85-94 | 5 | 1/10 = 10% | 100% | 90-99 |
| 6 | 5% | 100% | 95-99. | 6 | 0/10 = 0% | 100% | 00 - 00. i.e,<br>↳ ignored |

## Statement of Simulated Data.

| Trial | Random No | Time of Schedule | Time of Arrival | Service Start | Consumption of Time. | Departure | Length of Waiting line | Waiting Time Customer | Waiting Time Clerk |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 64, 18 | 11. | 11.04 | 11.04 | 02 | 11.06 | – | X | 4 |
| 2 | 04, 36 | – | 11.05 | 11.06 | 03 | 11.09 | 1 | 1 | * |
| 3 | 02, 38 | – | 11.06 | 11.09 | 03 | 11.12 | 1 | 3 | * |
| 4 | 70, 07 | – | 11.10 | 11.12 | 01 | 11.13. | 1 | 2 | – |
| 5 | 03, 08 | – | 11.11 | 11.13 | 01 | 11.14 | 2 | 2 | – |
| 6 | 60, 59 | – | 11.15 | 11.15 | 03 | 11.18 | 0 | – | 1 |
| 7 | 16, 53 | – | 11.17 | 11.18 | 03. | 11.21 | 1 | 1 | – |
| | | | | | 16 | | 6 | 9 | 5 |

Length of Waiting line = How many Customers Standing in front of us for getting service. (हम से पहले कितने customer wait कर रहे है line में)

Average lenght of Waiting line = $\frac{\text{Length of waiting line}}{\text{Total customer}}$ , $\frac{6}{7}$ = 85.71%.

Average waiting time of Customer = $\frac{9}{7}$ = 1.285 mint

Average Service time = $\frac{16}{7}$ = 2.28 mint.

Time spent by customer = $\frac{\text{waiting time + service Time}}{7} = \frac{9+16}{7} = \frac{25}{7} = 3\cancel{.143}$

= 3.57 mint.

classmate

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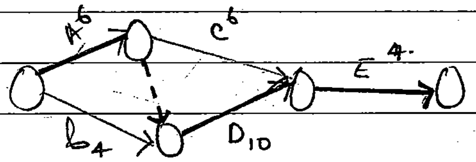
[Page-181]

## Statement of Random No. Range

| Activity | Event | Prob. | Cumm Prob | Range. |
|---|---|---|---|---|
| A | 6 days | 50% | 50% | 00-49 |
| | 8 days | 50%. | 100%. | 50-99. |
| b | 4 days | 30%. | 30% | 00-29 |
| | 5 days | 70%. | 100%. | 30-99 |
| C | 6 days | 50%. | 50%. | 00-49 |
| | 8 days | 50% | 100% | 50-99 |
| D. | 8 days | 30%. | 30%. | 00-29 |
| | 10 days. | 70%. | 100%. | 30-99 |
| E. | 2 days | 20%. | 20%. | 00-19 |
| | 4 days | 80%. | 100%. | 20-99. |

### First Trial

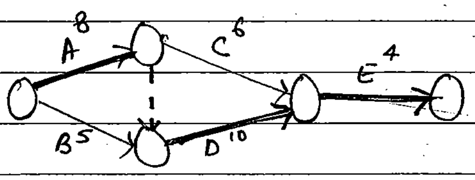
| Activity | Random No. | Days. |
|---|---|---|
| A | 11 | 6 days |
| b | 16 | 4 days. |
| C | 23 | 6 days |
| D | 72 | 10 day |
| E | 94 | 4 days. |



Critical path = A-C-E = 16 days
→ A-D-E = 20 days
b-D-E = 18 days

### TRIAL-II.

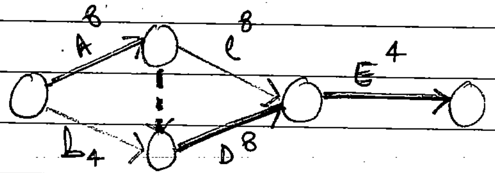
| Activity | Random No. | Days. |
|---|---|---|
| A | 83 | 8 days |
| L | 83 | 5 days |
| C | 02 | 6 days |
| D | 97 | 10 days |
| E | 99 | 4 days. |



Critical Path = A-D-E = 22 days.

Trial - III

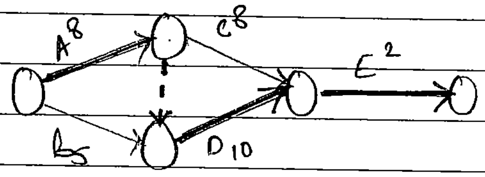
| Activity | Random No. | Days. |
|---|---|---|
| A | 83 | 8 days |
| B | 10 | 4 days. |
| C | 93 | 8 days |
| D | 4 | 8 day |
| E | 33 | 4 days |



Critical Path = A-D-E= 20 Days.

Trial IV

| Activity | Random No. | Days |
|---|---|---|
| A | 53 | 8 days |
| B | 49 | 5 days |
| C | 94 | 8 days |
| D | 37 | 10 days |
| E | 7 | 2 days |



Critical Path = A-D-E = 20 days.

(15) [Page-176] **Working** If we have < OR > type for analysing the event, then we should consider average.

Statement of Random No Range.

| Demand | Prob | Cumm Prob | Range. |
|---|---|---|---|
| 150 | 32% | 32% | 00-31. |
| 450 | 18% | 50% | 32-49 |
| 750 | 15% | 65% | 50-64 |
| 1050 | 25% | 90% | 65-89 |
| 1350 | 10% | 100% | 90-99. |

Statement of Simulated Data. [Newstand]

| TRIAL. | Random No. | Demand. | Contracted Units | Sales | Lost Sales [Demand > Contracted unit] | Profit |
|---|---|---|---|---|---|---|
| 1 | 62 | 750 | 750 | 750 | – | 7500 [750x10] |
| 2 | 83 | 1050 | 750 | 750 | 300 | 7500 |
| 3 | 96 | 1350 | 750 | 750 | 600 | 7500 |
| 4 | 69 | 1050 | 750 | 750 | 300 | 7500 |
| 5 | 27 | 150 | 750 | 150 | — | 1500 - (600x4) = (900) |
| 6 | 15 | 150 | 750 | 150 | — | [1500 - (600x4)] = (900) |
| 7 | 53. | 750 | 750. | 750. | — | 7500. |
|  |  |  |  |  | 1200 units | 35700 |

Loss on lost sales = 1200 units x ₹ 10 = 12000 ₹

(3)
(Page.169)

Statement of Random No. Range

| Demand | Probability | Cumm. Prob | Range. |
|---|---|---|---|
| 00 | 01% | 01% | 00-00 |
| 20 | 15% | 16% | 01-15 |
| 25 | 30% | 46% | 16-45 |
| 35 | 40% | 86% | 46-85 |
| 40 | 10% | 96% | 86-95 |
| 45. | 04% | 100% | 98-99. |

Statement of simulated Data.

| Days | Random No | Opening stock | Purchase | Demand | Sale | Closing stock | Profit | Lost onwards | Net Profit |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 58 | 5 | 35 | 35 | 35 | 5-5=0 | 140 $\left(\begin{smallmatrix}35\times\\4\end{smallmatrix}\right)$ | 05×16<br>=90 | 50. |
| 2 | 80 | 0 | 35 | 35 | 35 | 0 | 140 $\left[\begin{smallmatrix}35\\\times\\4\end{smallmatrix}\right]$ | — | 140 |
| 3 | 51 | 0 | 35 | 35 | 35 | 0 | 140 | — | 140. |
| 4 | 09 | 0 | 35 | 20 | 20 | 15 | 80 $\left[\begin{smallmatrix}20\\\times\\4\end{smallmatrix}\right]$ | ~~15×16~~ | 80 |
| 5 | 47 | 15 | 35 | 35 | 35 | 0 | [35×4]<br>= 140 | [15×16]<br>= 240 | (100) |
| 6 | 26 | 0 | 35 | 25 | 25 | 10 | [25×4]<br>= 100 | — | 100 |
| 7 | 64 | 10 | 35 | 35 | 35 | 0 | [35×4]<br>= 140 | [10×16]<br>= 160 | (20) |
| 8 | 43 | 0 | 35 | 25 | 25 | 10 | [25×4]<br>100 | — | 100 |
| 9 | 86 | 10 | 35 | 40 | 40 | 0 | [40×4]<br>= 160 | [5×16]<br>= 80 | 80. |
| 10. | 35. | — | 35. | 25 | 25 | 10 | [25×4]<br>= 100 | — | 100 |

Profit ⇒ 680

# ASSIGNMENT

16th class
DATE 04 01 2017

**ASSIGNMENT :-** If a Company has different Worker with different efficiency & different Job (Work Station), our purpose is to assign a particular Job to a particular Worker.

So that,

(1) Total Time (Cost) to be Minimized.
(2) No Worker will remain idle.
(3) No Jobs remain Un assigned.

We should develope a system which ensures that the above objective could be fullfilled, that system is known as "Assignment Proocedure."

Assignment Proocedure will be applied When
(1) Where No. of Worker = No. of Jobs.
(2) Where We have 1:1 "(One is to one) relation."
One is to one relation mean if a ~~worker~~ particular Job is assigned to a particular Worker, then that particular Worker would never do another Work. And that particular Job need not to be assigned to any other Worker.

[eg.] Assignment Proocedure

$$\begin{array}{c|ccc} & Job_1 & Job_2 & Job_3 \\ \hline W_1 & \boxed{4\text{ Hour.}} & 5\text{ Hours} & 7\text{ Hours} \\ W_2 & 10\text{ Hours} & \boxed{12\text{ Hours}} & 4\text{ Hours.} \\ W_3. & 5\text{ Hours} & 6\text{ Hours} & \boxed{2\text{ Hours.}} \end{array}$$

Step-No. 1

Row operation

$$\begin{array}{c|ccc} & Job_1 & Job_2 & Job_3. \\ \hline Worker_1 & 0 & 1 & 3 \\ Worker 2 & 6 & 8 & 0 \\ Worker_3. & 3 & 4 & 0 \end{array}$$

classmate

Step. No. 2

Coloumn operation

$$\begin{array}{c|ccc} & \underline{Job_1} & \underline{Job_2} & \underline{Job_3} \\ \hline \underline{Worker_1} & 0 & 0 & 3 \\ \underline{Worker_2} & 6 & 7 & 0 \\ \underline{Worker_3} & 3 & 3 & 0 \end{array}$$

## Assignment Proceedure Step.

**Step-1** Row Operation.

Identify the Minimum element from 1st Row & subtract it from all the element of that Row, repeat the proceedure for all rows.

**Step-2** Coloumn Operation

Identify the minimum element from 1st coloumn (Reduced Matrix) & subtract it from all the element of that coloumn repeat the proceedure for all coloumn.

**Step-3** Line Operation

Draw minimum No. of lines to cover all the zero, lines may be Horizontal or Vertical.

If No. of Minimum lines becomes equal to No. of order. of matrix, We can apply final assignment proceedure Step i.e Step No. 4. However if No. of lines fall short of No of order, we should improve the solution till No. of lines becomes equal to No. of order.

How to Draw the line :- Identify the row or coloumn having maximum No. of zero (0), Draw the line in that row / coloumn (firstly). If we have equal No. of Zero (Max) in any Row/coloumn then select the Row/coloumn so that (a) all zero (0) will be covered and

(b) Lines must be at Minimum.

~~Step. 4~~ How to do Improvement

Identify minimum element from Uncovered elements & subtract it form all uncovered element and Add it on Intersection.

## Step-4 Final Assignment Step

Select the Row/Coloumn having exactly 1 (one) Zero (0), put an assignment $\boxed{\phantom{0}}$ in that cell (If any Row/Column has more than one zero, leave it). Starting from Row₁, Row₂, Row₃ ----- & $c_1, c_2, c_3$.

$$\begin{array}{cccc} & J_1 & J_2 & J_3 \\ W_1 & 0 & 0 & \boxed{0} \\ W_2 & \boxed{0} & 2 & 5 \\ W_3 & 5 & \boxed{0} & 9 \end{array}$$

$Row_1$ = Two zero left

$Row_2$ : Single Zero, Assigned

$Row_3$ : Single Zero Assigned.

$W_1$ $J_3$

$W_2$ $J_1$

$W_3$ $J_2$

DATE

eg.2

(eg)

$$\begin{array}{c|ccc} & Job_1 & Job_2 & Job_3 \\ \hline Worker_1 & 6\ 2 & 8\ 6 & 9\ 8 \\ Worker_2 & 12\ 1 & 5\ 5 & 3\ 7 \\ Worker_3 & 9\ 4 & 12\ 2 & 10\cdot 1 \end{array}$$

Sol **Step-1** Row Operation

$$\begin{array}{c|ccc} & Job_1 & Job_2 & Job_3 \\ \hline Worker_1 & 0\ 0 & 2\ 4 & 3\ 6 \\ Worker_2 & 9\ 0 & 2\ 4 & 0\ 6 \\ Worker_3 & 0\ 3. & 3\ 1 & 1.0 \end{array}$$

**Step.2** Coloumn operation

$$\begin{array}{c|ccc} & Job_1 & Job_2 & Job_3 \\ \hline Worker_1 & 0\ 0 & 0\ 3 & 3\ 6 \\ Worker_2 & 9\ 0 & 0\ 3 & 0\ 6 \\ Worker_3 & 0\ 3 & 1\ 0 & 1\ 0 \end{array}$$

**Step 3** Lines operation

$$\begin{array}{c|ccc} & Job_1 & Job_2 & Job_3 \\ \hline Worker_1 & 0\ 0 & 0\ 3 & 3\ 6 \\ Worker_2 & 9\ 0 & 0\ 3 & 0\ 6 \\ Worker_3 & 0\ 7 & 1\ 0 & 1\ 0 \end{array}$$

classmate

**eg.3**

$$\begin{array}{c|ccc} & J_1 & J_2 & J_3 \\ W_1 & 2 & 9 & 12 \\ W_2 & 3 & 15 & 20 \\ W_3 & 1 & 2 & 4. \end{array}$$

**sol** **Step1 Row Operation**

$$\begin{array}{c|ccc} & J_1 & J_2 & J_3 \\ W_1 & 0 & 7 & 10 \\ W_2 & 0 & 12 & 17 \\ W_3 & 0 & 2 & 3 \end{array}$$

**Step-2 Coloumn Operation**

$$\begin{array}{c|ccc} & J_1 & J_2 & J_3 \\ W_1 & 0 & 6 & 7 \\ W_2 & 0 & 11 & 14 \\ W_3 & 0 & 0 & 0 \end{array}$$

**Step.3 Line operation**

$$\begin{array}{c|ccc} & J_1 & J_2 & J_3. \\ W_1 & 0 & 6 & 7 \\ W_2 & 0 & 11 & 14 \\ W_3 & 0 & 0 & 0 \end{array}$$

**Improvement$_1$**

$$\begin{array}{c|ccc} & J_1 & J_2 & J_3 \\ W_1 & 0 & 0 & 1 \\ W_2 & 0 & 5 & 8 \\ W_3 & 0 & 0 & 0 \end{array}$$

**Improvement$_2$**

$$\begin{array}{c|ccc} & J_1 & J_2 & J_3. \\ w_1 & 0 & 0 & 1 \\ w_2 & 0 & 5 & 8 \\ w_3 & 0 & 0 & 0 \end{array}$$

DATE

Now we Can apply <u>Step 4</u>

<u>Step 4</u> Final Assignment

$$\begin{array}{c|ccc} & J_1 & J_2 & J_3 \\ \hline W_1 & 0 & \boxed{0} & 1 \\ W_2 & \boxed{0} & 5 & 8 \\ W_3 & 0 & 0 & \boxed{0} \end{array}$$

Statement of Optimum Time.

| | | |
|---|---|---|
| $W_1$ | $J_2$ | 9 |
| $W_2$ | $J_1$ | 3 |
| $W_3$ | $J_3$ | 4 |
| | | 16 Hrs |

(3) Page-9

$$\begin{array}{c|ccc} & A & b & c. \\ 1 & 120 & 100 & 80 \\ 2 & 80 & 90 & 110 \\ 3 & 110 & 140 & 120. \end{array}$$

**Step-1** Row operation

$$\begin{array}{c|ccc} & A & b & c \\ 1 & 40 & 20 & 0 \\ 2 & 0 & 10 & 30 \\ 3 & 80 & 30 & 40. \end{array}$$

**Step2** Coloum Operation

$$\begin{array}{c|ccc} & A & b & c \\ 1 & 40 & 10 & 0 \\ 2 & 0 & 0 & 30 \\ 3 & 0 & 20 & 10. \end{array}$$

**Step-3** Line Operation

$$\begin{array}{c|ccc} & A & b & c \\ 1 & 40 & 10 & 0 \\ 2 & 0 & 0 & 30 \\ 3 & 0 & 20 & 10. \end{array}$$

Line = No. of Order. So we can apply Final Assignment Step-4.

**Step-4** Final Assignment step

$$\begin{array}{c|ccc} & A & b & c \\ 1 & \cancel{40} & \cancel{10} & \boxed{0} \\ 2 & 0 & \boxed{0} & 30 \\ 3 & \boxed{0} & \cancel{20} & \cancel{10}. \end{array}$$

Statement of optimum Time.

| | | |
|---|---|---|
| 1 | c | 80 |
| 2 | b | 90 |
| 3 | A | 110 |
| | | 280 |

## ㊡ Adjustment No. 1 Prohibited Route / Restricted.

$$\begin{array}{cccc} & J_1 & J_2 & J_3 \\ A & 4 & 2 & 1 \\ B & 5 & 8 & \infty/- \\ C & 6 & 7 & 9 \end{array}$$

$\infty/-$ means that cell will take Very higher Value of Time i.e. Could not be applied. It means we should ignore that cell during the assignment Proceedure i.e in all step (~~1 to 4~~) (1 to 4)

## Adjustment No. 2 Unbalanced Matrix.

If we have Unbalanced Matrix (Row ≠ coloumn), before applying assignment proceedure, we should balance such Matrix by introducing Dummy row OR Dummy coloumn. (Dummy Row/coloumn having zero element)

eg

$$\begin{array}{ccccc} & J_1 & J_2 & J_3 & J_4 \\ W_1 & 4 & 5 & 2 & 0 \\ W_2 & 1 & 6 & 7 & 0 \\ W_3 & 5 & 3 & 9 & 0 \\ W_4 & 10 & 12 & 15 & \boxed{0} \end{array}$$

Dummy coloumn

If No. of Row is > No. of Coloumn, Introduce Dummy coloumn

If No of Coloumn > No of Row, Introduce Dummy Row.

## Adjustment No. 3 Profit / Revenue / Production Maximization.

If we have any Maximization Matrix (Revenue / Profit / Production) We should convert such Maximization Matrix to Minimum Matrix by subtracting all the element of Matrix from the ~~highest element of Matrix~~

eg

Profit

$$\begin{array}{cccc} & A & B & C \\ S_1 & 100 & 90 & 80 \\ S_2 & 70 & 60 & 50 \\ S_3 & 40 & 30 & 20 \end{array}$$

If we had applied assignment Proceedure in this Matrix, optimum would have been as. 20, 30, 40, 50, 60, 70, 80, 90, 100

Ist last.

But we want

100 90 80 70 60 50 40 30 20 10

Ist 2nd 3rd 4th last

If we subtract all element from highest, then

0, 10, 20, 30, 40, 50, 60, 70, 80.

I II III last.

**Minimization Matrix**

|  |  |  |
|---|---|---|
| 0 | 10 | 20 |
| 30 | 40 | 50 |
| 60 | 70 | 80. |

**Adjustment No. 4** If we have Unbalanced & Maximization problem occurs simuntaneously, First we Convert unbalanced to Balanced & then Max to Minimum.

**Adjustment No.5** **Arbitrary** Suppose in final step.

|  | J1 | J2 | J3 |
|---|---|---|---|
| W1 | 4 | 5 | 6 |
| W2 | 7 | 8 | 9 |
| W3 | 4 | 5 | 19. |

|  | J1 | J2 | J3 |
|---|---|---|---|
| W1 | 2 | 3 | $\boxed{0}$ |
| W2 | 0 | 0 | 5 |
| W3 | 0 | 0 | 18. |

~~Step~~

If in step No. 4. (final Assignment step) No Coloumn/Row having single zero exist, apply arbitrary.

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$$\begin{array}{c|cccc} & R_1 & R_2 & R_3 & R_4 \\ C_1 & 4000 & 5000 & - & - \\ C_2 & - & 4000 & - & 4000 \\ C_3 & 3000 & - & 2000 & - \\ C_4 & - & - & 4000 & 5000. \end{array}$$

Step 1 Row operation

$$\begin{array}{c|cccc} & R_1 & R_2 & R_3 & R_4 \\ C_1 & 0 & 1000 & - & - \\ C_2 & - & 0 & - & 0 \\ C_3 & 1000 & - & 0 & - \\ C_4 & - & - & 0 & 1000 \end{array}$$

Step-2 Coloum Operation

$$\begin{array}{c|cccc} & R_1 & R_2 & R_3 & R_4 \\ C_1 & 0 & 1000 & - & - \\ C_2 & - & 0 & - & 0 \\ C_3 & 1000 & - & 0 & - \\ C_4 & - & - & 0 & 1000. \end{array}$$

Step-3 Line operation

$$\begin{array}{c|cccc} & R_1 & R_2 & R_3 & R_4 \\ C_1 & 0 & 1000 & - & - \\ C_2 & - & 0 & - & 0 \ \checkmark \\ C_3 & 1000 & - & 0 & - \\ C_4 & - & - & 0 & 1000 \end{array}$$

Lines ≠ order it needs improvement.

$$\begin{array}{c|cccc} & R_1 & R_2 & R_3 & R_4. \\ C_1 & 0 & 0 & - & - \\ C_2 & - & 0 & - & 0 \\ C_3 & 1000 & - & 0 & - \\ C_4 & - & - & 0 & 0 \end{array}$$

**Step-4** Final Assignment Step.

$$\begin{array}{c|cccc} & R_1 & R_2 & R_3 & R_4 \\ \hline C_1 & \boxed{0} & 0 & & \\ C_2 & & \boxed{0} & & 0 \\ C_3 & 4000 & & \boxed{0} & \\ C_4 & & & 0 & \boxed{0} \end{array}$$

Statement of Optimum Mix.

| | | |
|---|---|---|
| $C_1$ | $R_1$ | 4000 |
| $C_2$ | $R_2$ | 4000 |
| $C_3$ | $R_3$ | 2000 |
| $C_4$ | $R_4$. | 5000 |
| | | 15000 |

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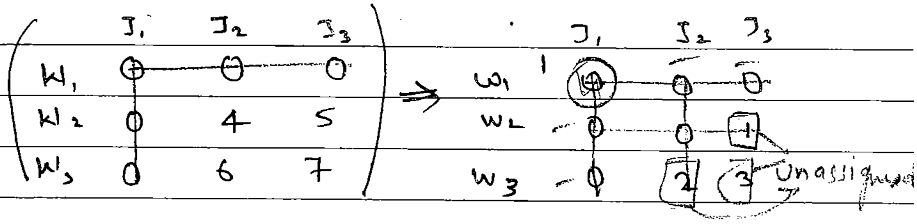
[Pag

17th class

DATE 06 01 2017

If we assigned at intersection, then Multiple option for Job/ Worker will not be utilized which results one worker or one Job may results unassigned.

for eg:-



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Working Notes.

| | Profit |
|---|---|
| Operator 'P' — Product A 1/6 x 420 = 70 Unit | 210 |
| Total Time = 420 mintes (7 Hr x 60 mint) | |
| 'P' — B = 1/10 x 420 = 42 Unit | 84 |
| 'P' — C = 1/14 x 420 = 30 Unit | 120 |
| — D = 1/12 x 420 = 35 Unit. | 35 |
| 'Q' — Product A = 1/7 x 420 = 60 Unit | 180 |
| B = 1/5 x 420 = 8 4 Unit | 168 |
| C = 1/3 x 420 = 140 Unit | 560 |
| D = 1/4 x 420 = 105 Unit | 105 |
| 'R' — Product - A = 1/6 x 420 = 70 Unit | 210 |
| B = 1/7 x 420 = 60 Unit | 120 |
| C = 1/10 x 420 = 42 Unit | 168 |
| D = 1/10 x 420 = 42 Unit | 42 |
| 'S' Product A = 1/20 x 420 = 21 Unit | 63 |
| B = 1/10 * 420 = 42 Unit | 84 |
| C = 1/15 x 420 = 28 Unit | 112 |
| D = 1/15 x 420 = 28 Unit. | 28 |

Product Matrix

$$\begin{array}{c|cccc} & A & B & C & D. \\ \hline P & 210 & 84 & 120 & 35 \\ Q & 180 & 168 & 560 & 105 \\ R & 210 & 120 & 168 & 42 \\ S. & 63 & 84 & 112 & 28. \end{array}$$

Minimization Matrix

$$\begin{array}{c|cccc} & A & B & C & D. \\ \hline P & 350 & 476 & 440 & 525 \\ Q & 380 & 392 & 0 & 455 \\ R & 350 & 440 & 392 & 578 \\ S & 497 & 476 & 448 & 532 \end{array}$$

Step-1 Row Operation

$$\begin{array}{c|cccc} & A & B & C & D. \\ \hline P & 0 & 126 & 90 & 175 \\ Q & 380 & 392 & 0 & 455 \\ R & 0 & 90 & 42 & 168 \\ S & 49 & 28 & 0 & 84 \end{array}$$

Step-2 Coloum Operation

$$\begin{array}{c|cccc} & A & B & C & D. \\ \hline P & 0 & 98 & 90 & 91 \\ Q & 380 & 364 & 0 & 371 \\ R & 0 & 62 & 42 & 84 \\ S & 49 & 0 & 0 & 0 \end{array}$$

Step-3 Line Operation

$$\begin{array}{cc|cccc} & & A & B & C\ \cancel{D.} & D \\ \hline & P & 0 & 98 & 90 & 91 \\ & Q & 380 & 364 & 0 & 371 \\ & R & 0 & 62 & 42 & 84 \\ 1 & S & 49 & 0 & 0 & 0 \\ & & 2 & & 3 & \end{array}$$

DATE

Here, No. of lines are not equal to No. of order. It needs to be improve.

**Improvement. 1**

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline P & 0 & 36 & 90 & 29 \\ Q & 380 & 302 & 0 & 309 \\ R & 0 & 0 & 42 & 22 \\ S & 111 & 0 & 62 & 0 \end{array}$$

Uncover में से Minimum को
Uncovered का बाकी No. से
Less करेंगे और लाइनों की
Intersection में Add कर देंगे

Now No. of line = No. of order.

$$\begin{array}{c|cccc} & A & B & C & D. \\ \hline P & \boxed{0} & 36 & 90 & 29 \\ Q & 380 & 302 & \boxed{0} & 309 \\ R & 0 & \boxed{0} & 42 & 22 \\ S & 111 & 0 & 62 & \boxed{0} \end{array}$$

**Statement of Profit**

| | | |
|---|---|---|
| P | A | 210 |
| Q | C | 560 |
| R | B | 120 |
| S | D | 28 |
| | Profit | 918 |

If we have Time Matrix & Profit/unit, then always first of all we should Construct Profit Matrix and then apply the system of Assignment Procedure to acheive Maximization over all profit.

If we had applied assignment procedure in Time Matrix., then although ~~It~~ assignment would have been for the cell having minimum time which result Maximum production but due to low profit ~~Profi~~ per unit, it may result overall low profit.

classmate

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## Revenue Matrix

$$\begin{array}{ccccccc} & & & A & b & c & D. \\ 14 & P & \left(\right. & 420000 & 336000 & 294000 & 462000 \\ 9 & Q & & 270000 & 216000 & 189000 & 297000 \\ 11 & R & & 330000 & 264000 & 231000 & 363000 \\ 8 & S. & & 240000 & 192000 & 168000 & 264000 \left.\right) \end{array}$$

## Minimization Matrix

$$\begin{array}{ccccc} & A & b & c & D \\ P & \cancel{252000} & \cancel{168000} & \cancel{126000} & \\ Q & \cancel{102000} & \cancel{48000} & \cancel{21000} & \\ R & \cancel{162000} & \cancel{96000} & \cancel{63000} & \\ S & \cancel{72000} & \cancel{24000} & \cancel{0} & \end{array}$$

## Minimization Matrix

$$\begin{array}{cccccc} & & A & b & c & D \\ P & \left(\right. & 42000 & 126000 & 163000 & 0 \\ Q & & 192000 & 246000 & 273000 & 165000 \\ R & & 132000 & 198000 & 231000 & 99000 \\ S & & 2,22000 & 270000 & 294000 & 198000 \left.\right) \end{array}$$

Highest में से
Minimum को less
कर दिये।

## Low operation

$$\begin{array}{cccccc} & & A & b & c & D. \\ P & \left(\right. & 42000 & 126000 & 168000 & 0 \\ Q & & 27000 & 81000 & 108000 & 0 \\ R & & 33000 & 99000 & 132000 & 0 \\ S & & 24000 & 72000 & 96000 & 0. \left.\right) \end{array}$$

## Coloumn operation & line operation

$$\begin{array}{cccccc} & & A & b & c & D. \\ P & \left(\right. & 18000 & 54000 & 72000 & 0 \\ Q & & 3000 & 9000 & 12000 & 0 \\ R & & 9000 & 27000 & 36000 & 0 \\ S & & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \left.\right) \end{array}$$

Improvement 1

$$\begin{array}{c|cccc} & A & B & C & D. \\ \hline P & 15000 & 51000 & 69000 & 0 \\ Q & 0 & 6000 & 9000 & 0 \\ R & 6000 & 24000 & 33000 & 0 \\ S & 0 & 0 & 0 & 3000. \end{array}$$

Improvement No. 2

$$\begin{array}{c|cccc} & A & B & C & D. \\ \hline P & 9000 & 45000 & 63000 & 0 \\ Q & 0 & 6000 & 9000 & 6000 \\ R & 0 & 18000 & 27000 & 0 \\ S & 0 & 0 & 0 & 9000 \end{array}$$

Improvement No. 3

$$\begin{array}{c|cccc} & A & B & C & D. \\ \hline P & 9000 & 39000 & 57000 & 0 \\ Q & 0 & 0 & 3000 & 6000 \\ R & 0 & 12000 & 21000 & 0 \\ S. & 6000 & 0 & 0 & 15000 \end{array}$$

Since No. of line = No. of Order

Final Assignment step

$$\begin{array}{cc|cccc|c} & & A & B & C & D. & \\ & & 2 & 2 & & & \\ \hline P & 1 & 9000 & 39000 & 57000 & \boxed{0} & 1 \\ Q & 3 & 0 & \boxed{0} & 3000 & 6000 & 3 \\ R & 2 & \boxed{0} & 12000 & 21000 & 0 & 2 \\ S & & 6000 & 0 & \boxed{0} & 15000 & \\ \hline & & 2 & 3 & & 1 & \end{array}$$

Statement of Profit

| | | |
|---|---|---|
| P | D | 462000 |
| Q | B | 216000 |
| R | A | 330000 |
| S | C | 168000 |
| ~~Profit~~ | | 11,76,000 ⇒ Maximum Sales |

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$$\begin{array}{cccc} & W & x & y \\ K & 10 & 10 & 30 \\ L & 30 & 10 & 20 \\ M & 20 & 30 & 20. \end{array}$$

Row operation

$$\begin{array}{cccc} & W & x & y \\ K & 0 & 0 & 20 \\ L & 20 & 0 & 10 \\ M & 0 & 10 & 0. \end{array}$$

Coloum operation

$$\begin{array}{cccc} & W & x & y \\ K & 0 & 0 & 20 \\ L & 20 & 0 & 10 \\ M & 0 & 10 & 0 \end{array}$$

Lines operation

$$\begin{array}{cccc} & w & x & y \\ k & 0 & 0 & 20 \\ L & 20 & 0 & 10 \\ M & 0 & 10 & 0 \end{array}$$

∴ Line = order

Final Assignment

$$\begin{array}{ccccc} & w & x & y. & \\ K & \boxed{0} & 0 & 20 & 2 \\ L & 20 & \boxed{0} & 10 & 1 \\ M & 0 & 10 & \boxed{0} & \\ & 2 & 1 & & \end{array}$$

(i) & (ii) Statement of optimum Cost

| Nurse | Patient | Cost/Hr | 40 Hrs (For) |
|---|---|---|---|
| K | W | 10 | 400 |
| L | x | 10 | 400 |
| M | Y | 20 | 800 |

₹ 1600 Ans

(ii) Revised Materix

$$\begin{array}{c|cccc} & W & x & Y & Z \\ \hline K & 10 & 10 & 30 & 40 \\ L & 30 & 10 & 20 & 40 \\ M & 20 & 30 & 20 & 40 \\ N & 50 & 50 & 50 & 50 \end{array}$$

low operation

$$\begin{array}{c|cccc} & W & x & y & z \\ \hline K & 0 & 0 & 20 & 30 \\ L & 20 & 0 & 10 & 30 \\ M & 0 & 10 & 0 & 20 \\ N & 0 & 0 & 0 & 0 \end{array}$$

Coloumn operation

$$\begin{array}{c|cccc} & W & x & y & z \\ \hline K & 0 & 0 & 20 & 30 \\ L & 20 & 0 & 10 & 30 \\ M & 0 & 10 & 0 & 20 \\ N & 0 & 0 & 0 & 0 \end{array}$$

(iii) Statement of Optimum Cost

| Nurse | Patient | Cost/Hr | 40 Hrs (for) |
|---|---|---|---|
| K | W | 10 | 400 |
| L | x | 10 | 400 |
| M | Y | 20 | 800 |
| N | Z | 50 | 2000 |
| | Cost ⇒ | | 3600 |

Line operation

$$\begin{array}{c|cccc} & W & x & Y & z \\ \hline K & 0 & 0 & 20 & 30 \\ L & 20 & 0 & 10 & 30 \\ M & 0 & 10 & 0 & 20 \\ N & 0 & 0 & 0 & 0 \end{array}$$

Line = order.

(iii) Comment

As new Row, as well as Coloumn have maximum Cost as compare to existing It means it will have no effect on existing Assignment i.e we can directly assign to cell $C_4 \times R_4$. instead of analyzing / applying whole procedure again.

Final ~~Statem~~ Assignment.

$$\begin{array}{c|cccc|c} & W & x & Y & Z & \\ \hline K & \boxed{0} & 0 & 20 & 30 & 2 \\ L & 20 & \boxed{0} & 10 & 30 & 1 \\ M & 0 & 10 & \boxed{0} & 20 & 3 \\ N & 0 & 0 & 0 & \boxed{0} & \\ \hline & 1 & 1 & 3 & 4 & \end{array}$$

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$$\begin{array}{c|ccccc} & I & II & III & IV & V \\ T_1 & 4 & 2 & 5 & 0 & 0 \\ T_2 & 6 & 3 & 3 & 0 & 3 \\ T_3 & 4 & 0 & 0 & 0 & 1 \\ T_4 & 0 & 0 & 5 & 3 & 0 \\ T_5 & 0 & 0 & 3 & 3 & 0 \end{array}$$

On analyzing the above situation we can easily obcerve that '3' element has been added at the Intersection Point of $C_{5,3}$ & $C_{54}$, because before adding such element was Zero (0) as Dummy Row has been introduced Just before drawing line operation.

Matrix before given Results

$$\begin{array}{c|ccccc} & I & II & III & IV & V \\ T_1 & 7 & 5 & 5 & 0 & 3 \\ T_2 & 9 & 6 & 3 & 0 & 6 \\ T_3 & 7 & 3 & 0 & 0 & 4 \\ T_4 & 0 & 0 & 2 & 0 & 0 \\ T_5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Matrix before line operation & before Dummy.

$$\begin{array}{c|ccccc} & I & II & III & IV & V \\ T_1 & 7 & 5 & 5 & 0 & 3 \\ T_2 & 9 & 6 & 3 & 0 & 6 \\ T_3 & 7 & 3 & 0 & 0 & 4 \\ T_4 & 0 & 0 & 2 & 0 & 0 \end{array}$$

(ii) Task'2' will be the most difficult Task for operation No. 'I', 'II' & 'V' due to highes cunsumption of time as Compared to others

(iii) Operator No. 4 was the most efficient operator due to less Consumption of time as compared to other

(iv) We should solve further the given Matrix

$$\begin{array}{c|ccccc|c} & I & II & III & IV & V & \\ T_1 & 4 & 2 & 5 & 0 & 0 & \\ T_2 & 6 & 3 & 3 & 0 & 3 & \\ T_3 & 4 & 0 & 0 & 0 & 1 & \checkmark \\ T_4 & 0 & 0 & 5 & 3 & 0 & \checkmark \\ T_5 & 0 & 0 & 3 & 3 & 0 & \checkmark \\ & & \checkmark & & \checkmark & \checkmark & \end{array}$$

Line= Order

Final Assignment

$$\begin{array}{cccccc|c} & I & II & III & IV & V & \\ T_1 & 4 & 2 & 5 & 0 & \boxed{0} & \\ T_2 & 6 & 3 & 3 & \boxed{0} & 3 & 1 \\ T_3 & 4 & 0 & \boxed{0} & 0 & 1 & 3 \\ T_4 & 0 & 0 & 5 & 3 & 0 & \\ T_5 & 0 & 0 & 3 & 3 & 0 & \\ \hline & & & 3 & 1 & 2 & \end{array}$$

Either Operator 'I' or 'II' remain Idle due to an arbitrary solution.

(v) Dummy Row/Coloum should be introduced at the begining i.e before Row & coloumn operation Hence he Cannot go with his Assignment for Correct optimum assignment.

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(i)

$$\begin{array}{c|cccc} & R_1 & R_2 & R_3 & R_4. \\ \hline C_1 & 9 & 14 & 19 & 15 \\ C_2 & \boxed{7} & 17 & 20 & 18 \\ C_3 & 9 & 18 & 21 & 16 \\ C_4 & 10 & \boxed{12} & 18 & 19 \\ C_5 & 10 & 15 & 21 & 16. \end{array}$$

Balance Matrix

$$\begin{array}{ccc} & R_3 & R_4 \\ C_1 & 19 & 15 \\ C_3 & 21 & 16 \\ C_5 & 21 & 16. \end{array}$$

This is unbalanced matix. so it will be converted in balance Matrix.

$$\begin{array}{c|ccc} & R_3 & R_4 & R_5(\text{Dummy}) \\ \hline C_1 & 19 & 15 & 0 \\ C_3 & 21 & 16 & 0 \\ C_5 & 21 & 16 & 0 \end{array}$$

Row operation

$$\begin{array}{c|ccc} & R_3 & R_4 & R_5 \\ \hline C_1 & 19 & 15 & 0 \\ C_3 & 21 & 16 & 0 \\ C_5 & 21 & 16 & 0. \end{array}$$

Coloumn operation

$$\begin{array}{c|ccc} & R_3 & R_4 & R_5 \\ \hline C_1 & 0 & 0 & 0 \\ C_3 & 2 & 1 & 0 \\ C_5 & 2 & 1 & 0 \end{array}$$

Line operation

$$\begin{array}{c|ccc} & R_3 & R_4 & R_5 \\ \hline C_1 & 0 & 0 & 0 \\ C_3 & 2 & 1 & 0 \\ C_5 & 2 & 1 & 0 \end{array}$$

Improvement

$$\begin{array}{c|ccc} & R_3 & R_4 & R_5 \\ \hline C_1 & 0 & 0 & 0 \\ C_3 & 1 & 0 & 0 \\ C_5 & 1 & 0 & 0 \end{array}$$

Line = order.

Final Assignment

$$\begin{array}{c|ccc} & R_3 & R_4 & R_5 \\ \hline C_1 & \boxed{0} & 0 & 1 \\ C_3 & & \boxed{0} & 0 \\ C_5 & & 0 & \boxed{0} \end{array}$$

Arbitrary

## Statement of Optimum Cost

|  |  | Cost (₹) lakh |  |
|---|---|---|---|
| $C_1$ | $R_3$ | 19 | lakh |
| $C_2$ | $R_1$ | 7 | " |
| $C_3$ | $R_4$ | 16 | " |
| $C_4$ | $R_2$ | 12 | " |
| $C_5$ | Dummy. | – |  |
|  |  | 54 lakh |  |

(ii) If Contractor$_5$ ($C_5$) offer Discount more than ~~3 lakh to achieve Road~~.

- ⊙ 3 lakh to acheive Road 1
- ⊙ 3 lakh to achive Road 2
- ⊙ 2 lakh to achive Road 3
- ⊙ Any Amt of ₹ to acheive Road 4.

(iii) Over all Discount (54-50) = 4 lakh.

$\%$ of Disount = $\frac{4}{54} \times 100 = 7.\dot{4}0\dot{7}\ \%$

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$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ R_1 & 0 & xx & xx \\ R_2 & xx & 0 & xx \\ R_3 & xx & xx & xx \end{array}$$

<u>Circumstances-1</u> = If the value exist in $R_3C_3$ becomes low as compared to other value exist in $R_3C_1$, $R_3C_2$.

<u>Circumstances-2</u> = if the value exist in $R_3C_3$ becomes low (after Row operation in $R_3$) as compared to other value exist in $C_3R_1$, $C_3R_2$.

(22) [Page-19] (i) After Row and Coloumn operation has Two (2) zero, then we can say ① First & 1st zero will occur due to Row operation
② 2nd zero will occur due to coloumn operation

But it's not necessary to say that It occur due to two (2) Corresponding value will ~~zero(s)~~ equal in original Matrix.

(ii) Matrix

$$\begin{array}{c|cccc} & \underline{C_1} & \underline{C_2} & \underline{C_3} & \underline{C_4} \\ R_1 & A_{11} & \cancel{A_{12}} & A_{13} & \cancel{A_{14}} \\ R_2 & \cancel{A_{21}} & \cancel{A_{22}} & \cancel{A_{23}} & \boxed{\cancel{A_{24}}} \\ R_3 & \cancel{A_{31}} & \boxed{\cancel{A_{32}}} & \cancel{A_{33}} & \cancel{A_{34}} \\ R_4 & A_{41} & \cancel{A_{42}} & A_{43} & \cancel{A_{44}} \end{array}$$

<u>Remaining Assignment in Final Assignment</u>

$$\begin{array}{c|cc} & \underline{C_1} & \underline{C_3} \\ R_1 & A_{11} & A_{13} \\ R_2 & & \\ R_3 & & \\ R_4 & A_{41} & A_{43} \end{array}$$

We can apply arbitrary assignment to the balance matrix i.e,
(#) if we assign $A_{11}$, then next assignment will be applicable for $A_{43}$ and if we
(#) if we assign $A_{13}$, then next assignment will be applicable for $A_{41}$.

(8) [Page-8]

ORiginal Matrix

$$\begin{array}{c|ccccc} & P & Q & R & S & T \\ \hline P & - & 5 & 14 & 20 & 2 \\ Q & 17 & - & 8 & 23 & 5 \\ R & 23 & 20 & - & 11 & 20 \\ S & 35 & 11 & 17 & - & 14 \\ T & 2 & 8 & 5 & 23 & - \end{array}$$

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Row operation

$$\begin{array}{c|ccccc} & P & Q & R & S & T \\ \hline P & - & 3 & 12 & 18 & 0 \\ Q & 12 & - & 3 & 18 & 0 \\ R & 12 & 9 & - & 0 & 9 \\ S & 24 & 0 & 6 & - & 3 \\ T & 0 & 6 & 3 & 21 & \cancel{0} \end{array}$$

Coloumn operation

$$\begin{array}{c|ccccc} & P & Q & R & S & T \\ \hline P & - & 3 & 9 & 18 & 0 \\ Q & 12 & - & 0 & 18 & 0 \\ R & 12 & 9 & - & 0 & 9 \\ S & 24 & 0 & 3 & - & 3 \\ T & 0 & 6 & 0 & 21 & \cancel{0} \end{array}$$

Line operation

$$\begin{array}{c|ccccc} & P & Q & R & S & T \\ \hline P & - & 3 & 9 & 18 & 0 \\ Q & 12 & - & 0 & 18 & 0 \\ R & 12 & 9 & - & 0 & 9 \\ S & 24 & 0 & 3 & - & 3 \\ T & 0 & 6 & 0 & 21 & - \end{array}$$

Line = order.

Final Assignment

$$\begin{array}{c|ccccc|c} & P & Q & R & S & T & \\ \hline P & - & 3 & 9 & 18 & \boxed{0} & 1 \\ Q & 12 & - & \boxed{0} & 18 & 0 & 2 \\ R & 12 & 9 & - & \boxed{0} & 9 & 3 \\ S & 24 & \boxed{0} & 3 & - & 3 & 4 \\ T & \boxed{0} & 6 & 0 & 21 & - & 5 \\ \hline & 5 & 4 & 3 & 2 & 1 & \end{array}$$

Final Assignment

| Start city | Destination city | Amt |
|---|---|---|
| P | T | 2000 |
| Q | R | 8000 |
| R | S | 11000 |
| S | Q | 11000 |
| T | P | 2000 |
| | | 34000 |

as above solution do not satify the given Condition hence we should analyse other option (next best given) which satisfy the Condition i.e.

Combination I

Start from 'P' & reaches to the next best city to 'T' (i.e. Q.)

| | | |
|---|---|---|
| P —— Q | | 5000 |
| Q —— R | | 8000 |
| R —— S | | 11000 |
| & S —— To next best of Q is 'T' | | 14000 |
| T —— P. | | 2000 |
| | | 40000 |

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Combination - II

| | |
|---|---|
| Q — R | 8000 |
| R — S | 11000 |
| S — T (next best) | 14000 |
| T — P (next best) | 2000 |
| P — Q. | 5000 |
| | 40,000 |

Ist & IInd Combination will provide the same cost & both satisfy our condition. hence we can select any one.

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(i) Original Matrix

$$\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline R_1 & 0 & x & x & x \\ R_2 & 0 & x & x & x \\ R_3 & 0 & 0 & 0 & x \\ R_4 & x & x & 0 & 0 \end{array}$$

which is Wrong.

← Horizontal line

He had drawn 4 lines in Incorrect Manner if he apply next final step then, One Job/worker will remain unassigned in order to have all proper assignment he should draw three (3) lines (Minimum) to cover all zero which is correct Manner and then apply improvement procedure till No. of line = No of order.

(ii) If $R_2C_2$ is lesser element in $R_1$ & $R_2$ non zero Values, we should subtract $R_2C_2$ from all element exist in $R_1$ & $R_2$ (non zero values) and Add on Intersection of $R_3C_1$, $R_4C_1$ & then drew the line (Minimum) to cover all zero, such Procedure should be continued to apply till No. of lines becomes equal to No. of order.

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## Preferences Ranking

$$\begin{array}{c|ccccc} M & M_1 & M_2 & M_3 & M_4 & M_5 \\ \hline 301 & - & 4 & 2 & - & 1 \\ 302 & 1 & 1 & 5 & 1 & 2 \\ 303 & 2 & - & 1 & 4 & - \\ 304 & 3 & 2 & 3 & 3 & 3 \\ 305 & - & 3 & 4 & 2 & - \end{array}$$

## Row operation

$$\begin{array}{c|ccccc} & M_1 & M_2 & M_3 & M_4 & M_5 \\ \hline 301 & - & 3 & 1 & - & 0 \\ 302 & 0 & 0 & 4 & 0 & 1 \\ 303 & 1 & - & 0 & 3 & - \\ 304 & 1 & 0 & 1 & 1 & 1 \\ 305 & - & 1 & 2 & 0 & - \end{array}$$

## Coloumn operation

$$\begin{array}{c|ccccc} & M_1 & M_2 & M_3 & M_4 & M_5 \\ \hline 301 & - & 3 & 1 & - & 0 \\ 302 & 0 & 0 & 4 & 0 & 1 \\ 303 & 1 & - & 0 & 3 & - \\ 304 & 1 & 0 & 1 & 1 & 1 \\ 305 & - & 1 & 2 & 0 & - \end{array}$$

No. Line = order

## Final Assignment

$$\begin{array}{c|ccccc|c} & M_1 & M_2 & M_3 & M_4 & M_5 & \\ \hline 301 & 0 & 3 & 1 & - & \boxed{0} & 1 \\ 302 & \boxed{0} & 0 & 4 & 0 & 1 & 4 \\ 303 & 1 & - & \boxed{0} & 3 & - & 2 \\ 304 & 1 & \boxed{0} & 1 & 1 & 1 & 3 \\ 305 & - & 1 & 2 & \boxed{0} & - & \\ \hline & 4 & 3 & 2 & & 1 & \end{array}$$

Statement of Optimum Assignment

| | | Ranking |
|---|---|---|
| 301 | - M5 | 1 |
| 302 | - M1 | 1 |
| 303 | - M3 | 1 |
| 304 | - M2 | 2 |
| 305 | - M4. | 2 |
| | | 7 |

(10) page-12 Chennai based

Chennai to Mumbai — Mumbai to Chennai (Return)

A1 6 A.M — 8. A.M.
Rest 5 Hours
Report 1. P.M

| | b1 | Layover |
|---|---|---|
| 19 Hrs. | 8. A.M | 24 Hrs. |
| 20 Hrs | b2 9. AM | 25 Hrs |
| 1 Hr | b3 2. P.M | 6 Hrs. |
| 6 Hrs | b4 7 P.M | 11 Hrs. |

A2 8. A.M — 10. A.M
Rest - 5 Hours
3. P.M

| | Return | Layover |
|---|---|---|
| 17 Hrs | b1 8. A.M | 22 Hrs |
| 18 Hrs | b2 9. A.M | 23 Hrs |
| 23 Hrs. | b3 2. P.M | 28 Hrs. |
| 4 Hrs. | b4. 7. P.M. | 9 Hrs. |

A3 2 P.M - 4. P.M
Rest 5 Hours
9. P.M

| | Return | Layover |
|---|---|---|
| 11 Hrs | b1 8. A.M | 16 Hrs |
| 12 Hrs | b2 9. A.M | 17 Hrs. |
| 17 Hrs. | b3 2. P.M | 22 |
| 22 Hrs. | b4. 7. P.M | 27 Hrs. |

A4 8 P.M 10 P.M
Rest 5 Hours
3. A.M.

| | Return | Layover |
|---|---|---|
| 5 Hrs | b1 8. AM | 10 Hrs |
| 6 Hrs | > b2 9. A.M | 11 Hrs |
| 11 Hrs | b3 2. P.M | 16 Hrs |
| 16 Hrs | b4 7. P.M. | 21 Hrs. |

If crew is Chennai based #

$$\begin{array}{c|cccc} \text{Matrix} & b_1 & b_2 & b_3 & b_4. \\ A_1 & 24 & 25 & 6 & 11 \\ A_2 & 22 & 23 & 28 & 9 \\ A_3 & 16 & 17 & 22 & 27 \\ A_4 & 10 & 11 & 16 & 21. \end{array}$$

Mumbai based

$b_1$ (go) 8 A.M - 10 A.M Return Layoves

Rest 5 Hrs $A_1$ Layoves

3 P.M $\xrightarrow{15 Hr}$ 6 A.M 20 Hrs

$\xrightarrow{17 Hrs}$ $A_2$ - 8 AM 22 Hrs

$\xrightarrow{23 Hr}$ $A_3$ 2 P.M 28 Hrs

$\xrightarrow{5 Hrs}$ $A_4$ 8 PM 10 Hrs.

$b_2$ 9 A.M 10 A.M

Rest 5 Hrs Return Layoves

4 P.M $\xrightarrow{14 Hr}$ $A_1$ 6 A.M 19 Hrs.

$\xrightarrow{16 Hr}$ $A_2$ 8 AM 21 Hrs

$\xrightarrow{22 Hr}$ $A_3$ 2 P.M 27 Hrs

$\xrightarrow{4 Hr}$ $A_4$ 8 P.M 9 Hrs.

$b_3$ 2 P.M 4 P.M

Rest 5 Hrs Return Layoves

9 P.M $\xrightarrow{9 Hr}$ $A_1$ - 6 AM 14 Hrs

$\xrightarrow{11 Hrs}$ $A_2$ 8 AM 16 Hrs

$\xrightarrow{17 Hr}$ $A_3$ 2 P.M 22 Hrs

$\xrightarrow{23 Hrs}$ $A_4$ 8 P.M 28 Hrs.

$b_4$ 7 P.M 9 P.M

Rest

Layoves

9

11

17

23.

If crew is Mumbai based

$$\begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ A_1 & 20 & 19 & 14 & 9 \\ A_2 & 22 & 21 & 16 & 11 \\ A_3 & 28 & 27 & 22 & 27 \\ A_4 & 10 & 9 & 28 & 23. \end{array}$$

Minimum layover Matrix (crew based either at the city)

$$\begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ A_1 & 20 & 19 & 6^{\#} & 9^{*} \\ A_2 & 22 & 21 & 16^{*} & 9^{\#} \\ A_3 & 16^{\#} & 17^{\#} & 22 & 17 \\ A_4 & 10 & 9 & 16^{\#} & 21^{\#} \end{array}$$

Row operation

$$\begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ A_1 & 14 & 13 & 0 & 3 \\ A_2 & 13 & 12 & 7 & 0 \\ A_3 & 0 & 1 & 6 & 1 \\ A_4 & 1 & 0 & 7 & 12. \end{array}$$

Coloumn operation

$$\begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ A_1 & 14 & 13 & 0 & 3 \\ A_2 & 13 & 12 & 7 & 0 \\ A_3 & 0 & 1 & 6 & 1 \\ A_4 & 1 & 0 & 7 & 12 \end{array}$$

line Operation

$$\begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ A_1 & 14 & 13 & 0 & 3 \\ A_2 & 13 & 12 & 7 & 0 \\ A_3 & 0 & 1 & 6 & 1 \\ A_4 & 1 & 0 & 7 & 12 \end{array}$$

No. of line = No. of order.

$$\begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ \hline A_1 & 14 & 13 & \boxed{0} & 3 \\ A_2 & 13 & 12 & 7 & \boxed{0} \\ A_3 & \boxed{0} & 1 & 6 & 1 \\ A_4 & 1 & \boxed{0} & 7 & 12 \end{array}$$

Statement of optimum Assignment

| Pair | Base city | Layover. |
|---|---|---|
| $A_1 - b_3$. | Chennai | 6 Hours |
| $A_2 - b_4$ | Chennai | 9 Hours. |
| $A_3 - b_1$ | Chennai | 16 Hours. |
| ~~$A_4 - b_2$~~ | Mumbai | 9 Hours |
| $b_2 - A_4$ | Total. | 40 Hours |

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Working :

| Typist | Job | Hr | Cost (₹) |
|---|---|---|---|
| A | P. | $\frac{199}{12} = 16.57 \cong 17$ | 85. |
| A | Q | $\frac{175}{12} \cdot 14.58 \cong 15$ | 75 |
| A | R | $\frac{145}{12} = 18.125 \cong 19.08$ | 65 |
| A | S | $\frac{298}{12} = 24.83 \cong 25$ | 125 |
| A | T | $\frac{178}{12} = 14.83 \cong 15$ | 75. |
| # b | P | $\frac{199}{14} = 14.21 \cong 15$ | 90 |
| | Q | $\frac{175}{14} = 12.5 \cong 13$ | 78. |
| | R | $\frac{145}{14} = 10.35 \cong 11$ | 66 |
| | S | $\frac{298}{14} = 21.28 \cong 22$ | 132 |
| | T. | $\frac{178}{14} = 12.71 \cong 13$ | 78. |
| # c | P | $\frac{199}{8} = 24.875 \cong 25$ | 75 |
| | Q | $\frac{175}{8} = 21.875 \cong 22$ | 66. |
| | R | $\frac{145}{8} = 18.125 \cong 19$ | 57 |
| | S | $\frac{298}{8} = 37.25 \cong 37$ | 114 |
| | T | $\frac{178}{8} = 22.25 \cong 23$ | 69. |

D P $\frac{199}{10} = 19.9 \cong 20$ 80

Q $\frac{175}{10} = 17.5 \cong 18$ 72

R $\frac{14.5}{10} = 14.5 \cong 15$ 60

S $\frac{298}{10} = 29.8 \cong 30$ 120.

T. $\frac{178}{10} = 17.8 \cong 18$ 72

E P $\frac{199}{11} = 18.09 \cong 19.$ 76

Q $\frac{175}{11} = 15.90 \cong 16$ 64

R $\frac{145}{11} = 13.19 \cong 14$ 56

S $\frac{298}{11} = 27.09 \cong 28$ 112

T $\frac{178}{11} = 16.18 \cong 17$ 68

Cost Matrix

$$\begin{array}{c|ccccc} & P & Q & R & S & T \\ A & 85 & 75 & 65 & 125 & 75 \\ B & 90 & 78 & 66 & 132 & 78 \\ C & 75 & 66 & 57 & 114 & 69 \\ D & 80 & 72 & 60 & 120 & 72 \\ E & 76 & 64 & 56 & 112 & 68 \end{array}$$

Row operation

$$\begin{array}{c|ccccc} & P & Q & R & S & T \\ A & 20 & 10 & 0 & 60 & 10 \\ B & 24 & 12 & 0 & 66 & 12 \\ C & 18 & 9 & 0 & 57 & 12 \\ D & 20 & 12 & 0 & 60 & 12 \\ E & 20 & 8 & 0 & 56 & 12 \end{array}$$

Column Operation

$$\begin{array}{c|ccccc} & P & Q & R & S & T. \\ A & 2 & 2 & 0 & 4 & 0 \\ B & 6 & 4 & 0 & 10 & 2 \\ C & 0 & 1 & 0 & 1 & 2 \\ D & 2 & 4 & 0 & 4 & 2 \\ E & 2 & 0 & 0 & 0 & 2 \end{array}$$

## Line Operation

$$\begin{array}{c|ccccc} & P & Q & R & S & T. \\ A & 2 & 2 & 0 & 4 & 0 \\ B & 6 & 4 & 0 & 10 & 2 \\ C & 0 & 1 & 0 & 1 & 2 \\ D & 2 & 4 & 0 & 4 & 2 \\ E & 2 & 0 & 0 & 0 & 2 \end{array}$$

No. of line ≠ No. of order. , need to be improved.

$$\begin{array}{c|ccccc} & P, & Q & R & S & T. \\ A & 3 & 2 & 1 & 4 & 0 \\ B & 6 & 3 & 0 & 9 & 1 \\ C & 0 & 0 & 0 & 0 & 1 \\ D & 2 & 3 & 0 & 3 & 1 \\ E & 3 & 0 & 1 & 0 & 2. \end{array}$$

line ≠ No. of order , need to , be improved.

## Improvement 2

$$\begin{array}{c|ccccc} & P. & Q & R & S & T. \\ A & 3 & 2 & 2 & 4 & 0 \\ B & 5 & 2 & 0 & 8 & 0 \\ C & 0 & 0 & 1 & 0 & 1 \\ D & 1 & 2 & 0 & 2 & 0 \\ E & 3 & 0 & 2 & 0 & 2 \end{array}$$

No. of line ≠ No of order

## Improvement 3

$$\begin{array}{c|ccccc} & P & Q & R & S & T. \\ A & 2 & 1 & 2 & 3 & 0 \\ B & 4 & 1 & 0 & 7 & 0 \\ C & 0 & 0 & 2 & 0 & 2 \\ D & 0 & 1 & 0 & 1 & 0 \\ E & 3 & 0 & 3 & 0 & 3 \end{array}$$

Final Assignment

$$\begin{array}{c|ccccc|c}
 & P & Q & R & S & T & \\
A & 2 & 1 & 2 & 3 & \boxed{0} & 1 \\
B & 4 & 1 & \boxed{0} & 7 & 0 & 2 \\
C & 0 & \boxed{0} & 2 & 0 & 2 & \\
D & \boxed{0} & 1 & 0 & 1 & 0 & \\
E & 3 & 0 & 3 & \boxed{0} & 3 & \\
 & 3 & & 2 & & 1 &
\end{array}$$

Statement of Cost

| | | ₹ |
|---|---|---|
| A | T | 75 |
| B | R | 66 |
| C | Q | 66 |
| D | P | 80 |
| E | S | 112 |

Least Cost ⇒ 399 Ans

(23) 'A' would be possible when we have either

Page. 20

(i) 3 Row & 5 coloumn

OR

(ii) 5 Row & 3 Coloumn.

'B' Could not be possible because difference of Requirement & supply should be Covered by Single Row / Coloum.

'C & D' 'C' Could not occurr in assignment Dummy Row / Coloumn should be introduced only when No. of Row < No. of coloumn i.e, Difference = 1. If One (1) Row, One (1) Coloumn short then its already balanced.

'D' Not possible in Transprotation because differente of Requirement & supply Compencated by either by Dummy Row / Coloum (Single).

# Chapter-2

## Transportation

19th class

DATE 11 01 2017

(#) Transportation approach should be applied where one is to one base does not exist.

(#) Transportation is an enlarge Method of assignment.

(#) Assignment is a short Method while Transportation is a long proceedure.

(eg)

| | Warehouse₁ | Warehouse₂ | Warehouse₃ | Capacity |
|---|---|---|---|---|
| Factory₁ | 2 | 4 | 3 | 100 |
| Factory₂ | 1 | 5 | 6 | 100 |
| Factory₃ | 3 | 2 | 5 | 100 |
| Required= | 100 | 100 | 100 | 300 |

Here assignment can be applied but transportation should not be hold good. i.e. if capacity & Demand of each Resources are equal, we should apply Assignment proceedure instead of Transportation. However if capacity of each factory & Demand of each Warehouse are not equal, we should apply transportation.

(#) Transportation Proceedure is Divided into Two (2) Parts.

$$\text{Basic solution} \longleftarrow \text{Transportation Proceedure} \longrightarrow \text{Test of Optimality}$$

1) Least Cost Method
2) North West Corner Rule
3) Vogel Approx. Method (VAM)

In the absence of Instruction always we should follow (VAM) for Basic Solution.

## (I) Least Coct Method. ⇒ Capacity of Requirement should be same.

eg

expenses of transport

|  | W₁ | W₂ | W₃. | Capacity |
|---|---|---|---|---|
| F₁ | 4 | 5 [200] | 9 | 200/0 |
| F₂ | 2 | 7 [70] | 8 [230] | 300/230/0 |
| F₃ | 1 [500] | 6 | 19. | 500/0 |
| Requirement | 500<br>0 | 2700<br>70<br>0 | 230<br>0 | 1000 |

Statement of Cost.

| | | | |
|---|---|---|---|
| F₁ | W2 | 200X5 = | 1000 |
| F₂ | W2 | 70x7 = | 490 |
| F₂ | W3 | 230x8 = | 1840 |
| F₃ | W₁ | 500x1 = | 500 |

Cost ⇒ 3830

(eg)

|  | W₁ | W2 | W3 | Capacity |
|---|---|---|---|---|
| F₁ | 6 [100] | 7 | 9 | 100 |
| F₂ | 8 [110] | 9 | 20 [90] | 200/90 |
| F₃ | 12 | 5 [290] | 6 [10] | 300/10/0 |
| Requirement | 210<br>110<br>0 | 290<br>0 | 100 | 600 |

(Q-1) Calculate Basic Cost from the following.

|  | W₁ | W₂ | W₃ | Capacity |
|---|---|---|---|---|
| f₁ | 9 | 8 [50] | 7 | 50/0 |
| f₂ | 6 [100] | 10 [50] | 11 | 150/50 |
| f₃ | 12 | 15 [10] | 2 [90] | 100/0 |
| Requirement | 100<br>0 | 110<br>60<br>10<br>0 | 90<br>0 | 300 |

Statement of Cost

| | | | |
|---|---|---|---|
| f₁ - W₂ | | 50x8 = | 400 |
| f₂ | W₁ | 100x6 = | 600 |
| f₂ | W₂ | 50x10 = | 500 |
| f₃ | W₂ | 10x15 = | 150 |
| f₃ | W₃ | 90x2 = | 180 |

1830 Answer

## (#) Steps for least Cost Method.

Step 1 : Identify the least Cost from the matrix & allocate the possible unit to that allocation. Possible unit here means least of Capacity, demand.

Step-2 : Exhaust either Row or/& coloumn & write Balance unit to allocate in next step.

Step-3 : Identify the Minimum element from balance Matrix & repeat the step 1 & step-2

Above proceedure should be followed till all capacity / requirement exhaust.

## [II] North West Corner Rule. [like Z]

| | W1 | W2 | W3 | Capacity |
|---|---|---|---|---|
| F1 | 14 100 | 12 | 10 — 1 | 100 |
| F2 | 5 150 | 6 50 | 8 — 3 | 200 / 50 |
| F3 | 7 | 3 100 | 2. 200 | 300 / 200 |
| Requirement | 2<br>250<br>150<br>0 | 4<br>150<br>100<br>0 | 200 | 600 |

### Statement of Cost

$F_1 - W_1 = 100 \times 14 = 1400$

$F_2 - W_2 = 50 \times 6 = 300$

$F_2 - W_1 \quad 150 \times 5 = 750$

$F_3 - W_2 \quad 100 \times 3 = 300$

$F_3 - W_3 \quad 200 \times 2 = 400$

Total Cost ⇒ 3150

(3)
Page-38

|  | $S_1$ | $S_2$ | $S_3$ | Capacity |
|---|---|---|---|---|
| $F_1$ | 8 ⌊50 | 7 ⌊10 | 3 | 60/0. |
| $F_2$ | 3 | 8 ⌊50 | 9 ⌊20 | 70/20 |
| $F_3$ | 11 | 3 | 5 ⌊60 | 60 |
| Required | 50<br>0 | 60<br>50<br>0 | 80<br>60<br>0 |  |

Statement of Cost

1350

|  |  |  |  |
|---|---|---|---|
| $F_1 - S_2$ | 10x7 | = | 70 |
| $F_2 - S_2$ | 50x8 | = | 400 |
| $F_2 - S_3$ | 20x9 | = | 180 |
| $F_3 - S_3$ | 60x5 | = | 300 |
| $F_1 - S_1$ | 50x8 |  | 400 |
|  | Total cost ⇒ |  | 1350 |

## (#) Steps for North West Corner Rule

Step 1 : First allocation should be made for $C_{11}$, if demand exhaust, Coloumn delete.
if capacity exhaust, Row delete.
Adjust the allocation.
Allocate only Possible unit.

Step 2 Next allocation should be applied to the cell $C_{12}$ (if possible) & $C_{13}$ & $C_{21}$ & $C_{22}$ & $C_{31}$, $C_{32}$ & $C_{33}$.....

## [III] Vogel Approx. Method.

(3) Page-38

|  | $S_1$ | $S_2$ | $S_3$. | Capacity | Penalty$_1$ | Penalty$_2$ |
|---|---|---|---|---|---|---|
| $F_1$ | 8 | 7 | $3^{\boxed{60}}$ | 60/0. | 4 | 4 |
| $F_2$ | $3^{\boxed{50}}$ | 8 | $9^{\boxed{20}}$ | ~~70~~ 70/20/0 | 5 | 1 |
| $F_3$ | 11 | $3^{\boxed{60}}$ | 5 | 60/0 | 2 | 2 |
| Required | 50/0 | 60 | 80/20. | 190 |  |  |
| Penalty$_1$ | 5↑ | 4 | 2 |  |  |  |
| Penalty$_2$ | x | 4↑ | 2 |  |  |  |

### Statement of Cost

$F_1 - S_3$ = 60×3 180

$F_2 - S_1$ = 50×3 150

$F_2 - S_3$ = 20×9 180

$F_3 - S_2$ = 60×3 180

Total Cost ⇒ 690

(Q-1) [Page-37]

376

|  | i | ii | iii | iv | Availability | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|---|---|---|---|
| A | $7^{\boxed{6}}$ | $3^{\boxed{25}}$ | $5^{\boxed{3}}$ | 5 ④ | 34/9/3/0 | 2 | 2 | 0 | 0 |
| B | $5^{\boxed{15}}$ | 5 | 7 | 6 ① | 15/0 | 0 | x | x | x |
| C | 8 | 6 | $6^{\boxed{14}}$ | $5^{\boxed{17}}$ | 31/14 | 1 | 1 | 1. | 1 |
| Demand | 21/6 ③ | 25/0 ② | 17/14 | 17 | 80 0 |  |  |  |  |
| $P_1$ | 2↑ | 2 | 1 | 0. |  |  |  |  |  |
| $P_2$ | 1 | 3↑ | 1 | 0 |  |  |  |  |  |
| $P_3$ | 1↑ | x | 1 | 0 |  |  |  |  |  |
| $P_4$ | x | x | 1↑ | 0 |  |  |  |  |  |

### Statement of Cost

[A - i] 7×6 = 42 [C - iii] 6×14 = 84

[A - ii] 3×25 = 75 [C - iv] 5×17 = 85

[A - iii] 5×3 = 15 Total Cost 376

[B - i] 5×15 = 75

# (#) Steps for Vogel Approx Method

**Step 1** ⇒ Calculate the Penalty for each Row & Coloum,
Penalty is the difference of first Minimum & IInd Minimum value for each Row.
[If Ist Minimum value & IInd Minimum value are same, then value of Penalty becomes Zero (0)]

**Step-2** Allocate Possible unit to the least Cost Cell of the Coloumn (or) Row having highest Penalty
Highest Penalty should be identify from all Penalties.
[If again, value of two highest Penalty are same, we can select any higher Penalty as Arbitrary]

**Step-3** Delete the Row if Capacity exhaust
Delete the Coloumn if Requirement exhaust.

**Step-4** Repeat the Process of calculating the Penalty & Allocation for the balance Matrix till all requirement & Capacity exhaust.

(9) [Page-41] Cost Matrix

|  | I | II | III | Capacity | $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|---|---|---|---|
| A | ~~14+6 =20~~ | ~~14+4 =18~~ | ~~14+4 =18~~ ① $\boxed{100}$ | 100/0 | 0 | X | X |
| B | 15+5 =21 | 16+6 $\boxed{35}$ =22 | 16+7= $\boxed{90}$ 23 | 125/90/0 | 1 | 1 | 1 |
| C | ~~15+3~~ $\boxed{60}$ =18 ② | 15+4 =19 $\boxed{115}$ | ~~15+6~~ ③ =22. | 175/115/0 | 1 | 1 | 2 |
| Required | 60/0 | 150/35/0 | 190/90 | 400 |  |  |  |
| $P_1$ | 2 | 1 | 3. ↑ |  |  |  |  |
| $P_2$ | 3 ↑ | 3 | 2 |  |  |  |  |
| $P_3$ | X | 3 ↑ | 2 |  |  |  |  |

Statement of Cost

| | | |
|---|---|---|
| A - III | 18 × 100 | = 1800 |
| B - II | 22 × 35 | = 770 |
| B - III | 23 × 90 | = 2070 |
| C - I | 18 × 60 | = 1080 |
| C - II | 19 × 115 | = 2185 |
| | Variable Cost | 7905 |

Total Cost = 7905 + (7000 + 4000 + 5000)

= 7905 + 16000

= 23,905 Answer

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## Adjustment No. 1 ⇒ Restricted Routes

(Not Minus)
Das

If any Cell is represented by $-/\infty$, it mean we should not allocate any Qty to that Cell i.e, ignore that Cell during Transportation Procedure.

## Adjustment No. 2 ⇒ Unbalanced Problem

If we have Capacity = Requirement, It is a balanced Problem. Other wise it is called Unbalanced Problem

First of all we should introduce a Dummy Row/coloumn to make it balance. All element of Dummy Row/Colowmn should be Zero.

'0' indicates cost of Transportation factory to factory

eg

$$\begin{array}{c|cccc|c} & W_1 & W_2 & W_3. & \overset{\underline{\text{Dummy}}}{W_4} & \text{Capacity} \\ \hline F_1 & 4 & 5 & 6 & 0 & 100 \\ F_2 & 2 & 1 & 8 & 0 & 200 \\ F_3 & 5 & 9 & 12 & 0. & 400 \\ \hline \underline{\text{Required}} & 300 & 100 & 200. & & 700. \end{array}$$

W4 located in factory.

## Adjustment No. 3 ⇒ Maximization Problem

Factor ⇒ Profit, Revenue, Production. Return (Maximized).
⇒ Cost. Wastage. Time, Loss (Minimized).

If we have Maximum (Maximization) Problem, first of all (before applying Transportation Procedure), We should Convert such Maximization Problem to the Minimized/Minimization Problem by subtracting all element of Matrix from highest element of Matrix.

(eg)

$$\begin{array}{c c} & \begin{array}{ccc} \underline{\underline{x}} & \underline{\underline{y}} & \underline{\underline{z}} \end{array} \\ \begin{array}{c} A \\ b \\ c \end{array} & \begin{pmatrix} 100 & 90 & 80 \\ 70 & 60 & 50 \\ 40 & 30 & 20 \end{pmatrix} \end{array}$$

Transportation will provide

20 , 30, 40 . . . . . . . . . 100

<u>we required</u> $\textcircled{100}$ 90 , 80 . . ——— —20.

$\downarrow$

Make it
very low. by Subtracting from 100.

$$\begin{array}{c c} & \begin{array}{ccc} \underline{\underline{x}} & \underline{\underline{y}} & \underline{\underline{z}} \end{array} \\ \begin{array}{c} A \\ b \\ c \end{array} & \begin{pmatrix} 0 & 10 & 20 \\ 30 & 40 & 50 \\ 60 & 70 & 80 \end{pmatrix} \end{array} \Rightarrow \underline{\text{Minimization Matrix}}$$

## Adjustment No. 4 ⇒

If we have Unbalanced Problem & Maximization Problem simuntaneously in a particular situation, First of all. we should Convert unbalanced Problem to balanced Problem & then Convert Maximization Problem to Minimization Problem, and then apply Transportation Procedure.

## [A] Test of Optimality

### [#] Steps for Test of Optimality

it is also called improvement procedure

**Step-1** = If No. of Allocation are equal to $M+n-1$, we can apply next step otherwise its called a case of Degenaracy. (Allocation $< m+n-1$)
If degeneracy occurs, we should remove degenaracy, then we can apply Next step.
$m$ = Row ; $n$ = Coloumn.

**Step-2** = Calculate the value of $u_i$ & $v_j$ for all occupied cell with help of relation.

$$C_{ij} = u_i + v_j \qquad [i = \text{Row} \mid j = \text{Coloumn}]$$

& Calculate $\Delta_{ij}$ for unallocated cell with the help of relation. [$\Delta$ means change in cost from present allocation to. New allocation]

$$\Delta_{ij} = C_{ij} - (u_i + v_j)$$

**Step-3** If all value of $\Delta_{ij}$ are Positive $(>0)$, then above solution is an optimum solution. (no need to improve) However if any value of $\Delta_{ij}$ become negative $(<0)$, above solution need to be improved with the Construction of loop.

If any value of $\Delta_{ij}$ becomes equal to '0' (zero) i.e. $\Delta_{ij} = 0$, It means alternate solution also exist without change in cost.

Steps for How to construction of [illegible]

## ㊖ How to Constouct the loop.

# Starting point of the loop should bea the Cell having highest value ~~of~~ in Negative of $\Delta ij$ (eg. $-10, -12$)

# loop must be Constoucted with Horizontal & Vertical line.

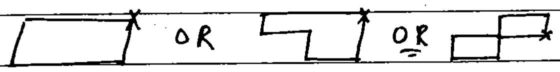
# Every Turning point of the loop must be an Allocated cell.

# Starting point of the loop must be an end point.

# Starting Point should be represented by $+\theta$ and
First Turning point should be Represented by $-\theta$
IInd Turning Point should be Represented by $+\theta$
& so on
i.e. No Two Consequtive $+\theta$ OR $-\theta$ would occur.

# Quantity to be shifted i.e. Minimum allocation of $\boxed{-\theta}$

# Loop may be in form of



after shifting the allocation apply again procedure of Test of optimality till all values of $\Delta ij$ becomes positive.

(eg) Contruction of loop.

Suppose we have IFBS &

| | $w_1$ | $w_2$ | $w_3$ |
|---|---|---|---|
| $f_1$ | 4 | $5^{20}$ | 1 |
| $f_2$ | 8 | $10^{30}$ | $12^{50}$ |
| $f_3$ | $15^{40}$ | $16^{50}$ | 17 |

$\Delta_{13}$ = Highest in Negative.

OR

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Test of Optimality (Improvement Procedure)

㊊ If we apply Test of optimality for the Initial basic feasible solution (IBFS) of Either least Cost Method.

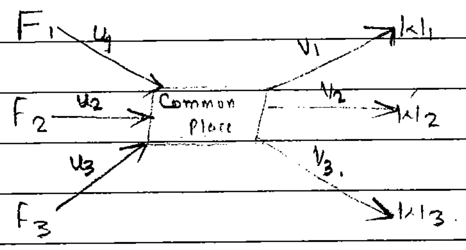
NWCR/VAM, always. optimum Answer will be same

(#) We should change the allocation so that overall Cost to be kept at Minimum level

(#) How to change the allocation ? → shift

$\begin{array}{cc} w_1 & w_2 \\ \times & \times \\ \times & \times \end{array}$ $C_{11}$ = Cost per unit from (Transportation) factory$_1$ to warehouse$_1$

Observation



$$\begin{array}{c|ccc|c} i & w_1 & w_2 & w_3 & \\ f_1 & 4^{100} & 5 & 6 & u_1 \\ f_2 & 7^{200} & 8^{200} & 9 & u_2 \\ f_3 & 10 & 11^{100} & 12^{\boxed{300}} & u_3 \\ & v_1 & v_2 & v_3. & \end{array}$$

(#) Distance of Factory to Warehouses is divided in two parts.

First factory to Commonplace i.e, Represented by $u_1, u_2, u_3$.

Second Commonplace to Warehouse is Represented by $v_1, v_2, v_3$.

In order to calculate break up of factory to warehouse, we presumed any factory is located near to common place i.e, either $u_1$, or $u_2$, or $u_3$, may be 0 (zero). [We Consider any value $u = 0$]

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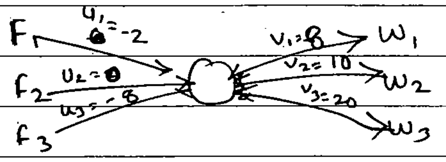
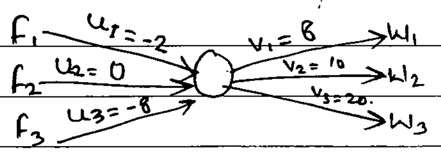
eg (Q.1) Calculate $u_i$ & $v_j$ from the following basic solution

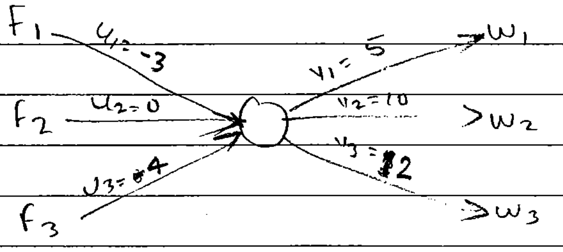
$$\begin{array}{ccccc} & w_1 & w_2 & w_3 & \\ f_1 & 6^{(10)} & 9 & 2 & u_1 = 2 \\ f_2 & 8^{(20)} & 10^{(10)} & 9 & = u_2 = 0 \\ f_3 & 3 & 2^{(50)} & 12^{(20)} & u_3 = -8 \\ & v_1 = 8 & v_2 & v_3\ 20 & \\ & & v_2 = 10 & & \end{array}$$





eg 2 Calculate the value of $u_i$ & $v_j$ from initial basic feasible solution of VAM.

$$\begin{array}{ccccc} & w_1 & w_2 & w_3 & \\ f_1 & 2^{(10)} & 6 & 8 & u_1 = -3 \\ f_2 & 5^{(10)} & 10^{(20)} & 12 & u_2 = 0 \\ f_3 & 12 & 6^{(10)} & 8^{(30)} & u_3 = -4 \\ & v_1 = 5 & v_2 = 10 & v_3 = 12 & \end{array}$$



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**eg-3** Calculate $u_i$ & $v_j$ from the following basic solution of VAM.

$$\begin{array}{c|ccc|l} & w_1 & w_2 & w_3 & \\ \hline f_1 & 12^{(10} & 6 & 8 & u_1 = -3. \\ f_2 & 15^{(10} & 8^{(20} & 12 & u_2 = 0 \\ f_3 & 12 & 16^{(10} & 19^{(30} & u_3 = 8 \\ \hline & v_1 = 15 & v_2 = 8 & v_3 = 11 & \end{array}$$

(eg) Calculate $U_j$ & $V_j$ from the following.

$$\begin{array}{c|ccc|c} & \underline{\underline{w_1}} & \underline{\underline{w_2}} & \underline{\underline{w_3}} & \\ f_1 & 14^{(10} & 5 & 2 & u_1 = 2. \\ f_2 & 12^{(20} & 10^{(10} & 12 & u_2 = 0 \\ f_3 & 6 & 15^{(10} & 18^{(20} & u_3 = 5 \\ & V_1 = 12 & V_2 = 10 & V_3 = 13 & \end{array}$$

$$\boxed{\Delta_{ij} = C_{ij} - (U_i + V_j)}$$

$\Delta_{12} = 5 - 12 = -7$ $[5 - (10+2)]$

$\Delta_{13} = 2 - 15 = -13.$ $[2 - (2+13)]$

$\Delta_{23} = 12 - 13 = -1$ $[12 - (0+13)]$

$\Delta_{31} = 6 - 17 = -11$ $[6 - (5+13)]$

(eg) Calculate the value of $U_j$ & $V_j$ from the following basic solution of VAM.

$$\begin{array}{c|ccc|c} P & w_1 & w_2 & w_3 & \\ f_1 & 14^{(10} & 5 & 4 & u_1 = 1. \\ f_2 & 13^{(20} & 12^{(10} & 12 & u_2 = 0 \\ f_3 & 16 & 16^{(10} & 19^{(20} & u_3 = 4 \\ & V_1 = 13 & V_2 = 12 & V_3 = 15 & \end{array}$$

$\Delta_{12} = [5 - (1+12)] = -8$

$\Delta_{13}$ $[4 - (1+15)] = -12$

$\Delta_{23}$ $[12 - (0+15)] = -3$

$\Delta_{31}$ $[16 - (4+13] = -1.$

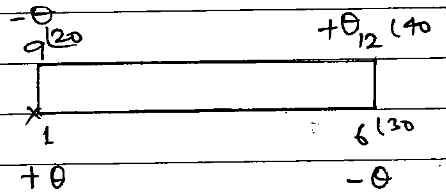
eg Contouration of loop &

suppose we have Ifbs. f.

$$\begin{array}{c|ccc|c} & w_1 & w_2 & w_3 & \\ \hline f_1 & 4 & 2^{(45} & 7 & 45 \\ f_2 & 9^{(20} & \leftarrow 8 \leftarrow & 12^{(40} & 60 \\ f_3 & \times 1 & \rightarrow 5^{(25} \rightarrow & 6^{(30} & 55 \\ \hline & 20 & 70 & 70 & \end{array}$$

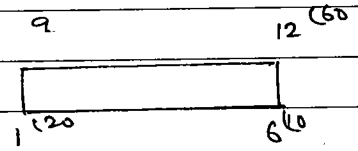
suppose $\Delta_{31}$ is Highest in Negation.

Loop



Qty to be shifted $-\theta\,(30,20)$ ~~420~~ $= \theta = 20$.

Revised Loop



Revised Matrix after Ist loop.

$$\begin{array}{c|ccc} & w_1 & w_2 & w_3 \\ \hline f_1 & 4 & 2^{(45} & 7 \\ f_2 & 9 & 8 & 12^{(60} \\ f_3 & 1^{(20} & 5^{(25} & 6^{(10} \end{array}$$

Q-12
Page-43

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $f_1$ | 3 (300 | 4 (100 | 4 (100 | 500 |
| $f_2$ | 9 | 6 (300 | 7 | 300 |
| $f_3$ | 4 | 6 | 5 (200 | 200 |
| | | | | 1000 |
| Demand | 300 | 400 | 300 | |

$\Delta_{21} = 8$

$\Delta_{23} = 2$

$\Delta_{31} = 0$

$\Delta_{32} = 2$

Explonation -1 $C_{21}$, $C_{31}$ are our unallocated cell because we have analysed value of $\Delta_{ij}$ i.e. $C_{11}$ is an unallocated cell i.e 300 allocation to $C_{11}$.

2 As $C_{32}$ is unallocated, hence $C_{12}$ would have 100 allocation

3) As $C_{23}$ is unallocated, Hence $C_{13}$ Would have 100 allocation.

~~Statement of Cost as per existing Optimum Solution.~~

Statement of Cost as per existing optimum solution.
(No. Value exist as Negative in Δ.)

$f_1 - D_1$ = 300×3 = 900
$f_1 - D_2$ = 100×4 = 400
$f_1 - D_3$ = 100×4 = 400
$f_2 - D_2$ = 300×6 = 1800
$f_3 - D_3$ = 200×5 = 1000
4500

To order to ~~solve~~ analyse alternate solution we should Construct a loop from $\Delta_{31}$.

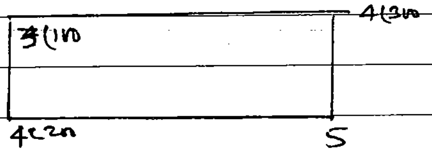
$-\theta$ $+\theta$

| | |
|---|---|
| 3 (300 | 4 (100 |
| 4 | 5 (200 |

$+\theta$ $-\theta$

Qty to be shifted = $Q(200, 300) = \theta = 200$.

Revised Loop



Revised Matrix after 1st loop.

$$\begin{array}{c|ccc} & D_1 & P_2 & P_3 \\ \hline F_1 & 3^{(100} & 4^{(100} & 4^{(300} \\ f_2 & 9 & 6^{(300} & 7 \\ P_3 & 4^{(200} & 6 & 5. \end{array}$$

## Statment of Cost

$f_1 - D_1 \quad 100 \times 3 = 300$

$f_1 - P_2 \quad 100 \times 4 = 400$

$f_1 - P_3 \quad 300 \times 4 = 1200$

$f_2 - P_2 \quad 300 \times 6 = 1800$

$F_3 - P_1 \quad 200 \times 4 = 800$

$4500$

| $P_5$ | $P_4$ | $P_3$ |
|---|---|---|
| X | $\overset{4}{\rightarrow}$ | 2 |
| 1 | 1 | 2 |
| 4 | 4 | 1 |

(7) **Profit Matrix**

[Page-40]

|  | $S_1$ | $S_2$ | $S_3$ | $S_4$ | Dummy<br>$S_6$ | $S_5$ | Capacity |
|---|---|---|---|---|---|---|---|
| Plant₁ | 9<br>30-(20+1) | 11<br>32-(20+1) | 6<br>31-(20+5) | 5 | 0 | 5 | 150 |
| Plant₂ | -1<br>30-(22+9) | 3<br>32-(22+7) | 1<br>31-(22+8) | 9 | 0 | 1 | 200 |
| Plant₃ | 8<br>[30-(18+4)] | 9<br>[32-(18+5) | 10<br>31-(18+3) | 14 | 0 | 4 | 125 |
| Demand | 80 | 110 | 75 | 45 | 50 (b/f) | 125 | 475 |

$\underbrace{80 \quad 110 \quad 75}_{425}$

We should Convert to Minimization Matrix.

| $P_2$ | $P_1$ | P | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | Capacity |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | Plant₁ | 5 150 | 3 110 | 8 | 9 | 9 | 14 | 150/50 |
| 2 | ~~6~~ | Plant₂ | 15 | 12 | 13 | 5 45 | 13 105 | 14 50 ⑥ | 200/155/50/0 |
| ~~1~~ | 4 | Plant-3 | 6 30 | 5 | 4 75 | 0 | 10 20 | 14 ⑤ | 125/50/20/0 |
|  |  | Demand | 80<br>30 | 110<br>0 | 75<br>0 | 45<br>0 | 125<br>105 | 50(b/f) | 475 |
|  |  |  | ④ | ③ | ② | ① |  |  |  |
|  |  | $P_1$ | 1 | 2 | 4 | 5 | 1 | 0 |  |
|  |  | $P_2$ | 1 | 2 | 4↑ | ~~7~~ | 1 | 0. |  |
|  |  | $P_3$ | 1 | 2↑ | x | x | 1 | 0 |  |
|  |  | $P_4$ | 1 | x | x | x | 1 | 0. |  |
|  |  | $P_5$ | 9↑ | x | x | x | 1 | 0. |  |

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## Test of Optimality

**VAM**

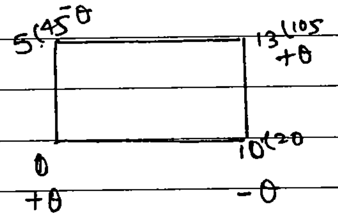
$$\begin{array}{c|cccccc|l}
 & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & \\
\hline
P_1 & 5^{(50} & 3^{(100} & 8 & 9 & 9 & 14 & u_1=-4 \\
P_2 & 15 & 11 & 13 & 5^{(45} & 13^{105} & 14^{(50} & u_2=0 \\
P_3 & 6^{(30} & 5 & 4^{(75} & 0 & 10^{(20} & 14 & u_3=-3 \\
\hline
 & v_1=9 & v_2=7 & v_3=7 & v_4=5 & v_5=13 & v_6=14 &
\end{array}$$

Step-1 = No. of allocated Cell = 8
Require = $m+n-1 = 3+6-1 = 8$.
Not a Case of Degeneracy, we can apply next step

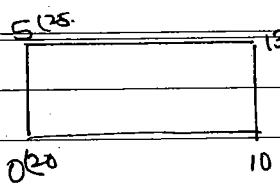
Step-2 Calculate value of $u_i$ & $v_j$ for all allocated cell

$C_{ij} = u_i + v_j$ & $\Delta_{ij}$ for unallocated cell.
$\Delta_{ij} = C_{ij} - (u_i + v_i)$.

$\Delta_{13} = 8 - (-4+7) = 5$
$\Delta_{14} = 9 - (-4+5) = 8$
$\Delta_{15} = 9 - (-4+13) = 0$
$\Delta_{16} = 14 - (-4+14) = 4$
$\Delta_{21} = 15 - (0+9) = 6$
$\Delta_{22} = 11 - (0+7) = 4$
$\Delta_{23} = 13 - (0+7) = 6$
$\Delta_{32} = 5 - (-3+7) = 1$
$\Delta_{34} = 0 - (-3+5) = -2$
$\Delta_{36} = 14 - (-3+14) = 3$.



Quantity to be Shifted $\theta(20, 45)$ i.e; 20



⇐ Revised loop

Revised Matrix

$$\begin{array}{c|cccccc|c} & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & \\ \hline P_1 & 5^{(50} & 3^{(100} & 8 & 9 & 9 & 14 & u_1 = -6 \\ P_2 & 15 & 11 & 13 & 5^{(25} & 13^{(125} & 14^{(50} & u_2 = 0 \\ P_3 & 6^{(30} & 5 & 4^{(75} & 0^{(20} & 10 & 14 & u_3 = -5 \\ \hline & v_1 = 11 & v_2 = 9 & v_3 = 9 & v_4 = 5 & v_5 = 13 & v_6 = 14 & \end{array}$$

**Step-1** No. of allocated cell = 8
Required = m+n-1 = 8. [∴ Not a Case of Degenaracy]

**Step-2** Calculation of $U_i$ & $V_j$ & $\Delta_{ij}$.

$\Delta_{13} = 8 - (-6+9) = 5$
$\Delta_{14} = 9 - (-6+5) = 10$
$\Delta_{15} = 9 - (-6+13) = 2$
$\Delta_{16} = 14 - (-6+14) = 6$
$\Delta_{21} = 15 - (0+11) = 4$
$\Delta_{22} = 11 - (0+9) = 2$
$\Delta_{23} = 13 - (0+9) = 4$
$\Delta_{32} = 5 - (-5+9) = 1$
$\Delta_{35} = 10 - (-5+13) = 2.$
$\Delta_{36} = 14 - (-5+14) = 5$

Since all Value of $\Delta ij$ are positive, hence we can say that above solution is optimum.

Statement of Profit

$P_1 S_1 = 50 \times 9 = 450$
$P_1 S_2 = 100 \times 11 = 1100$
$P_2 S_4 = 25 \times 9 = 225$
$P_2 S_5 = 125 \times 11 =$ 1[illegible]25
$P_2 S_6 = 50 \times 0 = 00$
$P_3 S_1 = 30 \times 8 = 240$
$P_3 S_3 = 75 \times 10 = 750$
$P_3 S_4 = 20 \times 14 = 280$

Profit ⇒ ~~4420~~ 3170 Ans

DATE ☐☐ ☐☐ ☐☐☐☐

(11) (i) Initial Transportation Matrix for Minimization.
Page 42

|  | A1 | A2 | A3 | A4 | A5. | Capacity |
|---|---|---|---|---|---|---|
| F1 | 9 | 11 | 6 | 5 | 5 | 110 |
| F2 | -1 | 3 | 1 | 8 | 1 | 240 |
| F3 | 8 | 9 | 10 | 10 | 4 | 125 |
| Demand | 80 | 100 | 75 | 45 | 125 | 475 |

$\underbrace{80 \quad 100 \quad 75 \quad 45 \quad 125}_{425}$

(ii) Balance Matrix

|  | A1 | A2 | A3 | A4 | A5 | A6. | Capacity |
|---|---|---|---|---|---|---|---|
| F1 | 9 | 11 | 6 | 5 | 5 | 0 | 110 |
| F2 | -1 | 3 | 1 | 8 | 1 | 0 | 240 |
| F3 | 8 | 9 | 10 | 10 | 4 | 0 | 125 |
| Demand | 80 | 100 | 75 | 45 ~~125~~ 125 | | 50 | 475 |

Minimization Matrix as under.

|  | A1 | A2 | A3 | A4 | A5 | A6 | Capacity | P1 | P2 | P3 | P4 | P5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F1 | $2^{30}$ | $0^{60}$ | 5 | 6 | $6^{20}$ | 11 | 110<br>50<br>20<br>0 | 2 | 2 | 2 | 4 ← | 5 |
| F2 | 12 | $8^{40}$ | 10 | $3^{45}$ | $10^{105}$ | $11^{50}$ | 240<br>200<br>155<br>50<br>0 | 5 ← | 2 | 2 | 1 | 1 |
| F3 | $3^{50}$ | 2 | $1^{75}$ | 1 | 7 | 11 | 125<br>50<br>0 | 0 | 1 | 1 | 4 | 4 |
| Demand | 80<br>50 | 100<br>60 | 75 | 45<br>0 | 125<br>105 | 50.<br>0 | | | | | | |
| P1 | 1 0 | 2 0 | 4 ↑ | 2 | 1 0 | 0 | | | | | | |

* F2 A2 is already booked for 80 unit. (This is due to earlier Commitment) Commitment) It does not mean atmost 40 unit should be transferred. It means ~~40~~ atleast 40 unit should be kept as Reserve.

* F2 A1 : Maximum allocation should be 30. units.

|  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|
| P2 | 1 | 2 | 4 ↑ | x | 1 | 0 |
| P3 | 1 | 2 ↑ | x | x | 1 | 0 |
| P4 | 1 | x | x | x | 1 | 0 |
| P5 | 9 ↑ | x | x | x | 1 | 0 |

classmate

* As per system $F_1A_1$ should have 50 allocation but due to restriction we should allocate only 30 units.

Statement of Profit (Basic solution).

$F_1\ A_1 = 30 \times 9 = 270$

$F_1\ A_2 = 60 \times 11 = 660$

$F_1\ A_5 = 20 \times 5 = 100$

$F_2\ A_2 = 40 \times 3 = 120$

$F_2\ A_4 = 45 \times 8 = 360$

$F_2\ A_5 = 105 \times 1 = 105$

$F_2\ A_6 = 50 \times 0 = 0$

$F_3\ A_1 = 50 \times 8 = 400$

$F_3\ A_3 = 75 \times 10 = 750$

Profit = 2765

(iii) * As per our basic solution $F_2A_1$ has no allocation. Hence it has no effects for that we are not able to transfer any unit to $F_2A_1$.

(10) North West Corner Rule
Page-42

$$\begin{array}{c|ccc|c}
 & A & B & C. & \text{Demand} \\
x & 3^{20} & 9 & 6 & 20/0 \\
Y & 4^{20} & 4^{20} & 6 & 40/20 \\
Z & 8 & 3^{30} & 5^{30} & 60/30 \\
\text{Supply.} & \frac{40}{\frac{20}{0}} & \frac{50}{\frac{30}{0}} & \frac{30}{0} & 120
\end{array}$$

Statement of Cost

A to x  20X3 = 60

A to Y  20X4 = 80

B to Y  20X4 = 80

B to Z  30X3 = 90

C to Z  30X5 = 150

Total Cost ⇒ 460

(ii) Vogel's Approximation Method

| | A | b | C | Demand | $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|---|---|---|---|
| x | ~~$3^{20}$~~ | ~~9~~ | ~~6~~ | 20/0 | 3 ← | x | x |
| y | $4^{20}$ | $4^{20}$ | 6 | 40/20/0 | 0 | 0 | 2 ← |
| z | 8 | $3^{30}$ | $5^{30}$ | 60 | 2 | 2 | 2 |
| Supply | 40/20 | 50<br>30. | 30. | 120 | | | |
| $P_1$ | 1 | 1 | 1 | | | | |
| $P_2$ | 4↑ | 1 | 1 | | | | |
| $P_3$ | x | 1 | 1. | | | | |

Statement of Cost.

| | | |
|---|---|---|
| A to x | 20x3 = | 60 |
| A to y | 20x4 = | 80 |
| B to y | 20x4 = | 80 |
| b to z | 30x3 = | 90 |
| C to z | 30x5 = | 150 |
| | | 460 |

(iii) Optimality by VAM

Step-1 No. of Allocation = 5

Require = m+n-1 = 3+3-1 = 5.

∴ Not a case of degeneracy. (So we can apply next step).

Step-2 Calculate $U_i, V_j$ for allocation cell $A_{ij}$ for unallocated cell. $C_{ij} = U_i + V_j$ ! $A_{ij} = C_{ij} - (U_i + V_j)$.

| | A | b | C | D |
|---|---|---|---|---|
| x | $3^{20}$ | 9 | 6 | $U_1 = -1.$ |
| y | $4^{20}$ | $4^{20}$ | 6 | $U_2 = 0$ |
| z | 8 | $3^{30}$ | $5^{30}$ | $U_3 = -1$ |
| | $V_1 = 4$ | $V_2 = 4$ | $V_3 = 6$ | |

$\Delta_{12} = 9 - (-1+4) = 6$ | $\Delta_{23} = 6 - (0+6) = 0$

$\Delta_{13} = 6 - (-+6) = 1.$ | $\Delta_{31} = 8 - (-1+4) = 5.$

All above Value are positive ($\Delta ij$) ∴ Above Solutim is optimum.

(iii) Analysis For above situatim We have same allocatim under North West Corner Rule as well as Vogel's Approximation Method hence Test of Optimality will provide the same Procedural Value & ultimate final Answer will had different allocations under these method (Initial basic feasible Solutim). final optimum Answer would have been same after applying Test of Optimality.

(19) [Page-50] Solutim as per Clerk Calculation

$$\begin{array}{c|cccc} & I & II & III & IV. \\ \hline A & 5 & 2^{12} & 4^{1} & 3^{9} \\ B & 4 & 8 & 1^{15} & 6 \\ C & 4^{7} & 6 & 7^{1} & 5 \end{array}$$

The Solutim provided by clerk would be an optimum when all value of $\Delta ij$ becomes positive in Test of Opitimality.

Step-1 No. of allocated cell = 6

Requirement = $m+n-1 = 3+4-1 = 6$

∴ Its not a Case of Degeneracy & we can apply Next Step.

Step-2 Calculate $U_i$ & $V_j$ for allocated Cell and $\Delta ij$ for unallocated Cell.

$[C_{ij} = U_i + V_j]$ $\Delta_{ij} = C_{ij} - (U_i + V_j)$

$$\begin{array}{c|cccc|c} & I & II & III & IV & \\ \hline A & 5 & 2^{12} & 4^{1} & 3^{9} & U_1 = 0 \\ B & 4 & 8 & 1^{15} & 6 & U_2 = -3 \\ C & 4^{7} & 6 & 7^{1} & 5. & U_3 = 3 \\ \hline & V_1 = 1 & V_2 = 2 & V_3 = 4 & V_4 = 3 & \end{array}$$

$\Delta_{11} = 5 - (0+1) = 4$

$\Delta_{21} = 4 - (-3+1) = 6$

$\Delta_{22} = 8 - (-3+2) = 9$

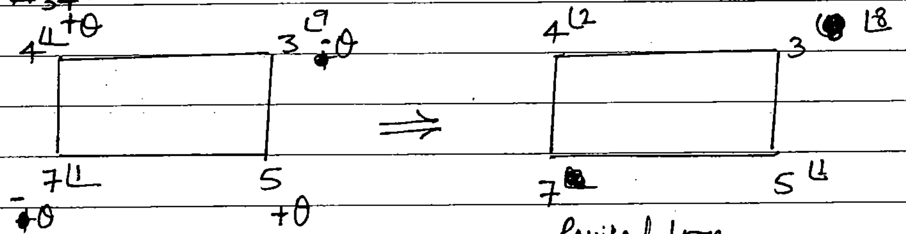
$\Delta_{32} = 6 - (3+2) = 1$

$\Delta_{34} = 5 - (3+3) = +1.$

$\Delta_{24} = 6 - (-3+3) = 6.$

Solution provided by Clerk is not an optimum

(ii) For Optimum solution (Schdule) we should Construct a loop from $A_{34}$.



Revised loop

Revised Matrix

$$\begin{array}{c|cccc|l} & I & II & III & IV & \\ A & 5 & 2^{12} & 4^{2} & 3^{8} & u_1=0 \\ B & 4 & 8 & 1^{15} & 6 & u_2=-3 \\ C & 4^{7} & 6 & 7 & 5^{1} & u_3=2 \\ & v_1=2 & v_2=2 & v_3=4 & v_4=3 & \end{array}$$

Test of optimality.

allocated cell: 6

Required: m+n-1: 3+4-1=6.

we can apply next step. i.e, Calculation of $U_i$ & $V_j$ & $A_{ij}$.

$A_{11} = 5-(0+2) = 3$

$A_{21} = 4-(-3+2) = 5$

$A_{22} = 8-(-3+2) = 9$

$A_{24} = 6-(-3+3) = 6$

$A_{32} = 6-(2+2) = 2$

$A_{3\cancel{3}} = 7-(2+4) = 1.$

Since all $A_{ij}$ are positive. Hence ~~cur~~ above. ~~Solution~~ Solution. is optimum.

Statement of Cost

M

A — II  12×2 =  24

A — III  2×4 =  8

A — IV  8×3 =  24

B — III  15×1 =  15

C — I  7×4 =  28

C = IV.  5×1 =  5

Total Cost = 104

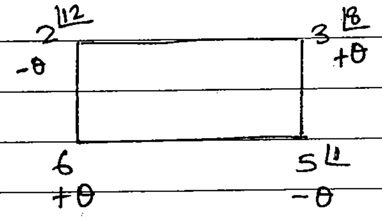
(iii) Rate for C-II be x/unit. so that.

(i) Total Cost becomes ≤ 104

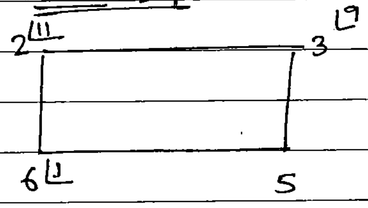
(ii) Total Capacity & Requirement exhaust

for this purpose we should Construct a loop.

self# [Not due to Negative value of Δij but due to Condition given]



Revised Loop



Revised Matrix

$$\begin{array}{c|cccc} & I & II & III & IV \\ \hline A & 5 & 2^{\boxed{11}} & 4^{2} & 3^{\boxed{9}} \\ B & 4 & 8 & 1^{\boxed{15}} & 6 \\ C & 4^{\boxed{7}} & 6^{\boxed{1}} & 7 & 5. \end{array}$$

Total Cost ≤ 104.

$$= (2\times11)+(3\times9)+(1\times15)\overset{+(4\times2)}{\wedge}+(4\times7)+(x\times1) \leq 104$$

$8+22+27+15+28+x = 104$

$x = 104-100$

$x \leq 4.$

If New Carrier offer a discount of ₹ atleast ₹ 2 p.u., we can offer him some business.

(22)
[Page-57]

**A**

$$\begin{array}{c|ccc|l} & P_1 & P_2 & P_3 & \\ D & \cancel{0}\,\boxed{20} & \cancel{0} & \cancel{0} & 20/0 \\ S_1 & \cancel{9}\,\boxed{80} & \cancel{27} & \cancel{18} & 80/0 \\ S_2 & 12\,\boxed{20} & 12 & 18 & 120/80/0 \quad 340 \\ S_3 & 24 & 10 & 15 & 140 \\ \hline & 120 & 150 & 90 & \underline{\underline{360}} \\ & 100 & 70 & & \\ & 20 & & & \end{array}$$

| Student | Cell Reference | Qty Allocation unit | Unit Cost at that cell. |
|---|---|---|---|
| A | $S_2P_1$ | 20 | 12 |
| b | $S_2P_2$ | 80 | 12. |
| C | $S_3P_2$ | 140 | 10. |
| D. | $S_2P_1$ | 40 | 12 |

**b**

$$\begin{array}{c|ccc|l} & P_1 & P_2 & P_3 & \\ S_1 & \cancel{9}\,\boxed{80} & \cancel{27} & \cancel{18} & 80/0 \\ S_2 & 12\,\boxed{40} & 12 & 18 & 120/80 \\ S_3 & 24 & 10 & 15 & 140 \\ D. & 0\,\boxed{20} & 0 & 0 & 20 \\ \hline & 120 & 150 & 90 & \underline{\underline{360}} \\ & 400 & & & \\ & 20 & & & \end{array}$$

**c**

$$\begin{array}{c|ccc|l} & P_1 & P_2 & P_3 & \\ S_1 & \cancel{9}\,\boxed{80} & \cancel{27} & \cancel{18} & 80/0 \\ S_2 & 12 & 12 & 18 & 120 \\ S_3 & \cancel{24} & \cancel{10}\,\boxed{140} & \cancel{15} & 140 \\ D & \cancel{0}\,\boxed{20} & \cancel{0} & \cancel{0} & 20/0 \\ \hline & 120 & 150 & 90. & 360 \\ & 100 & & & \\ & 20 & & & \end{array}$$

|  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
| **D** | $P_1$ | $P_2$ | $P_3$ |  | $P_1$ | $P_2$ | $P_3$ |
| $S_1$ | $9^{\boxed{80}}$ | ~~27~~ | ~~18~~ | 80 | 9 | $\overleftarrow{9}$ | x |
| $S_2$ | $12^{\boxed{40}}$ | 12 | 18 | 120\|80. | 0 | 0 | 0 |
| $S_3$ | 24 | 10 | 15 | 140 | 5 | 5 | 5 |
| D. | ~~0~~ | ~~0~~ | ~~0~~$^{\boxed{120}}$ | ~~20~~\|0 |  | x | x |
|  | $\frac{120}{40.}$ | 150 | $\frac{190}{70.}$ | $\underline{\underline{360}}$ |  |  |  |
| $P_1$ | 90 | 10 | 15.↑ |  |  |  |  |
| $P_2$ | 3 | 2 | 3 |  |  |  |  |
| $P_3$ | 12↑ | 2 | 3 |  |  |  |  |

22nd class
DATE 18 01 2017

(#) Degenaracy : Degenaracy Occurs when allocated cell falls below m+n-1 (It means we are not able to construct the loop due to Insufficient allocated cell) For this purpose we should create a Dummy cell as allocated dummy should be represented by alphabet 'e'. 'e' should be considered as equivalent to Zero.

Where to introduce 'e' :- 'e' should be introduced to the cell having ~~least Cost~~

(1) least Cost (Unallocated).
(2) Unallocated Independent Cell.

if least Cost Cell is not an Independent, we should Introduce 'e' to the next least Cost ~~Introduce~~ Independent Cell. Independent Cell means Cell from ~~f~~ where Loop Could not be Constructed.

Reason :- Least Cost

[I] We should Insert 'e' to the least Cost cell because, after Inserting 'e' It becomes an allocated Cell, & It may possible in next step, in Construction of loop, It becomes a Cell having exist at turning Point which means It becomes a part of Cost after shifting, & Cost should be minimized.

[II] If we Insert 'e' to the dependent Cell, It becomes an Independent loop & may possible that It would not become a part of next loop which results least Cost be ignored.

(17) [Page-44]

$$\begin{array}{l|ccccc|l}
 & & & & & & \text{Requirement} \\
 & 11 & 2^{(8} & 8 & 6^{(6} & 2^{(4} & 18 \; u_1=0 \\
 & 9^{(10} & 9 & 5 & 9 & 6 & 10 \; u_2=0 \\
 & 7 & 6 & 3^{(8} & 7 & 7 & 8 \; u_3=2 \\
 & 9^{(2} & 3 & 5^{(e} & 6^{(2} & 11. & 4 \; u_4=0 \\
\text{Availability} & 12 & 8 & 8 & 8 & 4 & \underline{40} \\
 & v_1=9 & v_2=2 & v_3=5 & v_4=6 & v_5=2 &
\end{array}$$

**Test of optimality**

**Step 1** allocated Cell = 7

Required = m+n-1 = 4+5-1 = 8.

∴ It is a Case of Degeneracy.

We should introduce 'e' to least cost Independent cell here we have least Cost cell is $C_{42}$ but It is dependent Cell. Hence we should identify next least Independent Cell i.e, $C_{43}$.

$\Delta_{11}=2$ $\Delta_{22}=7$ $\Delta_{24}=3$ $\Delta_{31}=0$ $\Delta_{34}=3$ $\Delta_{42}=1$

$\Delta_{13}=3$ $\Delta_{23}=7$ $\Delta_{25}=4$ $\Delta_{32}=6$ $\Delta_{35}=7$ $\Delta_{45}=9$

All $\Delta_{ij}$ are positive.

Statement of Cost.

$R_1C_2$ 8x2 = 16

$R_1C_4$ 6x6 = 36

$R_1C_5$ 4x2 = 08

$R_2C_1$ 10x9 = 90

$R_3C_3$ 8x3 = 24

$R_4C_1$ 2x9 = 18

$R_4C_3$ 5xe = 0

$R_4C_4$ 2x6 = 12

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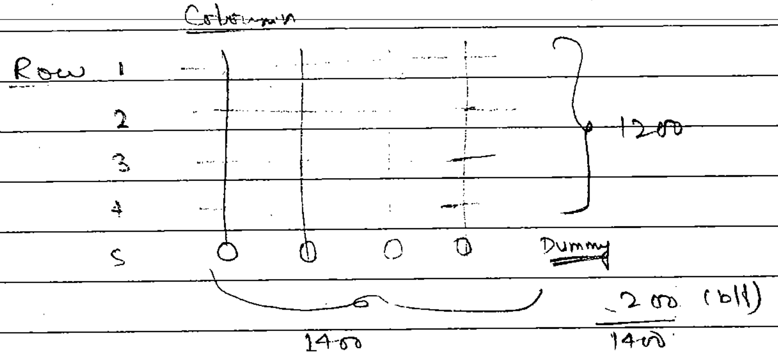
(13) [Page-43] In order to check the degeneracy we should first of all, we should Introduce a dummy Row/Coloumn to make it balance which means Now, if allocated Cell falls become less than M+n-1 [5+4-1] = 8, then it becomes a Case of Degeneracy.



(14) [Page-43] (i) The above solution is feasible when Total Requirement = Total Availability (balanced) i.e

Total Requirement = 55; Total Availability = 55

(ii) Allocation Cell = 6.

Requirement = m+n-1 = 3+4-1 = 6.

Its not a Case of Degeneracy.

(iii)

$$\begin{pmatrix} 5 & 10 & 4^{(10} & 5 \\ 6^{(20} & 8 & 7 & 2^{(5} \\ 4^{(5} & 2^{(10} & 5^{(5} & 7 \end{pmatrix} \begin{array}{l} u_1 = (1) \\ U_2 = 2 \\ U_3 = 0 \end{array}$$

$V_1 = 4 \quad V_2 = 2 \quad V_3 = 5 \quad V_4 = 0$

All Value of $\Delta ij$ are positive solution is optimum

$\Delta_{11} = 2 \quad \Delta_{14} = 6 \quad \Delta_{23} = 0$

$\Delta_{12} = 9 \quad \Delta_{22} = 4 \quad \Delta_{34} = 7$

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(16) Page-44

We Can't say that $R_2C_1$ always become a part of solution by North West Corner Rule because if after allocation of $R_1C_1$, first coloumn exhaust then Next allocation would be $R_1C_2$ & $R_2C_1$ would never becomes a part of solution in ~~word~~ other words we can say $R_2C_1$ would becomes a part of solution Only when $R_1$ exhaust after $R_1C_1$ allocation.

$$\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline R_1 & \checkmark & \checkmark & & \\ R_2 & \times\times & & & \\ R_3 & & & & \end{array} \quad 3\times4.$$

(5) [Page-39]

|  | A | B | C | D | E | ₹000" Available |
|---|---|---|---|---|---|---|
| 1 | 0·8 | 0·9 | 0·60 | 0·75 | 1·00 | 500 |
| 2 | 0·55 | 0·65 | 0·40 | 0·60 | 0·50 | 600 |
| 3 | 0·30 | 0·25 | 0·30 | 0·50 | 0·20 | 750 |
| 4 | 0·15 | 0·12 | 0·25 | 0·35 | 0·10· | 800 |
| Dummy Row. 5 | 0 | 0 | 0 | 0 | 0 | 1000 |
| Amt | 750 | 600 | 500 | 800 | 1000 | 3650 |

Minimezation Matrix.

|  | A | b | C | D | E | ₹ Available | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ~~20~~ | ~~10~~ | ~~40~~ | ~~25~~ | ~~$0^{(500}$~~ | 500/0 | 10 | X | X | x | x |
| 2 | ~~45~~ | ~~$35^{(600}$~~ | ~~60~~ | ~~40~~ | ~~50~~ | 600/0· | 5 | 5 | X | x | x |
| 3 | ~~70~~ | ~~75~~ | ~~70~~ | ~~$50^{(750}$~~ | ~~80~~ | 750/0 | 20 | 20 | 20↑ | x | x |
| 4 | $85^{(250}$ | 88 | $75^{(500}$ | $65^{(50}$ | 90· | 800/750/10 | 10 | 10 | 10 | 10 | 10 |
| 5 | $100^{(500}$ | 100 | 100 | 100 | $100^{(500}$ | 1000 | $\overset{250/0}{0}$ | 0 | 0 | 0 | 0 |
| Amt | 750 | 600/0 | 500 | 800/50· | 1000/500 | 3650 |  |  |  |  |  |
| $P_1$ | ~~25~~ | 25 | 20 | 15 | 50↑ |  |  |  |  |  |  |
| $P_2$ | 25 | 40↑ | 10 | 10 | 30 |  |  |  |  |  |  |
| $P_3$ | 15 | X | 5 | 15 | 10 |  |  |  |  |  |  |
| $P_4$ | 15 | X | 25 | 35↑ | 10 |  |  |  |  |  |  |
| $P_5$ | 15 | X | 25 | X | 10 |  |  |  |  |  |  |

## Test of Optimality -I

I No. of allocation = 8

~~No. of~~ Required = m+n-1

= 5+5-1 = 9.

It's a Case of Degenaracy. we should insert 'e' to the least Cost Independent Cell. i.e, $R_1C_2$.

$$\begin{array}{c|ccccc|l} & A & B & C & D & E & \\ 1 & 20 & 10^{(e} & 40 & 25 & 0^{(500} & U_1=-85 \\ 2 & 45 & 35^{(600} & 60 & 40 & 50 & U_2=-60 \\ 3 & 70 & 75 & 70 & 50^{(750} & 80 & U_3=(15) \\ 4 & 85^{(250} & 88 & 75^{(500} & 65^{(50} & 90 & U_4=0 \\ 5. & 100^{(500} & 100 & 100 & 100 & 100^{(500} & U_5=15 \\ & V_1=85 & V_2=95 & V_3=75 & V_4=65 & V_5=85 & \end{array}$$

II Calculate $U_i$ & $V_j$ for allocated Cell & $\Delta_{ij}$ for unallocated Cell.

$\Delta_{11} = 20$ $\quad$ $\Delta_{21} = 20$ $\quad$ $\Delta_{31} = 0$ $\quad$ $\boxed{\Delta_{42} = -7}$

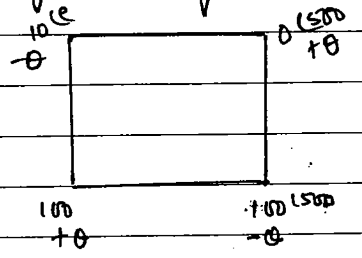
~~$\Delta_{12} = 0$~~ $\quad$ $\Delta_{23} = 45$ $\quad$ $\boxed{\Delta_{32} = (5)}$ $\quad$ $\Delta_{44} = 5.$

$\Delta_{13} = 30$ $\quad$ $\Delta_{24} = 35$ $\quad$ $\Delta_{33} = 10$ $\quad$ $\boxed{\Delta_{52} = -10}$

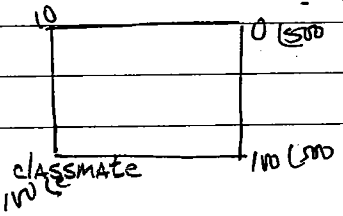
$\Delta_{14} = 45$ $\quad$ $\Delta_{25} = 25$ $\quad$ $\Delta_{34} = 10$ $\quad$ $\Delta_{53} = 10$

$\Delta_{54} = 20$

we should Start the loop from the $\Delta_{52}$ due to highest in Negative.



Qty to be shifted = θ = (500; e) [∵ e + Any value = Any value]
= θ = e. (e - e = Unallocated cell)
[e-Value = Not Applicable]



$\boxed{\text{Value} \pm e = \text{Value}}$

## Revised Matrix after First loop.

$$\begin{array}{c|ccccc|c} & A & B & C & D & E & \\ \hline 1 & 20 & 10 & 40 & 25 & 0^{(500} & U_1 = -85. \\ 2 & 45 & 35^{(600} & 60 & 40 & 50 & U_2 = -50 \\ 3 & 70 & 75 & 70 & 50^{(750} & 80 & U_3 = -15 \\ 4 & 85^{(250} & 88 & 75^{(500} & 65^{(50} & 90 & U_4 = 0 \\ 5 & 100^{(500} & 100^{(e} & 100 & 100 & 100^{(500} & U_5 = 15 \\ \hline & V_1 = 85 & V_2 = 85 & V_3 = 75 & V_4 = 65 & V_5 = 85 & \end{array}$$

## Test of Optimality - II

No. of allocated cell = ~~8~~ 9

Required = m+n-1 = ~~8~~ 9.

$\Delta_{11} = 20$ $\Delta_{21} = 10$ $\Delta_{31} = 0$ $\Delta_{42} = 3$

$\Delta_{12} = 10$ $\Delta_{23} = 35$ $\Delta_{32} = 5$ $\Delta_{45} = 5$

$\Delta_{13} = 50$ $\Delta_{24} = 25$ $\Delta_{33} = 10$ $\Delta_{53} = 10$

$\Delta_{14} = 45$ $\Delta_{25} = 15$ $\Delta_{35} = 10$ $\Delta_{54} = 20.$

All $\Delta_{ij}$ are positive.

Hence the solution is an optimum.

## Statement of Profit

| Year | Investment | Return |
|---|---|---|
| 1 | E | 500000 X 1 = 500000 |
| 2 | B | 600000 X 0.65 = 3,90,000 |
| 3 | D | 750000 X 0.5 = 3,75000 |
| 4 | A | 2,50,000 X 0.15 = 37500 |
| 4 | C | 450000 X 0.25 = 125000 |
| 4 | D | [illegible]50000 X 0.35 = [illegible] |
| 5 | A | 500000 X 0 = 0 |
| 5 | B | [illegible]e X 0 = 0 |
| 5 | E | 500 X 0 = 0 |
| | | 14,45,000 |

(#) if allocated Cells falls short by 2 from required value, We should introduce two 'e's.

[First 'e' will be considered as allocated cell for second 'e'.]

eg IBFS

$$\begin{pmatrix} 4^{10} & 5^{e} & 6^{e} \\ 7 & 8^{120} & 9 \\ 10 & 11 & 12^{170} \end{pmatrix}$$

Allocated cell = 3

Required = $m+n-1 = 3+3-1 = 5$.

First 'e' should be introduced to least Cost independent cell i.e, $R_1C_2$.

Second 'e' should be introduced to 2nd least independent cell i.e $R_1C_3$.

(8) [Page-41]

$$\begin{array}{c|cccc|c} & A & B & C & D. & \text{Supply} \\ A & 2.5 & 3 & 5 & 2.5 & 20000 \\ B & 4.00 & 2 & 3.5 & 2.5 & 38000 \\ C & 4.50 & 3.5 & 1.5 & 2.5 & 16000. \\ \text{Demand} & 10000 & 18000 & 22000 & 24000 & 74000 \end{array}$$

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Chapter = 3

# LINEAR PROGRAMMING

23rd Class
DATE 25 01 2017

## # LINEAR PROGRAMMING

Linear Programming is a Mathematical approach which is used to utilize the resource in optimum manner which are in short supply.

# Short supply :- Present Availability of the resources are not sufficient to meet the entire demand of all products & its not possible to purchase such resources form market.

Resources :- Material, labour, Machine Hours, sales.

# Linear Programming approach has been developed to analyse the optimum solution where we have multiple key factors among with multiple products.

# Revenue
Return
Profit
Production } → should be maximized

Cost
Time
Loss
Wastage } → should be minimized.

# Method

Procedure of Linear Programming is divided into two (2) Parts.

(1) FORMULATION
(2) Analysing optimum solution by
  (a) Simplex Method
  OR
  (b) Graphical Method.

(1) FORMULATION :- Converting the Word problem to the Equation Format.

Equation means Linear Equation but not Quadratic.

eg:- $ax_1 + bx_2$ = Linear equation

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Process of Formulation is divided into two (2) parts:—

1st ⇒ Objective function.
2nd ⇒ ~~Constraint funtion.~~ Constraint function.

1st ⇒ Objective function is always represented by 'Z'. either maximized or minimized.

eg:- (i) $\underset{(Max)}{Z} = 5X_1 + 6X_2$

| | | |
|---|---|---|
| Product ⇒ | A | B |
| Qty ⇒ | $X_1$ | $X_2$ |
| Contribution Per unit ⇒ | 5 | 6 |

or,

(ii) $\underset{(mini)}{Z} = 5x_1 + 6x_2$

| | | |
|---|---|---|
| Product → | A | B |
| Qty → | $X_1$ | $X_2$ |
| Cost/unit ⇒ | 5 | 6. |

How much Quantity of '$X_1$' & '$X_2$' should be analyzed so that.
(i) Total Contribution to be Maximized.
(ii) Total Cost to be minimized.

2nd ⇒ Subject to ⊕ Constraint function always to be expressed either ≤ or ≥

⊕ Consumption of resources always ≤ availability of that resources.

for eg

| Product | | A | B | |
|---|---|---|---|---|
| Contribution/unit. | | 5 | 6. | Total Hrs |
| Hours/unit | D1 | 6 | 8. | 1000 |
| | D2 | 4 | 3 | 2000. |

FORMULATION Let $x_1$ be the Qty of Product A
let $x_2$ be the Qty of Product B.

Objective function · $\underset{(max)}{Z} = 5X_1 + 6X_2$

Subject to $6X_1 + 8X_2 \leq 1000$. [Department I Constraint]
$4X_1 + 3X_2 \leq 2000$ [Department II Constraint].
$X_1, X_2 \geq 0$ [Non Negative Constraint].

(1) Page-84

Let $x_1, x_2$ be the Qty of chairs & Table respectively.

Max
$$Z = 10x_1 + 40x_2$$

Subject to
$3x_1 + 4x_2 \leq 40$ (M₁ Constraint)
$5x_1 + 6x_2 \leq 50$ (M₂ Constraint)
$2x_1 + 3x_2 \leq 60$. (M₃ Constraint).
$x_1, x_2 \geq 0$. (Non Negative Constraint).

(3) (Page-84)

Working

100 bottle/minute

8 ounce (ltr)

A = 100 bottles ——— 1 minute
1 bottle ——— $\frac{1}{100}$ minute. [8 hr × 60 mint × 5 day = 2400]

B 60 Bottle ——— 1 minute.
1 bottle ——— $\frac{1}{60}$ minute

16 ounce (ltr)

A 40 bottles ——— 1 minute
1 bottles ——— $\frac{1}{40}$ minutes

B 75 bottles ——— 1 minute
1 bottles ——— $\frac{1}{75}$ minute

Formulation Let $x_1, x_2$ be the Qty of 8 ounce & 16 ounce bottles respectively.

Max (Z) = $0.25x_1 + 0.35x_2$

Subject to
$\frac{1}{100}x_1 + \frac{1}{40}x_2 \leq 2400$ (Machine, 'A' Constraint)

$\frac{1}{60}x_1 + \frac{1}{75}x_2 \leq 2400$ (Machine 'B' Constraint).

$8x_1 + 16x_2 \leq 330000$ (Production Constraint).

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$x_1 \leq 25000$ [Market Constraint]
$x_2 \leq 7000.$

$x_1, x_2 \geq 0$ [Non Negative Constraint]

(20)
(Page-93) Let $x_1, x_2, \& x_3$ be the Qty of A, B & C respectively.

$$Max(Z) = [48-(15+18)]x_1 + [42-(14+16)]x_2 + [38-(16+6)]x_3 - \underset{WN}{15000}.$$

**Working** Calculation of fixed Cost

Fixed overhead = 50% of Direct labour Cost.
Labour Cost = 1.5 £/Hr X 20000 Hrs = 30000.
∴ Fixed Cost = 50% of 30000 = 15000

subject to
$x_1 \leq 1500$
$x_2 \leq 1200$ [Demand Constraint]
$x_3 \leq 900$

$1x_1 + 1x_2$ [illegible] $x_3 \leq 5800.$ [Special Component Constraint]

$$\frac{18}{1.5}x_1 + \frac{16}{1.5}x_2 + \frac{6}{1.5}x_3 \leq 20,000$$ [Labour Constraint].
→ or $18x_1 + 16x_2 + 6x_3 \leq 30000$ [Labour Cost Constraint].
$15x_1 + 14x_2 + 16x_2 \leq 30,000.$ [Material Constraint].

(10)
(Page-87) Working Note

Exposure = No. of Advertisement X Principal Buyer.
Let $x_1, x_2 \& x_3$ be the No. of Advertisement in Magazine A, B & C respectively.
$$Max(Z) = x_1(100000 \times 20\%) + x_2(60000 \times 15\%) + x_3(40000 \times 8\%)$$

subject to $8000x_1 + 6000x_2 + 5000x_3 \leq 100000$ (Budget Constraint)
$x_1 \leq 15$
$x_2 \geq 8$ [Policy Constraint]
$x_3 \geq 8$
$x_1, x_2, x_3 \geq 0$ [Non Negative Constraint]

(19) (i) Page-92 let $x_1$ be the No. of Deluxe Package
$x_2$ be the No. of Standard "
$x_3$ be the No. of Economy "

$$\underset{(Max)}{Z} = \left[10000 - (3500+4750)\right]x_1 + \left[7000 - (2200+2500)\right]x_2 + \left[6500 - (1900+2200)\right]x_3 - 250000$$

subject to $x_1 + x_2 + x_3 \leq 200$. (Market Constraint).

$x_1 \geqslant 10\%\ (x_1+x_2+x_3)$

$\left.\begin{array}{l} x_2 \geqslant 35\%\ (x_1+x_2+x_3) \\ x_2 \leq 70\%\ (x_1+x_2+x_3) \end{array}\right] \Rightarrow \text{or} \left[70\%\ (x_1+x_2+x_3) \geqslant x_2 \geqslant 35\%\ (x_1+x_2+x_3)\right]$

$x_3 \geqslant 30\%\ (x_1+x_2+x_3)$

$x_1 \leqslant 60$

$x_1 + x_2 \geqslant 120$.

$x_1, x_2, x_3 \geqslant 0$. (Non Negative).

(ii) $x_1 + x_2 + x_3 = 200$ (Market Constraint)

$x_1 \geqslant 10\%\ (200)$

$x_2 \geqslant 35\%\ (200)$

$x_2 \leq 70\%\ (200)$

$x_3 \geqslant 30\%\ (200)$

$x_1 \leq 60$

$x_1 + x_2 \geqslant 120$

$x_1, x_2, \& x_3 \geqslant 0$. (Non Negative)

Page 16(89) Working

| | Loan | Interest. |
|---|---|---|
| TNC | 60000 | 720 |
| Co-operative | 40000 | 480 |
| | 100000 | 1200 (for 1 month) |

Interest for March has been paid. For April only we will have TNC bank loan.

| April | Loan | Int |
|---|---|---|
| TNC | 60000 | 720 |
| Co-op. | — | — |
| | 60000 | 720 |

(16) (Page-89) Let $x_1, x_2, x_3$ be Qty of manual, electronic & Delux typewriter respectively

$$\text{Max }(Z) = (4100-2500)x_1 + (7500-4500)x_2 + (14600-9000)x_3 - 10{,}000 - 100000 \times 2\% - 720.$$

For April 1999.

Subject to

$15x_1 + 12x_2 + 14x_3 \le 3000$ (Machine Hours Constraint)

$4x_1 + 3x_2 + 5x_3 \le 1200$ (Assembly Hour Constraint)

$x_1 \geq 2$ ( Market Constraint)

$x_3 \geq 8$

Rough working

$$25000x_1 + 4500x + 9000x_3 \le 140000 + 55000\begin{pmatrix}\text{Debtor}\\\text{Receive}\end{pmatrix} - \overset{(\text{Salary})}{10000} - 720 - \underset{\text{loan}}{40000} - 2000\ (\text{Interest})$$

i.e $25000x_1 + 4500x_2 + 9000x_3 \le 137280$ (Cash Constraint)

$x_1, x_2, x_3 \geq 0$

If Interest out-standing

(i) Same treatment in Profit & loss statement)

(ii) Only Cash availability Reduced by 1200 i.e, 137280 - 1200.

(14) (Page-89) Let $x_1$ be the Amt to be invested in Govt bond.

let $x_2$ ———————— blue chip stock.

let $x_3$ ———————— Speculative stock

let $x_4$ ———————— Short term Deposit.

$$\text{Max }(Z) = \frac{14}{100}x_1 + \frac{19}{100}x_2 + \frac{23}{100}x_3 + \frac{12}{100}x_4$$

Subject to $x_1 + x_2 + x_3 + x_4 \le 20{,}00{,}000$ (Investment Constraint)

$x_4 \geq 200000.$

$x_3 \le 20\%$ of ~~2000000~~. $(x_1 + x_2 + x_3)$. [Policy Constraint].

$$\left[\frac{\frac{12}{100}x_1 + \frac{24}{100}x_2 + \frac{48}{100}x_3 + \frac{6}{100}x_4}{x_1 + x_2 + x_3} \le 42\right]$$

$x_1, x_2, x_3 \geq 0$ (Non Negative)

Working Note

Average Risk Factor

$$\left[\frac{\frac{12}{100}x_1 + \frac{24}{100}x_2 + \frac{48}{100}x_3 + \frac{6}{100}x_4}{x_1+x_2+x_3}\right] \leq 42.$$

| | | |
|---|---|---|
| 100 | 12 | |
| 100 | 24 | $= \frac{90}{400}$ = Average. |
| 100 | 48 | |
| 100 | 06 | |
| 400 | 90 | |

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(8) Page 84

Let $x_1$ be Amt to be invested in Mutual fund. XY.

$x_2$ be Amt to be invested in Mutual fund HN

$x_3$ be Amt to be invested in Money Market fund.

$x_4$ be Amt to be invested in Government bond.

$x_5$ be Amt to be invested in Share 'P'

$x_6$ be Amt to be invested in share 'Q'.

$$Max(z) = 15\% x_1 + 9\% x_2 + 8\% x_3 + 8.75\% x_4 + 17\% x_5 + 18\% x_6.$$

Subject to

$$x_4 \geq 40\% (x_1 + x_2 + x_3 + x_4 + x_5 + x_6)$$

$$x_5 + x_6 \leq 2,60,000.$$

$$x_3 \geq 25\% (x_1 + x_2 + x_3 + x_4 + x_5 + x_6)$$

$$x_5 + x_6 \leq (x_1 + x_2) \text{[illegible]})$$

$$x_1 \leq x_2$$

Investment Constraint

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ [Non Negative Constraint].

(6) (Page-85)

Let $x_1, x_2, x_3$ be the hectare of Land. to ~~[illegible]~~ be utilized in Radish Mutter, Potato respectively

$$Max(z) = 7072.5x_1 + 6775x_2 + 5572.5x_3$$

Subject to $x_1 + x_2 + x_3 \leq 126$ [Land Constraint]

$6x_1 + 5x_2 + 6x_3 \leq 500$ [Labour Constraint].

$x_1, x_2, x_3 \geq 0.$ [Non Negative Constraint].

P.T.O

Working Notes

**Working** Statement of Profit

| | Radish | Mutter | Potato. |
|---|---|---|---|
| Revenue | 1500kg × 5 = 7500 | 1800kg × 4 = 7200 | 1200kg × 5 = 6000. |
| Less:- Cost. | $\frac{12.5 \times 1500}{100} = 187.5$ | $\frac{12.5}{100} \times 1800 = 225.$ | $\frac{12.5}{80} \times 1200 = 187.5.$ |
| Labour. | 6 × 40 = 240 | 5 × 40 = 200 | 6 × 40. = 240 |
| Profit. | 7072.5 | 6775 | 5572.5 |

(#) **Graphical** :- Graphical Method should be applied only when we have two variable $(x_1, x_2)$.

If we have three (3) variable like $x_1, x_2, x_3$ or more than Its not possible to apply graphical, We should apply only simplex Method.

(31) Page-97 Let $x_1$ be the Qty of Table
$x_2$ " " " of Chair.

$\text{Max}(z) = 80x_1 + 60x_2$

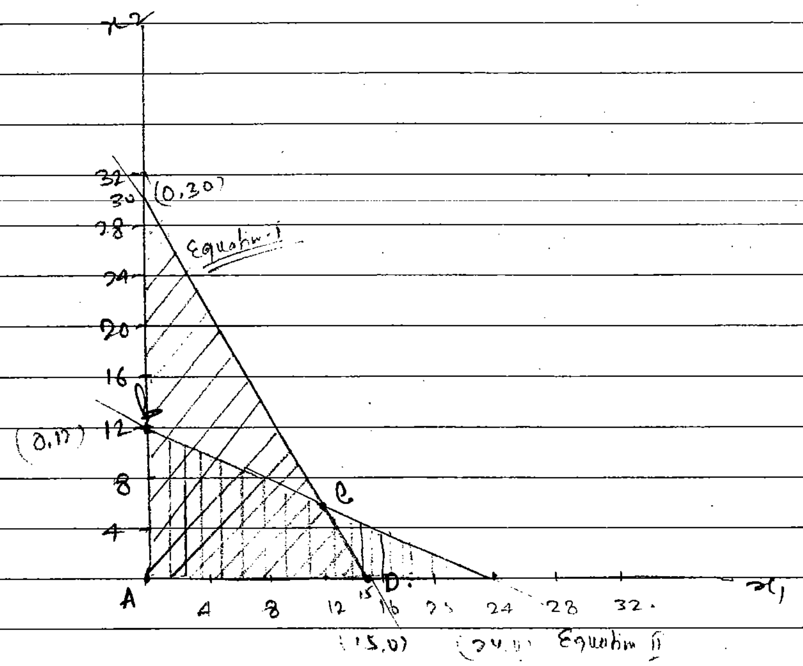
Subject to $4x_1 + 2x_2 \leq 60$
$2x_1 + 4x_2 \leq 48.$
$x_1, x_2 \geq 0.$

Step-1 For first Equation. $4x_1 + 2x_2 = 60$

| | | |
|---|---|---|
| $x_1$ | 0 | 15 |
| $x_2$ | 30 | 0 |

For Second Equation $2x_1 + 4x_2 \leq 48.$

| | | |
|---|---|---|
| $x_1$ | 0 | 24 |
| $x_2$ | 12 | 0 |



**Step-3** Consider any point (0,0) on L.H.S.

on L.H.S. Equation

$4x_1 + 2x_2 \leq 60.$

$4(0) + 2(0) \leq 60$

$0 \leq 60.$ True (Shade towards the point)

Consider (0,0) on L.H.S. of Equation

$2x_1 + 4x_2 \leq 48$

$2(0) + 4(0) \leq 48$

$0 \leq 48$ True.

Shade towards the point.

**Step 4** Identify the Common Area AbCD & extreme Common point.

'C' is the intersection point of equation I & II.

$x_1$

$4x_1 + 2x_2 = 60$ —— ×2

$2x_1 + 4x_2 = 48$ —— ×1

$8x_1 + 4x_2 = 120$

$\underline{\overset{-}{2x_1} \overset{-}{+} \overset{-}{4x_2} = 48}$

$6x_1 = 72$

$x_1 = 12.$

Putting $x_1 = 12$ in equation-I

$4(12) + 2x_2 = 60$

$x_2 = \frac{60-48}{2}$

$x_2 = 6$

$\therefore x_1 = 12$ & $x_2 = 6.$

DATE

## Statement of Comparative Profit

$Z = 80x_1 + 60x_2.$

A (0,0) $Z = 80\times0 + 60\times0 = 0.$

L(0,12) $Z = 80\times0 + 60\times12 = 720$

C (12,6) $= 80\times12 + 60\times6 = 1320$

D. (15,0) $= 80\times15 + 60\times0 = 1200.$

Optimum units be 12 unit of Table
6 unit of chair. Answer

(21)
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$\text{Max}(z) = 3x_1 + 4x_2.$

subject to $2x_1 + 3x_2 \leq 16$

$4x_1 + 2x_2 \leq 16$

$x_1, x_2 \geq 0.$

For Ist Equation

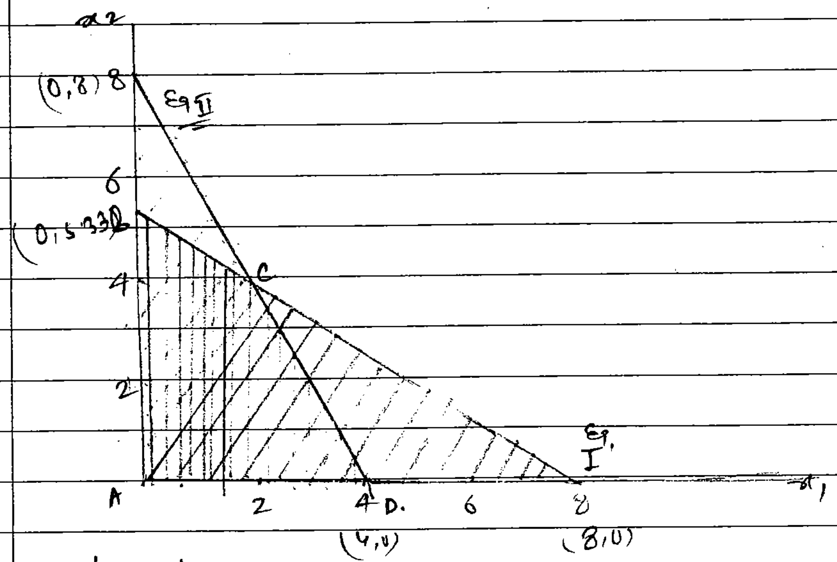
$2x_1 + 3x_2 = 16$

| | | |
|---|---|---|
| $x_1$ | 0 | 8 |
| $x_2$ | 5.33 | 0 |

for IInd Equation

$4x_1 + 2x_2 = 16.$

| | | |
|---|---|---|
| $x_1$ | 0 | 4 |
| $x_2$ | 8 | 0 |



Consider any point at 0,0 on LHS.

4

for eq. I = (0,0) = $2,(0)+3(0) \leq 16$

$0 \leq 16$ True

for equation II. (0,0) $4(0)+2(0) \leq 16$

$0 \leq 16.$ True

Common Area ABCD.

'C' is intersect point.

$2x_1 + 3x_2 \leq 16$

$4x_1 + 2x_2 = 16.$

$x_1 = 2$ & $x_2 = 4$

Statement of Comparative Profit

$Z = 3x_1 + 4x_2$

A (0,0) $Z = 0$

B = (0,5.33) $Z = 3(0)+4(5.33) = 21.33$

C = (2,4) $Z = 3(2)+4(4) = 22$

D = (4,0) . $Z = 3(4)+4(0) = 12.$

$\therefore$ Optimum $x_1 = 2$

$x_2 = 4.$ Ans

$x_1$

AKD- [illegible]

25th class

Always ask in exam    DATE 30 01 2017

## [#] Simplex

Basic purpose of Simplex is to identify the Qty of each product which will produce optimum profit (Maximum).

$Max(Z) = 2x_1 + 10x_2$    befor formulation

Subject to.

$30x_1 + 20x_2 \leq 300$

$5x_1 + 10x_2 \leq 110.$

$x_1, x_2 \geq 0.$

| | $x_1$ – A | $x_2$ – B | |
|---|---|---|---|
| | A($x_1$) | B($x_2$) | |
| Dept I | 30 | 20 | 300 |
| Dept II | 5 | 10 | 110. |

Step-1 Introduce Slack Variable (Spare Capacity) $s_1, s_2$.

Explanation :- Qty of ~~$x_1$, $x_2$~~ $\overset{x_1}{}$, $\overset{x_2}{}$ may be in such a manner so that total Consumption of resources falls below availablity & such differences represented by $s_1$ & $s_2$ (Spare Capacity)

$Max\ Z = 2x_1 + 10x_2 + 0s_1 + 0s_2$

$30x_1 + 20x_2 + s_1 = 300$    [If $x_1$ & $x_2 = 0$] = $s_1 = 300$

$5x_1 + 10x_2 + s_2 = 110.$    [If $x_1$ & $x_2 = 0$] = $s_2 = 110.$

$x_1, x_2, s_1, s_2 \geq 0.$

$x_1 = x_2 = 0$, $s_1 = 300$, $s_2 = 110.$

Basic Variable $\left[\begin{array}{l} x_1\ x_2 \text{ Unit} \rightarrow \text{Decision variable} \\ s_1\ s_2 \text{ Hours} \rightarrow \text{slack Variable} \end{array}\right]$

Step-2 Simplex Table I indicate that we should utilize our resources to produce either $x_1$ or $x_2$.

Step-3 Simplex Table-II idicate that $x_2$ may be reduced & released hours with spare hours may be utilized for $x_1$

Simplex Table - I

| $C_j$ ↓ | Basic variable | Qty | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Ratio (Possible unit) |
|---|---|---|---|---|---|---|---|
| | $S_1$ | 300 | 30 | 20. | 1 | 0 | |
| | $S_2$ | 110 | 5 | 10. | 0 | 1 | |

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Contribution / unit or Hour

## Simplex Table - I

| $C_j$ ↓ → | | | 2 | 10 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| | Basic Variable | Qty | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Ratio (Possible unit) |
| 0 | $S_1$ | 300 | 30 | 20<br>Corresponding Element. | 1 | 0 | $\frac{300}{20} = 15$ |
| Key Row → 0 | $S_2$ | 110 | 5 | 10<br>Key Element | 0 | 1 | $\frac{110}{10} = 11.$<br>⇓<br>Outgoing Variable |
| | | $Z_j$ | $[30 \times 0 + 5 \times 0]$<br>$= 0$ | $[20 \times 0 + 10 \times 0]$<br>$= 0$ | $[1 \times 0 + 0 \times 0]$<br>$= 0$ | $[0 \times 0 + 1 \times 0]$<br>$= 0$ | |
| | (Net) | $C_j - Z_j$ | $2 - 0 = 2$ | $10 - 0 = 10$ | $0 - 0 = 0$ | $0 - 0 = 0.$ | |
| | | | | ⇑<br>Key Column | | | |

Incoming Variable.

## Simplex Table - II

| $C_j$ ↓ → | | | 2 | 10 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| | Basic Variable | Qty | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Ratio (Possible unit) |
| 0 | $S_1$ | 80. | 20 | 0 | 1 | -2. | |
| [x Y Corresponding element] | | $[300 - (11 \times 20)]$ | $[30 - (\frac{5}{10} \times 20)]$ | $[20 - (\frac{10}{10} \times 20)]$ | $[1 - (0 \times 20)]$ | $[0 - (1/10 \times 20)]$ | |
| 10. | ← $x_2$ | 11 | $5/10 = 1/2$ | $10/10 = 1$ | $0/10 = 0$ | $1/10 = \frac{1}{10}.$ | |
| | | $\left[\frac{110}{10}\right]$ → key element | | | | | |
| | | $Z_j$ | $[20 \times 0 + \frac{1}{2} \times 10]$ | $[0 \times 0 + 1 \times 10]$ | $[1 \times 0 + 0 \times 10]$ | $[-2 \times 0 + \frac{1}{10} \times 10]$ | |
| | | | $= 5.$ | $= 10.$ | $= 0$ | $= 1$ | |
| | | $C_j - Z_j =$ | $2 - 5 = -3.$ | $10 - 10 = 0.$ | $0 - 0 = 0.$ | $0 - 1 = -1.$ | |

key element आएगा वो 1 & उसके ऊपर नीचे 0 करना होगा।

DATE

<u>Explanation</u> <u>Step-2</u> ⇒ ① Co-efficient of all variable to be placed in Ist Row of Ist equation of step one & so on.

Always $S_1, S_2$ will be apart of identity Matrix.

If 2 variab  If 3 variable

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

② In order to produce 1 unit of $x_1$, we should Release/divert 30 hours from $s_1$ & 5 hours from $s_2$

Contribution to be lost. $(Z_j) = 30 \times 0 + 5 \times 0 = 0$

In order to produce 1 unit of $x_2$, we should release/divert 20 Hours from $s_1$ & 10 Hours from $s_2$

Contribution to be lost $(Z_j) = 20 \times 0 + 10 \times 0 = 0$

③ Identify higher Positive in $C_j - Z_j$ Row. (Higher Net Contribution) & Called key Coloumn. i.e, Incoming variable i.e, All resources should be utilized for $x_2$.

④ Ratio = $\frac{\text{Qty Coloum}}{\text{key Coloumn}}$

⑤ Select least Positive in Ratio Coloumn

Key Row! Outgoing variable in next table. Incoming Variable replace outgoing variable.

<u>Explanation</u> <u>for Table-II.</u>

① In order to produce now, 1 unit of $x_1$ 20 hours to be released from $s_1$ & 1/2 unit to be sacrificed from $x_2$. $x_1$.

② Last Table :- if all value of $C_j - Z_j$ becomes Negative/Zero.

Optimum Product Mix

$x_1 = 0$

$x_2 = 11$

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$Z = 2x_1 + 10x_2$

$= 2 \times 0 + 10 \times 11 = 110$

Alternative solution

Zj in Table-II

Statement of Contribution

|  | Available | Spare | utilize | Cont/Hr | Total Cost. |
|---|---|---|---|---|---|
| $S_1$ | 300 | 80 | 220 | 0 | 0<br>(0x220) |
| $S_2$ | 110 | 0 | 110 | 1 | 110<br>(1x110) |
|  |  |  |  |  | 110 |

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Max $Z = 8x_1 + 2x_2$

Subject to

$60x_1 + 40x_2 \leq 600$

$10x_1 + 20x_2 \leq 220$

$x_1, x_2 \geq 0.$

| K.N | $x_1$ | $x_2$ |  |
|---|---|---|---|
| Dept I | 60 | 40 | 600 |
| Dept II | 10 | 20. | 220. |

Introducing slack variable $S_1, S_2$.

$Z = 8x_1 + 2x_2 + 0s_1 + 0s_2$

$60x_1 + 40x_2 + s_1 = 600$

$10x_1 + 20x_2 + s_2 = 220.$

$x_1, x_2, s_1, s_2 \geq 0$

$x_1, x_2 = 0 .\quad s_1 = 600 \ \& \ s_2 = 220.$

Simplex Table-I

| $C_j \rightarrow$ |  |  | 8 | 2 | 0 | 0 |  |
|---|---|---|---|---|---|---|---|
| $\downarrow$ | Basic variable | Qty | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Ratio. |
| 0 | $S_1$ | 600 | 60 | 40 | 1 | 0 | $\frac{600}{60} = 10$ |
| 0 | $S_2$ | 220 | 10 | 20 | 0 | 1. | $\frac{220}{10} = 22$ |
|  |  | $Z_j$. | $[60\times0+10\times0] = 0$ | $[40\times0+20\times0] = 0$ | $[1\times0+0\times0] = 0$ | $[0\times0+1\times0] = 0.$ |  |
|  |  | $C_j - Z_j =$ | $8-0=8$ | $2-0=02$ | $0-0=0$ | $0-0=0$ |  |

Key Row
Outgoing variable.

Key Column
Incoming variable

$x_1$ enters & $s_1$ out.

## Simplex Table - II

| | | 8 | 2 | 0 | 0 | |
|---|---|---|---|---|---|---|
| Basic variable | Qty | $x_1$ | $x_2$ | $S_1$ | $S_2$ | Ratio. |
| $x_1$ | 10. <br> $\boxed{\frac{600}{60}}$ | 60/60=1 | 4/6 | 1/60 | 0. | |
| $S_2$ | 120. <br> [220-(10×10)] <br> = 120. <br> [x × corresponding element] | 00 <br> [10×10×1] | 13.33. <br> [20-10×4/6] | -1/6. <br> [0-10×1/60] | 1 <br> [1-10×0] | |
| | $Z_j =$ | [1×8+0×0] <br> = 8 | [8×4/6+0×13.3] <br> = 5.33 | [1/60×8+-1/60×0] <br> 8/60 | [8×0+0×1] <br> = 0 | |
| | $C_j - Z_j =$ | 0 | -3.33 | -8/60 | 0. | |

$x_1 = 10 \quad x_2 = 0$

$Z = 8x_1 + 2x_2$

$= 8(10) + 2(0) = 80.$

### Alternative II

### Statement of Contribution

$Z_j$ in Table - II

| | Available | Spare | ~~Contribution~~ Utilize | Contribution | Total Contribution |
|---|---|---|---|---|---|
| S1 | 600 | — | 600 | 8/60 | 80 |
| S2 | 220 | 120 | 100 | 0 | 0 |
| | | | | | 80 |

$x_1$.

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Let $x_1$, $x_2$ & $x_3$ be the Qty of Product $x_1$, $x_2$ & $x_3$ respectively.

$Z = 8x_1 + 6x_2 + 5x_3.$

Subject to $4x_1 + 4x_2 + 6x_3 \leq 384$

$3x_1 + 2x_2 + 4x_3 \leq 288.$

$x_3 = 30.$

$x_1, x_2, x_3 \geqslant 0.$

$Z = 8x_1 + 6x_2 + 5x_3.$ [384 − (6×30)]

Revised $4x_1 + 4x_2 \leq 204 \rightarrow$ [384 − (6×30)]

$3x_1 + 2x_2 \leq 168. \rightarrow$ [288 − (4×30)]

Introduce slack variable.

$Z = 8x_1 + 6x_2 + 5x_3 + 0s_1 + 0s_2.$

~~4384~~

$4x_1 + 4x_2 + s_1 = 204$

$3x_1 + 2x_2 + s_2 = 168.$

$x_1 = x_2 = 0$ $s_1 = 204$ & $s_2 = 168.$

Simplex Table - I

| | | | 8 | 6 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| Cj ↓ | Basic variable | Qty | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Ratio. |
| 0 | $s_1$ | 204 | 4 | 4 | 1 | 0 | $\frac{204}{4} = 51$ |
| 0 | $s_2$ | 168 | 3 | 2 | 0 | 1. | $\frac{168}{3} = 56.$ |
| | | Zj. | [4×0+3×0]=0 | [4×0+2×0]=0 | [1×0+0×0]=0 | [0×0+1×0]=0. | |
| | | Cj−Zj. = | 8−0 = 8. | 6−0 = 6 | 0−0 = 0 | 0−0 = 0. | |

Simplex Table II.

| | | | 8 | 6 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| Cj | Basic variable | Qty | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Ratio. |
| 8 | $x_1$ | $\frac{204}{4} = 51$ | 1 | 1 | 1/4 | 0 | |
| 0. | $s_2$ | [168−(51×3)]<br>= 15 | 3−3×1<br>0. | 2−3×1<br>= −1 | [0−3×1/4]<br>= −3/4 | [1−3×0] = 1 | |
| | | Zj | [1×8+0×0]=8 | [1×8+−1×0]<br>= 8 | [1/4×8−(−3/4×0)]<br>= 2 | [(0×8)+(1×0)]<br>= 0. | |
| | | Cj−Zj = | 8−8 = 0 | 6−8 = −2 | 0−2 = −2 | 0−0 = 0. | |

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$x_1 = 51$
$x_2 = 0.$

$Z_1 = 8x_1 + 6x_2 + 5x_3$
$= 8 \times 51 + 6 \times 0 + 5 \times 30$
$= 408 + 0 + 150$
$= 558.$

Statement of Contribution

$Z_j$ in the final Table i.e Table. II.

| | Available | Spare | Utilize | Cont/Hr | Total Contribution |
|---|---|---|---|---|---|
| $S_1$ | 204 | — | 204 | 2 | 408. |
| $S_2$ | 168 | 15 | 153 | 0 | 0 |

$$\begin{array}{r} 408 \\ + \ 150 \\ \hline \end{array}$$

Total Contribution = $\underline{\underline{558}}$

$x_1.$

26th class

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eg) Max $Z = 40x_1 + 5x_2$

**Subject to**

$120x_1 + 80x_2 \leq 1200$

$20x_1 + 40x_2 \leq 440.$

$x_1, x_2 \geqslant 0.$

| **W.N** | $x_1$ | $x_2$ | |
|---|---|---|---|
| Department 1 | 120 | 80 | 1200 |
| Department 2 | 20 | 40 | 440. |

Introducing slack variable $s_1$ & $s_2$.

1

$Z = 40x_1 + 5x_2 + 0s_1 + 0s_2.$

$120x_1 + 80x_2 + s_1 = 1200$

$20x_1 + 40x_2 + s_2 = 440.$

$x_1 = x_2 = 0$ ~~$s_1, s_2 \geqslant 0$~~. $s_1 = 1200$ & $s_2 = 440.$

Simplex Table - I

| | | | 40 | 5 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| $C_j$ | Basic Variable. | Qty | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Ratio. |
| 0 | $s_1$ | 1200 | 120 | 80 | 1 | 0 | $\frac{1200}{120} = 10$ |
| 0 | $s_2$ | 440 | 20 | 40 | 0 | 1. | $\frac{440}{20} = 22.$ |
| | | $Z_j =$ | 0<br>$[120\times0 + 20\times0]$ | 0<br>$[(80\times0)+(40\times0)]$ | 0<br>$[(1\times0)+(0\times0)]$ | 0<br>$[(0\times0)+(0\times1)]$ | |
| | | $C_j - Z_j.$ | 40 | 5 | 0 | 0 | |

$x_1$ enter & $s_1$ out.

Simplex Table - II.

| $C_j$ | Basic Variable | Qty | $x_1$ | $x_2$ | $s_1$ | $s_2$ | Ratio. |
|---|---|---|---|---|---|---|---|
| 40 | $x_1$ | 10<br>$\left[\frac{1200}{120}\right]$ | 1<br>$\left[\frac{120}{120}\right]$ | $2/3$<br>$\left[\frac{80}{120}\right]$ | $1/120$<br>$\left[\frac{1}{120}\right]$ | 0<br>$\left[\frac{0}{120}\right]$ | |
| 0 | $s_2$ | 240<br>$[440-(20\times10)]$ | 0<br>$[20-(1\times20)]$ | 26.67<br>$[40-(\frac{2}{3}\times20)]$ | $-1/6$<br>$[0-(\frac{1}{120}\times20)]$ | 1.<br>$[1-(0\times20)]$ | |
| | | $Z_j =$ | 40<br>$[(1\times40)+(0\times0)]$ | 26.67<br>$[2/3\times40 + 26.67\times0]$ | 0.33<br>$[1/120\times40 + (-1/6\times0)]$ | 0.<br>$[(0\times40 + (1\times0)]$ | |
| | | $C_j - Z_j =$ | 0 | $-21.67$ | $-0.33$ | 0 | |

$x_1 = 10$

$x_2 = 0$

$Z = 40x_1 + 5x_2$

$= 40 \times 10 + 5 \times 0$

$= 400$ Rs

## Statement of Contribution

| | Available | Spare | Utilize | Cont/Hr. | Total Contribution |
|---|---|---|---|---|---|
| $S_1$ | 1200 | — | 1200 | 0.33 | 400 |
| $S_2$ | 440. | 240 | 200 | 0 | 0 |
| | | | | | 400 Rs |

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| | | | 50 | 40 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|
| $C_j \downarrow$ | Basic variable $\rightarrow$ | Qty | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $S_3$. | Ratio |
| 0 | $S_1$ | 150 | 3 | 5 | 1 | 0 | 0 | 50 |
| | | | | | | | | $\left[\frac{150}{3}\right]$ |
| 0 | $S_2$ | 20 | 0 | 1 | 0 | 1 | 0 | $\infty$ |
| | | | | | | | | $\left[\frac{20}{0}\right]$ |
| 0 | $S_3$ | 296 | 8 . | 5 | 0 | 0 | 1 | 37. |
| | | $Z_j=$ | 0 | 0 | 0 | 0 | | $\left[\frac{296}{8}\right]$ |
| | | $C_j - Z_j$. | 50 | 40 | 0 | 0. | | |

$x_1$

$\leq \Rightarrow$ in equation
$= \Rightarrow$ equation.

outgoing $\Rightarrow$ Space Capacity
Incoming $\Rightarrow$ Unit $(x_1, x_2 \& x_3)$

(i) $Z_{(max)} = 50x_1 + 40x_2$

Subject to $3x_1 + 5x_2 \leq 150$
$0x_1 + x_2 \leq 20$
$8x_1 + 5x_2 \leq 296.$
$x_1, x_2 \geq 0.$

(ii) Net Contribution per unit from $x_1$ will be 50/unit
Net Contribution per unit from $x_2$ will be 40/unit.

∴ If we select/bring 1 unit of $x_1$ the we will have to Sacrifice ₹40/unit from 1 unit of $x_2$ which is the benefit from Next best alternative i.e, Opportunity Cost of 1 unit of $x_1$.

(iii)

| Basic Variable | Present Hour | Utilize | Balance |
|---|---|---|---|
| $S_1$ | 150 | 12<br>(3x4). | 138 |
| $S_2$ | 20 | 0<br>[4x0] | 20 |
| $S_3$ | 296. | 32<br>(8x4) | 264 |

(iv) $Z = 50x_1 + 40x_2$
$Z = 50x_1 + 40 \times 4$
$Z = 50x_1 + 160.$

(V) Incoming Variable will be $x_1$ due to higher positive in $C_j - Z_j$ & Qty will be 37 due to least positive in Ratio (Possible unit) Coloumn.

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| | | | 4 | 5 | 0 | 0 |
|---|---|---|---|---|---|---|
| $C_j$ | Basic Variable | Qty | $x_1$ | $x_2$ | $S_1$ | $S_2$ |
| 5 | $x_2$ | 10 | 1 | 1 | 1 | 0 |
| 0 | $S_2$ | 3 | 1 | 0 | -1 | 1. |
| | | $Z_j$ = | 5 | 5 | 5 | 0 |
| | | $C_j \cdot Z_j$ = | -1 | 0 | -5 | 0. |

(i) Above solution is optimal because all value of $C_j - Z_j$ are Negative/zero.

(ii) $S_1$ is fully utilized, according to this solution because $S_1$ has no value in above table. (i.e final table)

(iii) If a Customer would like to have 1 unit of $x_1$ then we should charge ₹ 1 extra from Normal price for each unit of $x_1$ because we will have to suffer a loss of ₹ 1 in producing one unit of $x_1$

(#) Price (New) = Normal Price + ₹ 1/- (Extra)

(iv) Contribution/Hr from $S_1$ = 5

At Present 100% Utilize for $S_1$ Hours.

Now we can say if 2 Hours would be shut Down then loss Comes to = ₹ 5/Hrs X 2 Hrs = ₹ 10.

(V) According to this solution, we are producing 10 unit of $x$ & 0 unit of $x_1$.

~~$x_1$ = 10~~ $x_1 = 0$

$x_2 = 10$.

$Z = 4x_1 + 5x_2$

$= 4 \times (0) + 5 \times 10 = 50$.

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400, 400, 400

(45)
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(i) Any solution said to be feasible, when all values of basic variable becomes positive value. so that Qty could occur in positive term.

(ii) Table

| | | | 6 | 4 | 10 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|---|
| Cj ↓ | Basic Variable | Qty | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | Ratio. |
| 0 | $S_1$ | 400 | 0 | $4/3$ | 0 | 1 | $-1/3$ | 0 | $\frac{400}{4/3} = \frac{400 \times 3}{4} = 300$ |
| 6 | $x_1$ | 400 | 01 | $2/3$ | 2 | 0 | $1/3$ | 0 | $\frac{400}{2/3} = \frac{400 \times 3}{2} = 600$ |
| 0 | $S_3$ | 400 | 0 | $5/3$ | 0 | 0 | $-2/3$ | 1. | $\frac{400}{5/3} = \frac{400 \times 3}{5} = 240$ |
| | | $Z_j$ | 6 | 4 | 12 | 0 | 02 | 0 | |
| | | $C_j - Z_j$ | 0 | 0 | -2 | 0 | -2 | 0. | |

$S_3$ outgoing variable & $x_2$ Incoming variable.

$-\frac{1}{3} - (\frac{-2}{5} \times \frac{4}{3})$

$-\frac{1}{3} - \frac{-8}{15}$ -5+8

| | | | 6 | 4 | 10 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| Cj ↓ | Basic variable | Qty | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ |
| 0 | $S_1$ | 80 | 0 | 0 | 0 | 01 | $3/15$ | $-4/5$ |
| | | | $[0 - 0 \times 4/3)$ | $[4/3 - 1 \times 4/3]$ | $0 - 0 \times 4/3$ | $(1 - 0 \times 4/3)$ | $-1/3 - (-2/5 \times 4/3]$ | $0 - (3/5 \times 4/3)$ |
| 6 | $x_1$ | 240 | $1 - 0 \times 2/3$ | $2/3 - 1 \times 2/3$ | $2 - 0 \times 2/3$ | $0 - 0 \times 2/3$ | $1/3 - (-2/5 \times 2/3)$ | $0 - 3/5 \times 2/3$ |
| | | $400 - [240 \times 2/3]$ | = 1 | 0 | 2 | 0 | $3/5$ | $-2/5$ |
| 4 | $x_2$ | 240 | 0 | 1 | 0 | 0 | $-2/5$ | $3/5$ |
| | | | | | | | $[2/3 \div 5/3]$ | $[1 \div 5/3]$ |

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(iii) # Shadow Price : It should be analysed in respect of spare Capacity (Slack variable) thru Qty.

Contribution Per Hour (at Present) in system from $s_1 = 0$

" " " " " " " $s_2 = 2$

" " " " " " " $s_2 = 0$

if we release 1 Hour of $s_1$ from system, Contribution loss = 0

" " " 1 " " $s_2$ " " " " = 2

" " " 1 " " $s_3$ " " " " = 0.

} i.e Shadow Price

∴ Above Contribution loss is our shadow Price.

(ix) New Price = Normal Price + ₹ 2 per unit.

(v) The given table is our ~~2nd~~ Second (IInd) Table because only $x_1$ enter in Basic Variable as $x_1$, is our Incoming Variable Hence this has been arrived by dividing a value (key element), we know that coefficient of $s_1, s_2, s_3$ in IInd Row had been 0, 1, 0 in First Table & in IInd Table $s_2$ is 1/3 means all value have been divided by 3 to prepare second (II) table. Hence in order to get back we should Multiply by '3'. i.e, Co-efficient of $s_2$ in I table : 1200 $\underline{\underline{3}}$ $\underline{\underline{2}}$ $\underline{\underline{6}}$ $\underline{\underline{010}}$

in equation $3x_1 + 2x_2 + 6x_3 \leq 1200$.

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(47)
~~page-105~~

|  |  |  |  |  |  | KA-07 | KB-27 | KC-49 |  |
|---|---|---|---|---|---|---|---|---|---|
|  |  |  | 30 | 40 | 20 | 0 | 0 | 0 |  |
| $C_j$ ↓ |  |  | $x_1$ | $x_2$ | Z | $S_1$ | $S_2$ | $S_3$ | Ratio |
| 30 | $x$ | 250 | 1 | 0 | $-\frac{26}{16}$ | $10/16$ | $-\frac{12}{16}$ | 0 |  |
| 1 |  |  |  |  |  |  |  |  |  |
| 40 | y | 652 | 0 | 1 | $\frac{31}{16}$ | $-7/16$ | $10/16$ | 0 |  |
| 0 | $S_3$ | 125 | 0 | 0 | $11/16$. | $-3/16$ | $1/8$ | 1 |  |
|  |  | $Z_j$= | 30 | 40 | 28.75 | 1.25 | 2.5 | 0 |  |
|  |  |  |  |  | $\left[30\times-\frac{26}{16}+\frac{31}{16}\times40+0\right]$ | $\left[30\times\frac{10}{16}+40\times-7/16+0\right]$ | $\left[30\times-\frac{12}{16}+40\times\frac{10}{16}+0\right]$ | $\left[0\times30+0\times40+1\times0\right]$ |  |
|  |  | $C_j-Z_j$. | 0 | 0 | $-35/4$ | $-5/4$ | $-5/2$ | 0 |  |
|  |  |  |  |  | i.e −8.75 | −1.25 | −2.5 |  |  |

(i) Above Solution is an optimum because all value of $C_j - Z_j$ are Negative and 0.

(ii) If any Non Basic Variable have Zero value in $C_j - Z_j$ Row for final Table, we can say alternate solution exist without changing the total Profit. Here also Non basic variable i.e ⓩ $Z_1$, $S_1$, $S_2$ all have negative Value Hence no Alternate exist

(iii) Basic Variables in optimum solution have Positive Value (> 0) Hence we can say Solution is feasible. (i.e fullfill our Condition)

(IV) When two (2) value becomes same in Ratio Coloumn. (in any table of simplex) then we should consider any one as arbitrary & in next table any one basic variable becomes Zero (0), This situation is called Degenaracy in simplex. In above Degenaracy not exist.

(v) <u>Objective function</u>

$Z(\text{Max}) = 30x + 40y + 20z$

(vi) $x = 250$

$y = 652$

$z = 0$

Profit $Z = 30 \times 250 + 40 \times 652 + 20 \times 0$

$= 7500 + 26080$

$= ₹\ 33580$ Ans

(vii) KA 07 & KB 27 are being utilized as full Capacity.

(viii) We Can Pay $\overset{1.25}{5/4}$ per hour for one extra hour of $S_1$ because this One we are earning from $S_1$ in the system.

We Can pay $\overset{2.5}{5/2}$ per hour for one extra hour of $S_2$ because this One we are earning from $S_2$ in the system.

& for $S_3 = 0$ | Hr.

(ix) If Company wishes to expand the production Capacity, then we should increase the Capacity having higher Contribution/Hr.

i.e $S_2 = 2.5$ | Hr.

(x) Loss = 16 Hr × ₹ 2.5 | Hr

Loss = ₹40

(xi) Profit of Z = Normal Price + 35/4

= Normal Price + 8.75 Ans

(xii) If a new Product is proposed to be introduced, we will have to suffer loss = $S_1$ (KA-07) = 4 Hr × 5/4 = 5

$S_2$ (KB-27) : 2 Hr × 5/2 = 5

$S_3$ (KC-47) = 4 Hr × 0 = $\underline{0}$

$\underline{10}$

Profit from New Product 12

∴ Excess Profit = 2 (12-10)

better to Produce.

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Each Production Run ⇒ Two types of Process.

Older Process & New Process.

Let $x_1$ be the No. of times to be run for old process

Let $x_2$ be the No. of time to be run for New Process.

Max $Z = 60A + 90B$.

$= 60(9x_1 + 5x_2) + 90(7x_1 + 9x_2)$

Subject to.

$9x_1 + 5x_2 \geq 500$ (Market Constraint)

$7x_1 + 9x_2 \geq 300$. ( " )

$5x_1 + 3x_2 \leq 1500$ (Consumption Constraint)

$7x_1 + 9x_2 \leq 1900$ ( " )

$2x_1 + 4x_2 \leq 1000$ ( " )

$x_1 . x_2 \geq 0$.

| Rough | | | | Produce | |
|---|---|---|---|---|---|
| | Q | P | R | A | B |
| Older Process | 5 | 7 | 2 — | 9 | 7 |
| New Process. | 3 | 9 | 4 — | 5 | 9 |
| Available | 1500 | 1900 | 1000 | 500<br>atleast | 300<br>atleast. |

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(7) Page-85 **Foundary Rept**

Model x Either 160 unit (OR) Model Y 320 unit In Available Resources.

It mean Consumption of time for 'Y' will be half (1/2) of 'x'.

Let the total Resources be = 320 [L.C.M of x & Y] कुछ भी Assume कर लेंगे।

[∴ Model x require 2 Hour/unit & Y require 1 Hour/unit].

Qty of x = $x_1$

Qty of Y = $x_2$

$2x_1 + x_2 \leq 320.$

**Machine Shop**

Model x Either 200 unit (OR) Model Y 400 Unit In available Resources.

Qty of x = $x_1$

Qty of y = $x_2$

Let the Resource be = 400 Hours. [L.C.M of x & Y)

[∴ Model x Requires 2 Hours/unit & Y requires 1 Hour/unit).

$2x_1 + x_2 \leq 400.$

## Self Note If value of Total Resources are not given, we should presume (assume) Resource = [L.C.M of Both or 1] [or Any value which could be divided from Both factor]

Fair Let $x_1, x_2$ be the Qty of Product x & Y respectively

Max $Z = 120x_1 + 90x_2$ [$Z = [120x_1 + 90x_2]$]

Subject to $2x_1 + x_2 \leq 320$ [Production Constraint]

$2x_1 + x_2 \leq 400$ [ " ]

$x_1 \leq 140.$ (Material Constraint)

classmate $x_1, x_2 \leq 0$

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(13) ~~Page-88~~ Let $x_1$ $x_2$ be the Qty of Type A & Type B unit respectively.

Max $Z = 90x_1 + 60x_2$.

Subject to

$2x_1 + x_2 \leq 800$

$x_1 \leq 175$

$x_2 \leq 350$.

$3x_1 + x_2 \leq 600$

$x_1, x_2 \geq 0$.

(15) ~~Page-89~~ Working

| | A | b | C |
|---|---|---|---|
| Hr Per unit | 6 | 3 | 2. |

Total Resource = 3000 X 6 = 18000 Hours.

$6x_1 + 3x_2 + 2x_3 \leq 18000$.

fair Let $x_1$ $x_2$ $x_3$ be the Qty of Product A, B & C respectively.

Max $Z = 50x_1 + 50x_2 + 80x_3$.

Subject to $6x_1 + 3x_2 + 2x_3 \leq 18000$.

$3x_1 + 4x_2 + 5x_3 \leq 5000$ ( Raw Material Constraint)

$5x_1 + 3x_2 + 5x_3 \leq 7500$ ( " )

$x_1 \geq 600$ ( Market Constraint)

$x_2 \geq 650$ ( " " )

$x_3 \geq 500$. ( " " ).

Ratio = 2: 3: 4.

$\therefore$ $x_1 : x_2 : x_3 :: 2:3:4$.

$$\left[\frac{x_1}{x_2} = \frac{2}{3}\right]$$

$$\left[\frac{x_2}{x_3} = \frac{3}{4}\right]$$

$$\boxed{\frac{x_1}{x_3} = \frac{2}{4}.}$$

$3x_1 - 2x_2 = 0$

$4x_2 - 3x_3 = 0$

$4x_1 - 2x_3 = 0.$

$x_1, x_2, x_3 \geq 0.$

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| | | | | |
|---|---|---|---|---|
| Grade | | A 8. | b 6.5 | c. 5.5. |
| Crude oil | | | | |
| 9.5 | d | 5 ltr | 2.5 ltr | |
| 5.5 | e | 2.5 ltr. | 5 ltr | |
| 6.5. | f. | (1/1) 2.5 ltr. | 2.5 ltr. (1/f) | |
| | Assume ⇒ | 10 ltr | 10 ltr | 10 ltr |

d e f

Let $x_1, x_2, x_3$ be the Qty of d, e, f for grade A.

$y_1, y_2, y_3$ be the Qty of d, e, f for grade b.

$z_1, z_2, z_3$ be the Qty of d, e, f for grade c.

Profit

Max Z = Profit from A + Profit from b + Profit from C.

$$= \left[8(x_1+x_2+x_3) - (9.5x_1 + 5.5x_2 + 6.5x_3)\right]$$
$$+$$
$$\left[6.5(y_1+y_2+y_3) - (9.5y_1 + 5.5y_2 + 6.5y_3)\right]$$
$$+$$
$$\left[5.5(z_1+z_2+z_3) - (9.5z_1 + 5.5z_2 + 6.5z_3)\right]$$

Subject to

$x_1 \geq 50\%$ of $(x_1+x_2+x_3)$ (For Grade A Constraint)

$x_2 \leq 25\%$ of $(x_1+x_2+x_3)$. ( " " " )

$y_1 \geq 25\%$ of $(y_1+y_2+y_3)$ (For Grade b Constraint)

$y_2 \leq 50\%$ of $(y_1+y_2+y_3)$ ( " " " " )

$x_1 + y_1 + z_1 \leq 500$ (Available Constraint)

$x_2 + y_2 + z_2 \leq 500$ ( " " )

$x_3 + y_3 + z_3 \leq 300.$ ( " " )

$x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3 \geq 0.$ (Non Negative Constraint).

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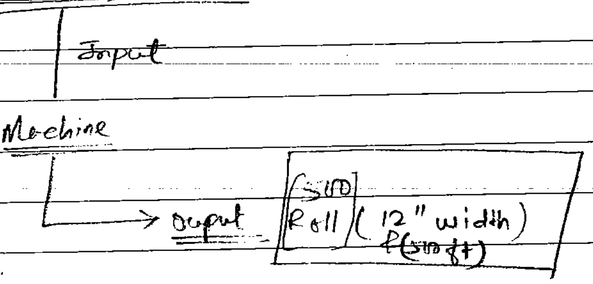
Working Note

Production Process



Let $x_1$ be the No. of Time to be operated for cutting Alternative I

let $x_2$ be the No of Time to be operated for Cutting Alternative-II

let $x_3$ " " " " " " " " " " Alternative III

let $x_4$ " " " " " " " " " " Alternative IV

let $x_5$ " " " " " " " " " " Alternative V

let $x_6$. " " " " " " " " " " Alternative VI.

Minimum Z = $0x_1 + 0x_2 + 1x_3 + 1x_4 + 1x_5 + 1x_6$
(wastage)

Subject to

$6x_1 + 0x_2 + 1x_3 + 0x_4 + 0x_5 + 4x_6 \leq 3000$ (1" Constraint)

$3x_1 + 3x_2 + 1x_3 + 0x_4 + 4x_5 + 2x_6 \leq 2000$ (2" Constraint)

$0x_1 + 2x_2 + 1x_3 + 2x_4 + 1x_5 + 1x_6 \leq 1500$ (3" Constraint)

$0x_1 + 0x_2 + 1x_3 + 1x_4 + 0x_5 + 0x_6 \leq 1000$ (5" Constraint)

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 500$ (Non Negative Constraint)

#) **Dual Concept** : Process is a Mathematical approach which is to be used to Convert either

Maximization Problem to Maximization

(OR)

Minimum Problem to Maximization problem

eg We have a problem of Max.

| | | | |
|---|---|---|---|
| Two products | A | b. | |
| Cost/unit. (contribution) | 100 | 200. | |
| **Dept** | | | |
| x | 4 Hr | 6 Hr | 10000 Hrs |
| Y | 5 Hr | 7 Hr | 20000 Hrs. |

Let $x_1$ $x_2$ be the Qty of A & b. respectively.

$Z(\text{Max}) = 100x_1 + 200x_2.$

Subject to

$4x_1 + 6x_2 \leq 10000$ (Dept - I)

$5x_1 + 7x_2 \leq 20000.$ (Dept - II)

$x_1, x_2 \geqslant 0.$

Now He wants to disContinue his manufacturing process & would like to lease out the space of Dept.

Now we should fix the rental Income (Rate/Hour/Per Unit of Resources) so that Rental Income would not become less than Present Income.

Let $Y_1$ be the Per Hour/Per unit of Resource of Dept-I

$Y_2$ be the Per Hour/Per unit of Resource of Dept-II.

$Z = 10000y_1 + 20000y_2$

subject to

[Explanation : Rate should be in such a manner so that Revenue in respect of Consumption of Resource/unit of 'A' should not be less than present Contribution from 1 unit of 'A']

'A' $4y_1 + 5y_2 \geq 100$

'B' $6y_1 + 7y_2 \geq 200.$

$y_1 , y_2 \geq 0.$

(eg) **Product**

$$\left[\begin{array}{ll} \text{Max } z = 100x_1 + 300x_2 + 400x_3. & \\ \text{Subject to} \quad 4x_1 + 5x_2 + 6x_3 \leq 10000 & = \text{Dept-I} \\ \quad 2x_1 + 3x_2 + 4x_3 \leq 40000 & = \text{Dept-II}. \\ \quad x_1 , x_2 , x_3 \geq 0. & \end{array}\right] \Rightarrow \text{PRIMAL i.e original}$$

**Dual**

Let $Y_1$ be the 'Rate per unit of Resource for Dept-I
Let $Y_2$ " " " " " " " Dept-II.

$Z = 10000 Y_1 + 40000 Y_2$

Subject to

$4Y_1 + 2Y_2 \geq 100$

$5Y_2 + 3Y_2 \geq 300$

$6Y_1 + 4Y_2 \geq 400.$

$Y_2 , Y_2 \geq 0.$

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**Dual**

Let $Y_1 , Y_2$ & $Y_3$ be the Rate Per unit of Resource of Dept-1, Constraint ⇒ Dept II & Dept-III respectively.

Constraint, Constraint

Mini $Z = 3Y_1 + 4Y_2 + 1Y_3$

Subject to

$1Y_1 + 2Y_2 + 3Y_3 \geq 90$

$2Y_1 + 1Y_2 + 1Y_3 \geq 60$

$3Y_3 + 1Y_2 + 2Y_3 \geq 80$

$Y_1 , Y_2 , Y_3 \geq 0.$

$1 < 2$ (After Multiplying by(-1))

$-1 > -2$ proved

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$\text{Max } z = 90x_1 + 60x_2 + 80x_3$

eg

Subject to $1x_1 + 2x_2 + 3x_3 \leq 3$

$2x_1 + 1x_2 + 1x_3 \leq 4$

$(-1)3x_1 + 1x_2 + 2x_3 \geq 1.(-1)$

(both side (-1) से Multiply करने से sign (≤, ≥) change हो जाता है।

Primal

$\text{Max } z = 90x_1 + 60x_2 + 80x_3$

Subject to $x_1 + 2x_2 + 3x_3 \leq 3$

$2x_1 + 1x_2 + 1x_3 \leq 4$

$-3x_1 - 1x_2 - 2x_3 \leq -1.$

$x_1, x_2, x_3 \geq 0.$

Ⓗ Point to be Remember while applying Dual Process ONLY.

1. If we have Maximization Problem which is required to Convert into minimization then All Constraint should be ≤ type. If any Constraint is ≥ type then first of all that Constraint should be Converted into ≤ type by multiplying (-1) both side & then apply Dual Process.

2. If we have Minimization Problem which is required to Convert into Maximization, then All Constraint should be ≥ type. If any Constraint is ≤ type then first of all that Constraint should be Converted into ≥ type by multiplying (-1) both side & then apply Dual Process

3. eg $\text{Max } Z = 100x_1 + 200x_2.$

Subject to

$4x_1 + 5x_2 \leq 10000$

$6x_1 + 7x_2 = 20000$

$x_1, x_2 \geq 0.$

It can be written as·

$6x_1 + 7x_2 = 20000.$

Ist $6x_1 + 7x_2 \geq 20000$

or

$6x_1 + 7x_2 \leq 25000.$

After Multiplying by -1

**PRIMAL**

$Z\,(\text{Max}) = 150x_1 + 200x_2$

**Subject to**

$4x_1 + 5x_2 \leq 10000$

$6x_1 + 7x_2 \leq 20000.$

$-6x_1 - 7x_2 \leq -20000.$

$x_1, x_2 \geq 0.$

**Dual**

$Z\,(\text{Mini}) = 10000y_1 + 20000y_2 - 20000y_3$

**Subject to** $4y_1 + 6y_2 - 6y_3 \geq 150.$

$5y_1 + 7y_2 - 7y_3 \geq 200.$

$y_1, y_2, y_3 \geq 0.$

If we have any Constraint = type, then first of all we should express such Constraint into $\leq$ type & $\geq$ type (both)

a) If we have originally maximization Problem, then $\leq$ type retain as it is & Contribution ~~#~~unit $\geq$ into $\leq$ by Multiply (-1) both sided.
& then apply Dual Process.

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## Statement of Cost & Qty Detail.

| S.P | Product ↓ | Raw material<br>Qty ((4) | A<br>100kg | B<br>180kg | C<br>120kg. | Forest Balance (other) | Total. |
|---|---|---|---|---|---|---|---|
| 32·60 | X | $x_1$ kg | 5% | 10% | 5%. | 80% | 100 kg |
| 34·8 | Y | $x_2$ kg | 5% | ~~5%~~ | 10% | 80% | 100 kg |
| 36. | Z | $x_3$ kg. | 20% | 5% | 10%. | 65% | |
| Cost | | | 64/kg | 16/kg | 40/kg | 16/kg. | |

Let Three(3) Product be called x, Y, Z
$x_1, x_2, x_3$ be the Qty of x, Y, & Z respectively.

Cost of 1 unit (kg) x: $\frac{5\text{kg} \times ₹64 + 10\text{kg} \times ₹16 + 5\text{kg} \times 40 + 80\text{kg} \times ₹16}{100\text{ kg}} = ₹19.60/\text{kg}$

Cost of 1 unit (kg) Y = $\frac{5\text{kg} \times ₹64 + 5\text{kg} \times 16 + 10\text{kg} \times 40 + 80\text{kg} \times 16}{100\text{ kg}} = ₹20.8/\text{kg}$

Cost of 1 unit (kg) Z = $\frac{20\text{kg} \times 64 + 5\text{kg} \times 16 + 10\text{kg} \times 40 + 65\text{kg} \times 16 =}{100\text{ kg}} = ₹28/\text{kg}$

Profit function

$$Z(\text{Max}) = (32.6 - 19.60)x_1 + (34.8 - 20.8)_{x_2} + (36 - 28)x_3$$

Subject to

$$\frac{5}{100}x_1 + \frac{5}{100}x_2 + \frac{20}{100}x_3 \leq 100.$$

$$\frac{10}{100}x_1 + \frac{5}{100}x_2 + \frac{5}{100}x_3 \leq 180.$$

$$\frac{5}{100}x_1 + \frac{10}{100}x_2 + \frac{10}{100}x_3 \leq 120$$

$$x_1, x_2, x_3 \geq 0.$$

$x_1 \leq 30.$

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Let $x_1$ be the Qty of Product 1

$x_2$ ———————— 2

$x$ ———————— Loan Amount

Profit (Max) $Z = (14-10)x_1 + (11-8)x_2 - \frac{20\%}{12} \times 3x_3$

Subject to
$$0.5x_1 + 0.3x_2 \leq 500$$
$$0.3x_1 + 0.4x_2 \leq 400$$
$$0.2x_1 + 0.1x_2 \leq 200.$$
$$x_3 \leq 200000.$$

Working Note

Surplus Cash in Hand after Production

$= 300000 - (10x_1 + 8x_2) + x_3$

Account Receivable (Assuming Credit Sale)

$= 14x_1 + 11x_2.$

If here asked
only surplus in
Cash then We
deduct Interest
from surplus.

nterest
ost not
onsidered
n calculating
surplus
because
t is a part
P/L statement
here ask
surplus after
Production.

Subject to
$$\frac{300000 + x_3 - (10x_1 + 8x_2) + 14x_1 + 11x_2}{x_3 + 0.05x_3} \geq 1.$$

Working Note

Needed Fund ⟶ Cash Available.

$10x_1 + 8x_2 + 0.05x_3 \leq 300000 + x_3$

Subject to $10x_1 + 8x_2 + 0.05x_3 \leq 300000 + x_3.$

## (#) Big (M) Method.

If any Constraint $\geq$ occur then We should apply Big (M) Method ($\geq$ may occur either in Maximization or Minimization Problem).

(eg) Max $Z = 62000x_1 + 124000x_2$

Subject to $3x_1 + 4x_2 \leq 260.$

$x_1 \geq 2$

$x_2 \geq 3$

$x_1, x_2 \geq 0.$

Step-1 = Introducing Slack Variable & surplus Variable.

$$\boxed{\begin{aligned}Z = 62000x + 1{,}24000x_2 \\ +0s_1 + 0s_2 - MA_1 - MA_2\end{aligned}}$$

$3x_1 + 4x_2 + s_1 = 260.$

$x_1 - S_2 + A_1 = 2.$

$X_2 - S_3 + A_2 = 3$

In First Table, we should Consider Artificial Variable & Slack Variable

p-T-O

Explanation

When $x_1, x_2 = 0$

$S_1 = 260$

$-S_2 = +2$

$S_2 = -2$

(#) We have to subtract $S_2$ in equation 2 to equate but by doing so Condition do not hold good. In order to maintain our Condition (all variable $\geq 0$), we should Add a Variable (Self.): Artificial Variable.

(#) Still Problem Could not be solved when $S_2$ become higher $A_1$ become lower

To solve this situation / Problem, we should take Co-efficient 'A' Very-Very high Called 'M'. 'M' is very very high Value.

(#) We should not Consider 'M' in decision because It is our Artificial Variable (Just to fullfill the Condition) if we add as +'M' in 'Z' then first of all it becomes a part of decision due to higher (in max) & we should ignore. Hence It's better to take -'M' Which results very low (automatically ignored)

Simplex Table I (Introduce, slack, Artificial)

| Cj | basic variable | Qty | 62000<br>$x_1$ | 124000<br>$x_2$ | 0<br>$S_1$ | 0<br>$S_2$ | 0<br>$S_3$ | -M<br>$A_1$ | -M.<br>$A_2$ | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 260 | 3 | 4 | 1 | 0 | 0 | 0 | 0 | $\frac{260}{4} = 65$ |
| -M | $A_1$ | 2 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | $\frac{2}{0} = \infty$ |
| -M. | $A_2$ | 3 | 0 | 1 | 0 | 0 | -1 | 0 | 1. | $\frac{3}{1} = 03.$ |
| | | $z_j$ | -M | -M. | 0 | M | M | -M | -M. | |
| | | $C_j - z_j$ | 62000+M | 124000+M | 0 | -M | -M | 0 | 0 | |

Incoming $x$ & outgoing $A_2$.

Simplex Table-II.

| Cj | basic variable | Qty | 62000<br>$x_1$ | 124000<br>$x_2$ | 0<br>$S_1$ | 0<br>$S_2$ | 0<br>$S_3$ | -M<br>$A_1$ | -M.<br>$A_2$ | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 248 | 3 | 0 | 1 | 0 | 4 | 0 | | $\frac{248}{3} = 82.\dot{6}$ |
| -M | $A_1$ | 2 | 1 | 0 | 0 | -1 | 0 | 1 | | $\frac{2}{1} = 2$ |
| 124000 | $x_2$ | 3 | 0 | 1 | 0 | 0 | -1 | 0 | | $\frac{3}{0} = \infty$ |
| | | $z_j =$ | -M | 124000 | 0 | M | -124000 | -M | | |
| | | $C_j - z_j.$ | 62000+M | 0 | 0 | -M | 124000 | 0 | | |

Working $260 - 3 \times 4 = 248$ Incoming $= x_1$

$3 - 4 \times 0 = 3$ & outgoing - $A_1$

$4 - 4 \times 1 = 0$

$1 - 4 \times 0 = 1$

$0 - 4 \times 0 = 0$

$0 - 4 \times -1 = 4$

$0 - 4 \times 0 \cdot 0$

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Statement of Simplex Table-III

| Cj | basic variable | Qty | 62000<br>$x_1$ | 124000<br>$x_2$ | 0<br>$S_1$ | 0<br>$S_2$ | 0<br>$S_3$ | Ratio. |
|---|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 242 | 0 | 0 | 1 | 3 | 4 | $\frac{242}{4} = 60.5$ |
| 62000 | $x_1$ | 2 | 2 1 | 0 | 0 | 0 -1 | 0 | $\frac{2}{0} = \infty$ |
| 124000 | $x_2$ | 3 | 0 | 1 | 0 | 0 | -1 | $\frac{3}{-1} = -3.$ |
| | | $z_j$ | 62000 | 124000 | 0 | -62000 | -124000 | |
| | | $c_j - z_j$ | 0 | 0 | 0 | 62000 | 124000 | |

Incoming $s_3$ & Outgoing $s_1$.

Simplex Table- IV.

| Cj. | basic variable | Qty | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $S_3$. |
|---|---|---|---|---|---|---|---|
| 0 | $S_3$ | $60.5(\frac{242}{4})$ | 0 | 0 | $1/4$ | $3/4$ | 1 |
| 62000 | $x_1$ | 2 | 1 | 0 | 0 | -1 | 0 |
| 124000 | $x_2$ | $3-(-\frac{1}{4}\times 60.5)$<br>$= 635$ | $0-[0\times -1]$<br>$= 0$ | $1-[0\times -1]$<br>1 | $0-[1/4\times -1]$<br>$1/4$ | $0-[3/4\times -1]$<br>$3/4$ | $-1-[1\times -1]$<br>$= 0$ |
| | | $z_j$ | 62000 | 124000 | 31000 | 31000 | 0 |
| | | $c_j - z_j$ | 0 | 0 | -31000 | -31000 | 0. |

$x_1 = 2$ & $x_2 = 63.5$

Profit : $Z = 62000 \times 2 + 124000 \times 63.5$

$= 124000 + 78,74000$

$= 79,98,000$

classmate

(39) Page-101

Max $z = 8x_1 + 4x_2 - 3x_3 + 10x_4 + 0s_1 + 0s_2 - MA_1 - MA_2.$

Subject to

$2x_1 - x_2 + x_3 + 2x_4 - s_1 + A_1 = 40.$

$3x_1 - x_3 + x_4 + S_2 = 90$

$2x_1 + x_2 + x_4 + A_2 = 60$

|  |  |  | 8 | 4 | -3 | 10 | 0 | 0 | -M | -M. |
|---|---|---|---|---|---|---|---|---|---|---|
| Cj | Basic variable | Qty | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ |
| - M | $A_1$ | 40 | 2 | -1 | 1 | 2 | -1 | 0 | 1 | 0 |
| 0 | $S_2$ | 90 | 3 | 0 | -1 | 1 | 0 | 1 | 0 | 0 |
| M. | $A_2$ | 60 | 2 | 1 | 0 | 1 | 0 | 0 | 0 | 1. |

Pointr (#) Constraint Variable in Constraint

≤ Slack Variable
(Spare Capacity) Always added in Constraint →

Pointr

| Constraint | Variable in Contraint | Maximization<br>Z | Minimization<br>Z |
|---|---|---|---|
| ≤ | slack Variable<br>(spare Capacity)<br>[Always added in Constraint] | 0<br>[Contribution from spare Capacity = Zero] | 0 |
| ≥ | Surplus variable<br>(excess Capacity)<br>[Always subtracted in Constraint to equalize & add Artificial variable to Fullfill the Condition] | Co-efficient of s = 0.<br>&<br>Co-efficient of $A_1$ = -M | Co-efficient of s = 0<br>Co-efficient of A = +M. |
| = | Only Artificial variable should be added. | - M | + M. |

(37) Page-100 Introduce Slack, Surplus & Artificial Variable.

Max $z = 100x_1 + 80x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4 - MA_1$.

Subject to

$3x_1 + 5x_2 + s_1 = 150$

$x_2 + s_2 = 20$

$8x_1 + 5x_2 + s_3 = 300$

$x_1 + x_2 - s_4 + A_1 = 25$.

Simplex Table-I.

| | | | 100 | 80 | 0 | 0 | 0 | 0 | -M. |
|---|---|---|---|---|---|---|---|---|---|
| $C_j$ | Basic Variable | Qty. | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $A_1$ |
| 0 | $S_1$ | 150 | 3 | 5 | 1 | 0 | 0 | 0 | 0 |
| 0 | $S_2$ | 20 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | $S_3$ | 300 | 8 | 5 | 0 | 0 | 1 | 0 | 0 |
| -M | $A_1$ | 25 | 1 | 1 | 0 | 0 | 0 | -1 | 1 |

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Max $Z = x_1 + 5x_2$

$6x_1 + 8x_2 \leq 12$

$5x_1 + 15x_2 \geqslant 10$

$x_1, x_2 \geqslant 0.$

Introduce slack, surplus & Artificial variable.

$Z = x_1 + 5x_2 + 0s_1 + 0s_2 - MA_1.$ ─ ①

Subject to

$6x_1 + 8x_2 + s_1 = 12$ ② $\Rightarrow s_1$ = Slack Variable (Spare Capacity)

$5x_1 + 15x_2 - s_2 + A_1 = 10$ ③ $\Rightarrow s_2$ = Surplus variab (Excess Capacity)

(A) Co-efficient of $s_1 = 0$

Co-efficient of $s_2 = 0$

Co-efficient of $A_1 = -M.$ ─ in equation ①.

Co-efficient of $s_1 = 0$

Co-efficient of $s_2 = -1$ / Not Applicable.

Co-efficient of $A_1 = 1$

(B) Slack Variable = $s_1$

Surplus Variable = $s_2$

(C) All variable belongs part of Simplex Table except Surplus as surplus is automatically covered from Artificial ie $s_1, A_1$.

(D) Minimize

$z = x_1 + 5x_2 + 0s_1 + 0s_2 + MA_1.$

Subject to

$6x_1 + 8x_2 + s_1 = 12$

$5x_1 + 15x_2 - s_2 + A_1 = 10.$

| In equation-1 | In equation-3 |
|---|---|
| Coefficient of $s_1 = 0$ | Co-efficient of $s_1 = 0$/ N/A |
| Co-efficient of $s_2 = 0$ | Co-efficient of $s_2 = -1$ |
| Co-efficient of $A_1 = M.$ | Co-efficient of $A_1 = 1$ |

DATE

(eg) Introduce slack, surplus & Artificial variable and set up Simplex Table -1 from the following formulation :-

Max $Z = 10x_1 + 20x_2$

Subject to

$2x_1 + 5x_2 \leq 100$

$3x_1 + 4x_2 \geq 200$

$x_1 + x_2 = 500$

$x_1, x_2 \geq 0.$

Solution Max $Z = 10x_1 + 20x_2$

Subject to

$2x_1 + 5x_2 + S_1 = 100$

$3x_1 + 4x_2 - S_2 + A_1 = 200$

$x_1 + x_2 + A_2 = 500$

|  |  |  | 10 | 20 | 0 | 0 | -M | -M |
|---|---|---|---|---|---|---|---|---|
| $C_j$ | Basic variable | Qty | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ |
| 0 | $S_1$ | 100 | 2 | 5 | 1 | 0 | 0 | 0 |
| -M | $A_1$ | 200 | 3 | 4 | 0 | -1 | 1 | 0 |
| -M | $A_2$ | 500 | 1 | 1 | 0 | 0 | 0 | 1. |

# Chapter-7

# Standard Costing

29th class
DATE 08 02 2017

## # Coverage

(1) Calculation of Variances
(2) Reconciliation statement
(3) Calculation of Variance with Equivalent Production.
(4) Planning & Operating Variances
(5) Balance score Card
(6) Single & Partial Planning

## # Calculation of Variances.

### I Material Cost Variances.

- Material Price variances.
  [CSR-AR) x AQ]
- Material usage / Qty Efficiency Variance.
  [(SQ-AQ) x SR]
  - Material Mix Variance
  - Material Yield Variance.

### II Labour Cost Variance

- Labour Rate Variance
- Labour efficiency Variance
  - Labour Mix (Group/Gang) Variance
  - Labour Yeild Variance (Sub-efficiency)
- Labour Idle Variance.

### III Variable Overhead Variance Cost

- Variable overhead Expenditure Variances.
- Variable overhead Efficiency Variance

### IV Fixed Overhead Cost Variance.

- Fixed Overhead Expenditure Variance
- Fixed overhead Volume Variance.
  - Fixed overhead Capacity variance.
    - Fixed overhead calender Variance.
    - Revised fixed overhead Capacity Variance.
  - Fixed overhead Efficiency Variance

### V Sales Variance

- Turnover Based.
- Margin Based.
  1) Absorbtion Approach
  2) Marginal Approach.

[In Absorption]

## 1) Turnover based Budgeted Profit Per unit

- **Sales Value Variance**
  - Sales Price Variance.
  - Sales Qty Variance.
    - **Sales Volume Variance**
      - Sales Market Size Variance
      - Sales Market share Variance.
    - Sales Mix Variance.

## 2) Marginal based

- **Sales Margin Value Variance.** (Contribution)
  - Sales Margin Price Variance (BSP − ASP) X AQS.
  - Sales Margin Qty Variance (BQ − AQ) X B Margin Per unit.
    - Sales Margin Yeild ~~Volume~~ Variance. (Total Std Qty − Total Act Qty) X WABMPU; (Std Ratio for Total Std Qty − Std Ratio for Total Actual Qty) X bud Margin Per unit
      - Sales Margin Market Size Variance. [bud Industry sale − Actual Industry sale] X B. Share X W.AM.P.V.
      - Sales Margin Market Share Variances. [B. Share − Actual Share] X Actual Industry sale X W.A.B.M.P.U.
    - Sales Margin Mix Variance (Std Ratio for Actual Qty − Actual Ratio for Actual Qty) X B. Margin Per unit.

㊓ Margin means "Contribution" (Marginal Approach)

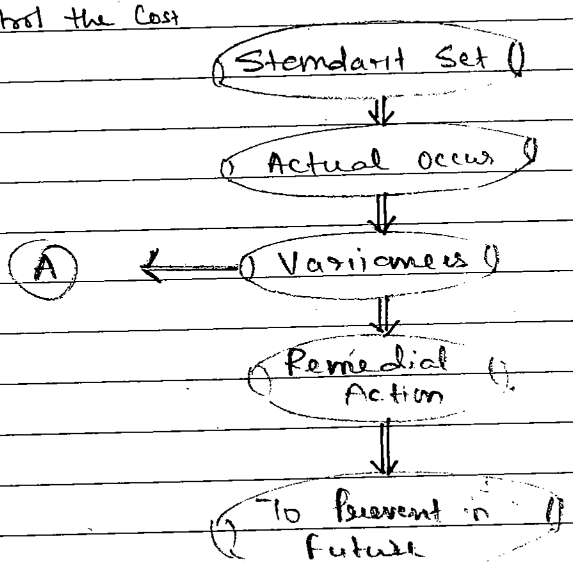
㊓ Margin Mean "Profit" (Absorbtion Approach)

(1) Calculation of Variance

(#) Standard Costing :- It is a Technique Which insures to Control the Cost by Comparing Actual Cost with Standard Cost. & differences of Actual Cost with Standard Cost is Known as Variance.

If Variances occur unfavourable (Adverse) then Management would like to take remedial action against such Variances.

Due to remedial action, manager would take appropriate remedial steps to Control the Cost



(#) Standard Cost The Cost Should be Incurred in Actual Working Condition i.e, Target Cost = 10,000

Standard Cost should be set 18000

Variance ⇒ 8000

Standard Cost Should be set not only on the basis of Past data but also Consider Future factor Which may Effect our Cost

eg

| Cricketer | Mr 'x' |
|---|---|
| Match-1 | - 98 |
| 2 | - 100 |
| 3 | - 105 |
| 4 | - 90 |
| 5 | - 92. |

What he should score in Next Match

| | |
|---|---|
| (#) Past Data ⇒ Average | 95 |
| ± Future Factor ⟹ | 30 |
| Standard ⇒ | 75 |
| Actual | XX |
| Variance | XX |

called Standard

(eg) -2 **Industry Data**

|  | Qwk | Standard | Actual |
|---|---|---|---|
| Material | 10000 $\xrightarrow{\text{Discount}}$ | 15000 |  |
| Labour | 20,000 $\xrightarrow{\text{Learning effect}}$ | 30 000 |  |
| fixed overhead. | 40, 000 $\xrightarrow{\text{Inflation}}$ | 50 000 |  |
| Cost $\Longrightarrow$ | 70, 000 | 95000 | 120000 |

$95000 \xrightarrow[\text{Variance}]{25000} 120000$

## # Material Cost Variance

Material Cost Comprises two Factors. Ⓐ Price Ⓑ Qty (Input)

Standard Cost = $\underset{\text{Qty}}{\underset{\Downarrow}{6\text{ kg}}} \times \underset{\text{Rate.}}{\underset{\Downarrow}{₹ 5/\text{kg}}} = ₹ 30.$

Actual Cost $8\text{ kg} \times ₹ 6/\text{kg} = ₹ 48.$

Actual Cost may differ from standard Cost due to the reasons which comprises Cost. Cost Comprises from two (2) Factors.
1) Price (Rate/unit of unit)
2) Qty.

[Cost = Per unit of output]

Now, we Can say differences of standard Cost & Actual Cost may occur due to
① change in Price called Price Variance
② Change in Qty Called Qty Variance.

# Purchase Manager will be held responsible for Price Variance.

# Production Manager will be held responsible for Qty Variance. (Usage)

# (#) Material Price Variance

[Purchase Manager]

Change in Price × Actual Qty

Change in Price = [Standard Price − Actual Price] × Actual Qty

[₹5/kg − ₹6/kg] × 8kg

₹1/kg × 8kg = $\boxed{\text{₹8 Adverse}}$

In order to get Value of Price Variance, Difference of Price should be multiplied with "Qty" but we have Standard Qty & Actual Qty.

(#) As per purchase Manager view :- the Qty should be Standard Instead of Actual because if "Production manager had worked effectively & efficiently" then Qty Consumed would have been 6kg (Standard) instead of 8kg (Actual) i.e, Why I suffer the loss due to ineffective working of Production Manager.

(#) As per Management Accountant View :- Difference of Price should be Multiplied with Actual Qty due to following Reasons.

(i) The duty of Purchase Manager is to Purchase all Qty (Standard & excess Qty) effectively & efficiently i.e, not only Standard Qty but also excess Qty

(ii) If Production Manager had worked more effectively & efficiently the Qty used would have been less than standard i.e. 5kg (eg), then Difference (loss) on price should be calculated on Actual Qty i.e, Actual loss.

(iii) If we Consider standard Qty, Purchase manager may Purchase Excess Qty at very high rate. which is not in the interest of Company.

## #) Material Usage/Qty Variance

Difference of Qty X Standard Price.

$$\boxed{\begin{matrix}\text{Standard Qty} & & \text{Actual Qty}\\ \text{for Actual} & - & \text{for Actual}\\ \text{Production} & & \text{Production.}\end{matrix}} \times \text{Standard Price}$$

$$(6kg - 8kg) \times 5$$

$$2kg \times 5 = \boxed{10 \text{ Adverse}}$$

In order to get value of Qty variance, Difference of Qty should be multiplied with Price. Price are of two (2) types (i) Standard Price (ii) Actual Price.

#) We should consider standard Price because excess Price on excess Qty had been already considered in Price Variance. Hence we should consider only standard Price.

SC = 6kg x 5|kg = ₹30

Ae = 8kg x 6|kg = ₹48

| Material Price Variance | Material Usage Variance |
|---|---|
| (5-6) X (6kg + 2kg) | (6-8) X ₹5\|kg |
| = 8 Adverse | 10 Adverse |

Difference of Qty

Input

100 finished Goods → 1000kg

Standard Input for Actual Production

→ 800 [Revised standard Cost]

Actual Qty for Actual Production

Actual 80 finished Good → 1000kg [Actual Cost]

(eg) 1 finished Goods Standard

1 finished Goods Actual

40000 kg × 20/kg = 800000
Qty Rate Cost

50,000kg × 30/kg = 150000

⇓

Material Cost Variance 700000 Adverse

MPV

(20-30) × (40000 kg + 10000 kg)

= 10 × 50000

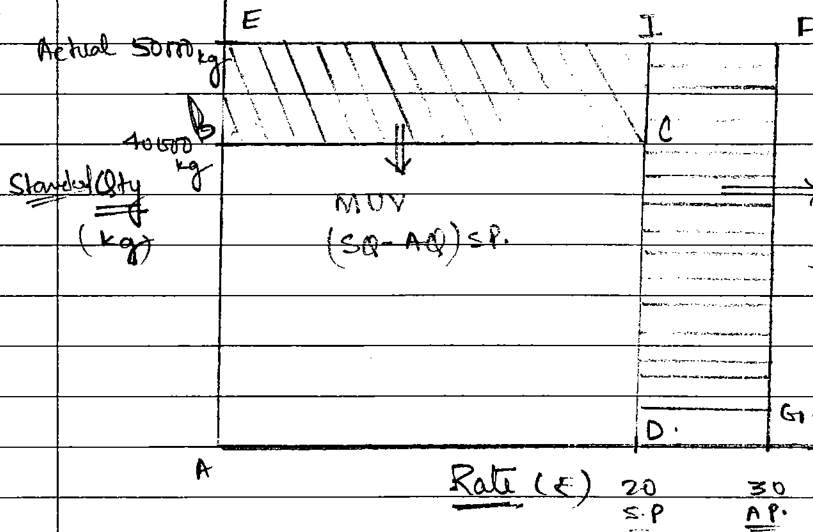
= 500000 Adverse

MUV

(40000-50000) × 20/kg

= 10,000 × 20

= 2,00,000 Adverse



④ Area DIFG = Price Variance

→ MPV = (SP-AP) AQ

5) Area BETC = Material Usage Variance.

1) Area ABCD = Standard Cost.

2) Area AEFG = Actual Cost.

3) Area BEFGDC = Cost Variance.

Budget ⇒ Target to be applied for a particular Company.

Standard ⇒ Target to be applied (set) for overall Industry. (Bench Mark).

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(9)

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## Data for Material Variance.

| | Standard | | | Revised Standard | | | Actual | | |
|---|---|---|---|---|---|---|---|---|---|
| | Qty(kg) | Rate(₹) | Amt(₹) | Qty(kg) | Rate(₹) | Amt(₹) | Qty(kg) | Rate(₹) | Amt(₹) |
| x | 30kg | 5 | 150 | 60 | 5 | 300 | 80 kg | 6 | 480 |
| y | 70kg | 10 | 700 | 140 | 10 | 1400 | 70 kg | 12 | 840 |
| Input 100 | | | 850. | 200 | | 1700 | 150 kg | | 1320 |
| Loss 10 | | | | Loss 20 | | | | | |
| Output 90 | | | | 180 | | 1700 | 180 kg | | 1320 |

W.N

x

90 ——— ~~30~~ 30

1 ——— $\frac{30}{90}$

180 ——— $\frac{30 \times 180}{90}$

= 60

y

90 ——— 70

1 ——— $\frac{70}{90}$

180 ——— $\frac{70 \times 180}{90}$

= 140.

∴ Material Price Variance = Se for

## Statement of Variances.

| SR.NO | Particular | Basis. | Amt |
|---|---|---|---|
| 1. | Material Cost Variance | Standard Cost for Actual output − Actual Cost for Actual output | 1700 − 1320<br>= 380 f. |
| 2. | Material Price Variance | [Standard Price − Actual Price] x Actual Qty.<br>x (5 − 6) x 80 = 80 A<br>y (10 − 12) x 70 = 140 A | 220 A. |
| 3. | Material Usage/Qty Variance | (Standard Qty − Actual Qty) x Stnd Rate<br>x (60 − 80) x 5 = 100 A<br>y (140 − 70) x 10 = 700 F | 600 f. |

(#) Point

(1) Material Cost Variance = [Material Price Variance + Material Usage Variance] only at Consumption.

(2) How to Calculate Consumption of Raw Material.

| Store | | Production | |
|---|---|---|---|
| Opening Stock | xx | Opening Stock | xx |
| + Purchase | xx | Received from store | xx |
| - Closing Stock | xx | (-) Closing Stock | xx |
| Transferred to Prod<sup>n</sup> | xx | Issued to Machine<br>i-e Consumption | xxx |

If either opening Stock and Closing stock of Raw Material at Production Department are same or Not available then issued to Production is our Consumption of Raw material.

(3) If Standard Output, & Actual output are not given, always we should presume Standard output = Actual output = 1. For such Case No need to prepare Revised Standard.

30th class
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⑧ Page.217

Material Price Variance = [Difference of Price X Actual Qty]

$= (SC - AC) \times AQ$

$= (8 - 10) \times 25000 + (8-10)\, 6000$

$= 50000 + 12000$

$= ₹\ 62000$ A.

Material Usage Variance = (Difference in Qty) X Standard Price

$(25000 - 36000) \times 8 + 6000 \times (10-13)$

$= 88000 + 18000$

$= 106000$ A.

Material Price Variance

At the time of Consumption
↓
$\left[\left(\text{Standard Price} - \text{Actual Price}\right) \times \text{Actual Qty Consumed}\right]$

at the time of Purchase
↓
$\left[\left(\text{Standard Price} - \text{Actual Price}\right) \times \text{Actual Qty Purchased}\right]$

Material Price Variance: (At the time of Purchase):-

- If Management decides to purchase in bulk for subsequent period.
- Purchase Manager may change at the time of Purchase of Material & at the time of usage of Material
- We should calculate MPV (at Purchase) only when it is required.

eg

| 1st April | | SR | AR |
|---|---|---|---|
| Opening Stock | = Nill | | |
| Purchase | = 1000 kg | 10 | 12 |
| Used | = 800 kg | | |
| Closing | = 200 kg | | |

MPV (At the time of Purchase) = (10-12) X 1000 = 2000 (A)

MPV (At the time of Consumption) = (10-12) X 800 = 1600 (A).

| 1st May | | SP | AP |
|---|---|---|---|
| Opening Stock | = 200 kg | | |
| Purchase | = 1200 kg | 12 | 15 |
| Used | = 1000 kg | | |
| Closing Stock | 400 kg | | |

MPV (at the time of Purchase) = (12-15) X 1200 = 3600 (A)

MPV (at the Consumption) = (10-12)X200 +(12-15)X 800 =

= 400+2400 = 2800 (A).

# If Actual Price of last Period is not given, We presume the Current period standard Price is equal to Actual Price of last Period because it is assumed that current Period Standards have been set (Prepared) on the basis of last Period Actual.

# If Standard Price of last Period is not given, we should Presume Standard Price of last Period = Standard Price of Current Period. because standards are not to be changed so frequently.

DATE

~~Working~~

(10)
[Page-217]

Rough

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| 01-04-2012 |  |  | SR |  | AR |
| Opening stock | A | 50kg | 2 |  |  |
|  | B | 60kg | 3 |  |  |
| Purchase | A | 800 kg | 2 | 4 | $\left[\frac{3200}{800}\right]$ |
|  | B | 1200kg | 3 | 5 | $\left[\frac{6000}{1200}\right]$ |
| Closing stock | A | 5kg |  |  |  |
|  | B | 50kg |  |  |  |
|  |  |  | SR | AR |  |
| ∴ Usage | A | 845 kg | 2 | 4 |  |
|  | B | 1210 kg | 3 | 5 |  |

## Data for Material Variance.

| Standard |  |  | Revised |  |  | Actual |  |  |
|---|---|---|---|---|---|---|---|---|
| Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rate | Amt. |
| Kg | ₹ | ₹ | Kg | ₹ | ₹ | Kg | ₹ | ₹ |
| A 50 | 2 | 100 | 1000 | 2 | 2000 | 845 |  | 3280 |
| B 50 | 3 | 150 | 1000 | 3 | 3000 | 1210 |  | 5930 |
| Input 100 |  | 250 | 2000 |  | 5000 | 2055 |  | 9210 |
| Loss 10 |  |  | 200 |  |  |  |  |  |
| Output 90 |  |  | 1800 |  |  | 1800 |  |  |

Material Price Variance = (SP-AP) AQ =

$$A = \left[\begin{matrix}(2-2)\times 50\\ +\\ (2-4)\times 795\end{matrix}\right] = 1590\ (A)$$

$$B = \left[\begin{matrix}(3-3)\times 60\\ +\\ (3-5)\times 1150\end{matrix}\right] = 2300\ (A)$$

∴ Material Price Variance = 3890 (A).

Material Usage Variance = (SQ - AQ) × SR

$A = (1000 - 845) \times 2 = 310\ (F)$

$B = (1000 - 1210) \times 3 = 630\ (A)$

classmate ∴ Material Usage Variance = 310 (A)

**Working**

| | **A** | | **B** |
|---|---|---|---|
| | Stock | | Stock |
| Opening stock | 50 | Opening stock | 60 |
| Purchase | 800 | Purchase | 1200 |
| Closing stock | (5) | less:- closing | 50 |
| Usage | 845 | Usage | 1210 |

| | | | |
|---|---|---|---|
| 50 kg | 795 kg | 60 kg | 1150 kg |
| out of opening | out of current Purchase | out of opening | out of current Purchase |
| X | X | X | × 5 |
| 2 [Std of last] | 4 | 3 | |
| = | = | = | = |
| ₹ 100 | ₹ 3180 | 180 | 5750. |
| = 3280. | | = 5930 | |

(6) [Page 276]

Material Price Variance = $(SP - AP) \times AQ$ purchase
(at the time of Purchase)

$x = (2-3) \times 4000 = 4000\ A$

$y = (3-3) \times 6000 = 0$ or Nill $\boxed{4000\ A}$

Material Usage Variance = $(SQ - AQ) \times SR$.
for Actual Production.

$x = (15000 - 6000) \times 2 = 18000\ F$

$y = (22000 - 4000) \times 3 = 54000\ F.$ $\boxed{72000\ F.}$

P-T-O

DATE

**Working**

**Standard** $x = \underbrace{1kg \times 3000}_{\text{For A}} + \underbrace{3kg \times 4000}_{\text{For b}}$.

$= 3000 + 12000$

$= 15000kg.$

$Y = \underbrace{2kg \times 3000}_{\text{For A}} + \underbrace{4kg \times 4000}_{\text{for- b.}}$

$= 6000 + 16000$

$= 22000kg.$

| | | | | |
|---|---|---|---|---|
| **Actual** | $x$ | | | $Y$ |
| | **Store** | | **Production** | |
| Opening | | Opening | 1000 | 1000 |
| +Purchase | | Receipt | 7000 | 5000 |
| (-) closing | | (-)Closing | 2000 | 2000 |
| Transferred to ~~Store~~ Production | 7000 | 5000kg Usage | 6000. | 4000 |

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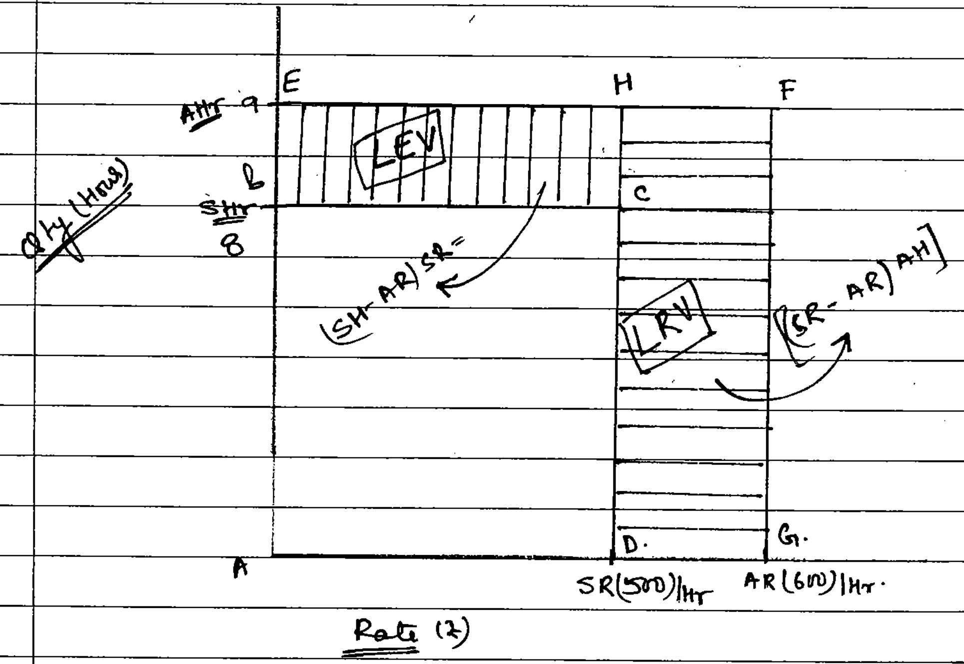
DATE

## # Labour Cost Variance

| Labour Rate Variance | Labour Efficiency Variance | Labour Idle Varia |
|---|---|---|
| [Personnel Manager] | [Production Management] | [~~Production~~ Managemnt |
| [(SR-AR) Actual Hour] | (SH-AH)XSR. | [(Idle Hr) X SR] |
| [~~for Actual payment~~] | [~~for Actual production~~] | for Actual Producti |



AREA - DHFG = Labour Rate Variance

AREA - bEHC = Labour Efficiency Variance

AREA = bEFGDC = Labour Cost Variance

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~~(eg)~~ Labour Cost = ?

Budgeted Hours = 10000 Hrs ⇒ [It is for each Worker] Always.

Labour = 10.

Labour Rate = 4 per Hour

Labour Cost = 10000 Hrs X 10 Labour X ₹4/Hr = ₹400000 Ans.

(#) Labour Cost = Rate / Hour X Qty [Labour Hour]

Standard Labour Cost = ₹500/Hr X (1 Labour X 8 Hour) = ₹4000

Actual Labour Cost = ₹600/Hr X (1 Labour X 9 Hr) = ₹5400

Actual Labour Cost may be different from standard Cost of Labour due to two Factor (Which comprise Labour Cost)

1) Change in Rate Called Labour Rate Variance ⇒ Personnel Manager

2) Change in Qty (Labour Hour) Called Labour Qty (Efficiency) Variance. ↳ Production Manager.

Labour Rate Variance = Difference of Rate X Actual Hour.

(SR - AR) X (SH + Excess Hour)

(500 - 600) X 9 Hr

= ₹900 A.

[Excess Hours also to be appointed effectively / efficiently. Hence Actual Hours Considered.]

Labour Efficiency Variance = Difference of Qty X Standard Price

= (SH - AH) X SP.

= (8 - 9) X 500

= 500 A.

DATE

## # Idle Time

LOSS

Normal loss
[Can't be reduced]

Abnormal loss
[Due to Negligence / in effectiveness]

Due to Hours [working] but not effectively / efficiently utilized.
⇓
[Labour Efficiency Variance]

Due to Hours which has not been utilized
⇓
[Idle]

**Normal loss** :- Normal loss which cannot be Controlled to some extent. eg loss of labour Hours from factory gate to work station & from work station to factory gate & settingup Time of Machine & lunch hours.

| | | |
|---|---|---|
| 8 Hr | 15/Hr | ₹120. |
| (−) 2 Hr [Normal loss] | | |
| 6 Hr | 20/Hr (b/f) | 120 |
| | ⇓ SR | |

# In Standard Rate Normal loss already adjusted.

**Idle Variance** :- Idle Variance occurs due to idle Hours. Idle Hours occurs means those Hours which have not been utilized due to.

1) Machine Break Down
2) Non Availability of Material.
3) Labour Strike
4) Power failure.
5) Any other Reasons.

IDLE Variance
⇓
Idle Hours × SR / Hr

Idle Hours = [Payment Hours − Working Hours]

[6 Hr − 5½ Hr]

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Efficiency Variance :- Efficiency Variance occurs due to [Advorne] those Working hours which have been utilized but not so effectively & efficiently

eg

SR | Hour = 20

AR | Hour = 30·

Standard time for 1 finished Goods = 4 Hr

Payment Hours (Job card) = 6 Hr

| | |
|---|---|
| Actual Working Hours· | 5½ Hrs· |
| Idle Hours | ½ Hrs· |

Labour Cost Variance = SC - AC

= 4×20 - 30×6

= 80 - 180

= 100 A·

Labour Rate variance = (SR - AR) × A Payment Hours

= (20-30) × 6

= 60 (A)

Labour Idle variance = Idle Hours × SR

= ½ × 20

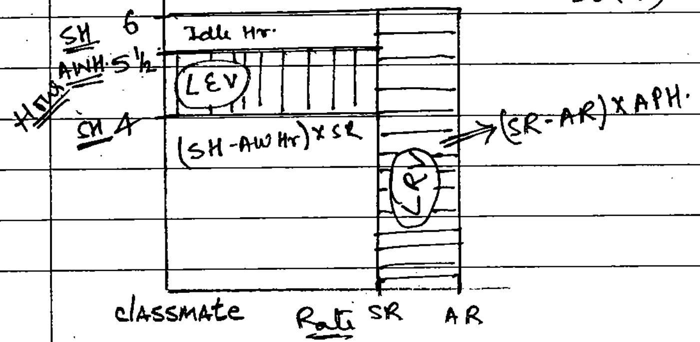
= 10 (A)

Labour Efficiency Variance = (SH - $\overset{\text{working}}{\text{AH}}$) × SR.

= (4 - 5½) × 20

= 1½ × 20

= 30 (A·)



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Data for Labour Variance

Standard Revised Actual

Hour Rate Amt Hr Rate Amt Pay Hr Rate Amt

Data for Labour Variance.

| | Standard [1000 FG] | | | Revised (960 unit) | | | Actual [960 FG] | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Hour | Rate | Amt | Hr | Rate | Amt | Payment Hour | Rate | Amt | Idle | Working Hr [APH-IH] |
| Men | 10X40 | 1 | 400 | 384 | 1 | 384 | 13X40=520 | 3 | 1560 | 26 [13X2] | 494 |
| Women | 5X40 | 2 | 400 | 192 | 2 | 384 | 4X40=160 | 4 | 640 | 8 [4X2] | 152 |
| Boys | 5X40 | 3 | 600 | 192 | 3 | 576 | 3X40=120 | 5 | 600 | 6 [3X2] | 114 |
| | 800 | | 1400 | 768 | | 1344 | 800 | | 2800 | | 760 |

Labour ~~Cost~~ Rate Variance = (SR-AR) APH.

Men = (1 - 3) X 520 = 1040 A

Women = (2 - 4) X 160 = 320 A

Boys = (3 - 5) X 120 = 240 A 1600 A

Labour Idle Variance = Idle Hour X SR

Men = 26 X 1 = 26 A

Women = 8 X 2 = 16 A

Boys = 6 X 3 = 18 A 60 A.

Labour Efficiency Variance = (SH - AWH) X SR

Men = (384 - 494) X 1 = 110 A

Women = (192 - 152) X 2 = 80 F

Boys = (192 - 114) X 3 = 234 F 204 F.

Labour Cost Variance = (SC - AC)

= 1344 - 2800

= 1456 (A)

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[Page 219]

Data for Labour Variance

| | Standard (1 Job) | | | Actual (1 Job) | | |
|---|---|---|---|---|---|---|
| | Qty (Labour hours) | Rate | Amt | Qty | Rate | Amt |
| x | 2000<br>[50×40] | 200 | 400000 | 3600<br>(80×45) | 300 | 1,08 0000 |
| y | 2400<br>[60×40] | 300 | 720000 | 4050<br>(90×45) | 400 | 1620000 |
| | 4400 | | 11,20000 | 7650 | | 2700000 |

Labour Cost Variance = ~~2~~ Standard Cost - Actual Cost

= 11,20,000 - 2700000

= 1580000 (A)

Labour Rate Variance = (SR - AR)AQ

= (300-200)×3600 + (300-400)×4050

= -360000 + -40,5000

= -765000

∴ 765000(A).

Labour Efficiency Variance = (SQ - AQ)×SR

= (SH - AH)×SR

= ∴

x = (2000 - 3600)×200 = 3,20,000 (A)

y = (2400 - 4050)×300 = 495000 (A)

815000 (A)

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## Data for Labour Variances.

| | Standard | | | Revised | | | Actual | | |
|---|---|---|---|---|---|---|---|---|---|
| | Labour Hour | Rate | Amt | Lab. Hr | Rate | Amt | Labour Hour | Rate | Amt |
| Grade x | 70000<br>[70 x 1000] | 1 | 70000 | 71750 | 1 | 71750 | 81000<br>[90 x 900] | 3 | 243000 |
| Grade Y | 80 000<br>[80 x 1000] | 2 | 1,60 000 | 82000 | 2 | 164000 | 54000<br>[60 x 900] | 4 | 216 000 |
| | 150 000<br>for 4000 unit | | 230 000 | 153750 | | 235750 | | | 4,59 000 |

| | | | |
|---|---|---|---|
| Budgted Production. | 5000 | Actual Gross Prodn | 5000 |
| Less:- Normal Loss @20%. | 1000 | Actual loss | 900 |
| Net Production | 4000 | Actual Production | 4100 |

Labour Cost Variance = SC − AC

= 235750 − 459 000

= 223250 (A)

Labour Rate Variance: (SR − AR) AH

Grade x = (1 − 3) × 81000 = 162000 (A)

Grade Y = (2 − 4) × 54000 = 108000 (A)

~~3[illegible]~~

270000 (A)

Labour Efficiency Variance = (SH − AH) × SR.

Grade x: (71750 − 81000) × 1 = 9250 A

Grade-Y = (82000 − 54000) × 2 = 56000 F

46750 (F)

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## # Variable overhead Variances

| Variable overhead [Price Effect] Expenditure Variance | Variable overhead Efficiency variance. |
|---|---|
| 1) For Indirect Material Rate variance $= \left[(SR - AR)\,AQ\right] = xx$ | for Indirect Material Usage variance $\left[(SQ - AQ) \times SR\right] = xx$ |
| 2) For Indirect Labour Rate variance & Indirect Expenses $= \left[\left(\frac{SR}{Hr} - \frac{AR}{Hr}\right) \times AH\right] = xx$ | For Indirect Labour efficiency variance & Indirect expenses $= \left[(SH - AH) \times \frac{SR}{Hr}\right] = \underline{xx}$ |
| Variable O/H Expenditure Var. $= \overline{\underline{xxx}}$ | Variable O/H efficiency var $= \underline{\underline{xxx}}$ |

Variable Overhead Consist Three (3) element.

1) Indirect Material: based on Quantity → behave like Material variance
2) Indirect Labour: based on Hours ⇒ behave like labour variance
3) Indirect expenses: based on Hours. ⇒ behave like labour variance

# In the absence of Instruction about break up ~~of~~ of Variable overhead (Indirect Material, Indirect labour & expenses), always we should Preassume that Variable overhead Consists Indirect labour & Indirect Expenses. based on Hours.

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(18) [Page. 220] Variable overhead Cost variance =

Variable overhead Expenditure Variance.

Indirect Material = (SR - AR) AQ
= (1000 - 1100) x 3 bottle
= -100 x 3 = 300 A

Indirect labour = (SR - AR) AH
= (2000 - 2500) x 30
= 15000 A.

Indirect expenditure = (SR - AR) x AH.
(500 - 600) x 30
= 3000 A.

18300 A.

Variable overhead Efficiency Variance

Indirect material = (SQ - AQ) SR
= (2 - 3) x 1000 = 1000 A.

Indirect labour - (SH - AH) x SR =
= (20 - 30) x 2000
= 20000 A

Indirect Expenditure = (SH - AH) x SR
= (20 - 30) x 500
= 5000 A.

26000 A.

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(19) [Page-221] Variable overhead expenditure Variance

$= (SR - AR) AH$

$= \left(\frac{10000}{8000} - \frac{22000}{11000}\right) \times 11000$

$= (1.25 - 2) \times 11000$

$= 8250$ (A)

Variable overhead Efficiency Variance

$= (SH - AH) SR$

$= (8000 - 11000) \times 1.25$

$= 3750$ (A)

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Data for Resource Variance.

| | Standard. | | | Revised(2000) | | | Actual. | | |
|---|---|---|---|---|---|---|---|---|---|
| ~~Qty~~ | Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rate | Amt |
| Material (kg) | 10 | 40 | 400 | 20000 | 40 | 800000 | 12000 | 50 | 600000 |
| labour (Hr) | 36 | 10 | 360 | 72000 | 10 | 720000 | 80000 | 12 | 960000 |
| Var. O/H. (Hr) | 36 | 20 | 720 | 72000 | 20 | 1440000 | 80000 | 25 | 2000000 |
| | | | 1480 | | | 2960000 | | | 3560000 |

Material Price Variance. = $(SR - AR) \times AQ$

$= (40 - 50) \times 12000 = 120000$ A

Labour Rate Variance = $(SR - AR) \times AH$

$= (10 - 12) \times 80000 = 160000$ A.

Variable O/H exp Variance = $(SR - AR) \times AH$

$= 20 - 25 \times 80000 = 400000$ A

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Material Usage Variance = (SQ - AQ) X SR

= (20000-12000) X 40 = 32000 F

Labour Efficiency Variance = (SH - AH) X SR

= (72000-80000) X 10 = 8000 A

=

~~Labour~~ Variable OH Efficiency Variance = (SH-AH) SR

= (72000-80000) X 20 = 160000 A.

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Data for Resource Variance.

| | Standard (1) | | | Revised (2000 | | | Actual (2000) | | |
|---|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rate | Amt |
| Material (kg) | 7 | 4 | 28 | 14000 | 4 | 56000 | 9000 | 5 | 45000 |
| Labour (Hr) | 12 | 3 | 36 | 24000 | 3 | 72000 | 2000 | 2.4 | 48000 |
| Variable OH (Hr) | 12 | 4 | 48 | 2400 | 4 | 96000 | 20000 | 1.2 | 24000 |
| | | | 112 | | | 224000 | | | 117000 |

Material Rate Variance = (SR - AR) AQ

= (4 - 5) X 9000 = 9000 A

Materiate Usage Variance = (14000-9000) X 4 = 20000 F.

Labour. Rate Variance = (SR - AR) X AH

= (3 - 2.4) X 20000 = 12000 F

Labour Efficiency Variance = (SH - AH) X SR

= (24000 - 20000) X 3 = 12000 F

Variable OH Expenditure Variance = (SR - AR) AH

= (4 - 1.2) 20000 = 56000 F

Variable OH Efficiency Variance = (SH-AH) X SR

= (24000-20000) X 4 = 16000 F

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## Statement of Bonus

a) 50% of Standard Cost of Material saved = 20000 × 50% = 10000

b) 30% of Standard Cost of Labour Hour saved = 12000 × 30% = 3600

~~Total Bonus Amt Payable ⟹ 13600~~

(29)
[Page-225]

### Data for Resource Variance

| | Standard (1 unit) | | | Revised | | | Actual (50000) unit | | |
|---|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rate | Amt |
| Material | 1.3 | 4 | 5.2 | 65000 | 4 | 260000 | 78000 | 4.2 | 561600 |
| Labour | 2.9 | 2.3 | 6.67 | 145000 | 2.3 | 333500. | 150000 | 2.5 | 375000 |
| Variable O/H | 2.9 | 1.5 | 4.35 | 145000 | 1.5 | 217500 | 150000 | 1.586 | 238000 |
| | | | 17.67 | | | 811000 | | | 11,74,600 |

① Material Price Variance = (SR − AR) × AQ
= (4 − 4.2) × 78000 = 15600 (A)

② Material Usage Variance = (SQ − AQ) × SR
= (65000 − 78000) × 4 = 52000 (A)

③ Labour Rate Variance = (SR − AR) × AH
= (2.3 − 2.5) × 150000 = 30000 (A)

④ Labour Efficiency Variance = (SH − AH) × SR
= (145000 − 150000) × 2.3 = 11500 (A)

⑤ Variable O/H Exp Var. = (SR − AR) × AH
= (1.5 − 1.586) × 150000 = 13000 (A)

⑥ Variable O/H Eff. Variance = (SH − AH) × SR
= (145000 − 150000) × 1.5 = 7500 (A)

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Data of Resource Variance

| | Standard (1000) | | | | Actual (1000) | | | |
|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Cost/unit | Qty | Rate | Amt | Cost/unit |
| Material (Kg) | 3800. | 3 | 11400<br>~~10000~~ | 11·4 | 4000 | 2·5 | 10000 | 10 |
| Labour (Hr) | 1600 | 5 | 8000 | 8 | 1800 | 5·25 | 9450 | 9·45 |
| Variable OH (Hr) | 1600 | 1·0 | 1600 | 1·6 | 1800 | 0·9.<br>CHH | 1620. | 11·62. |
| | | | | 21 | | | 21070[#2] | 21·07 [W.N.#1] |

Working Note ① over all Cost variance = 0·07

∴ $(\underset{\text{Unit}}{SC} - \underset{\text{unit}}{AC})$ = Overall Cost variance = 0·07/unit

∴ ~~(21 - 21·07) × AQ = 21 ×~~

~~0-0~~ (21 - AC/unit) = 0·07

∴ AC/unit 21·07-

② Overall Actual Cost = 21·07 × 1000 = 21070-

Ⓐ Labour Rate Variance = (SR - AR) × AH

= (5 - 5·25) × 1800

= 450 (A)

working

Ⓑ Raw Material Usage Variance = 600 A.

~~(SQ - AQ) × SR = 600 A~~

~~(3800 - AQ) × 3 = 600 A~~

~~11400 - 3AQ = 600 A~~

~~3AQ = 11400 + 600~~

~~AQ = $\frac{12000}{3}$ = 4000~~

Ⓑ Std. Quantity of R.M/unit = $\frac{3800}{1000}$

= 3.8 kg/unit

Ⓒ Labour Efficiency Variance = (SH - AH) SR

= (1600 - 1800) × 5

= 1000 (A)

(D) Variable O/H Efficiency Var. =

$(SH - AH) \times SR$

$(1600 - 1800) \times 1 = 200$ A

(E) Standard Direct Material Cost /unit

$= \frac{11400}{1000} = ₹ 11.4$ /unit

(F) Variable O/H. Expenditure Variance

$(SR - AR) \times AQ.$

$= (1 - 0.9) \times 1800$

$= 180$ (F.)

(G) Standard Direct labour cost/unit

$= \frac{8000}{1000} = 8$ /unit

(H) Standard Direct labour Rate / Hour $= \frac{₹ 8000}{1600 \text{ hr}} = ₹ 5$ / hr

(I) Material Price Variance =

$(SR - AR) \times AQ$

$= (3 - 2.5) \times 4000 =$

2000 (F)

(J) Standard total Material Cost for the output

$3800 \text{kg} \times 3 = ₹ 11400$

(K) Actual Total Direct labour Cost $= 1000 \times 9.45 = ₹ 9450 \cdot$ A

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~~Page-228~~

## Data for Cost Variances

| Standard | | | | Actual | | | |
|---|---|---|---|---|---|---|---|
| Qty | Rate | Amt | Cost/unit | Qty | Rate | Amt | Cost/unit. |

Working

① Labour Rate Variance = 1900 A.

$(SR - AR) \times AH = -1900$

$(20 - AR) \times AH = -1900.$

$(20 - AR) \times 2400 = -1900$

$48000 - 2400\,AR = -1900$

④ labour $AR = \frac{48000+1900}{2400} = 20.791\dot{6}$

② Labour Efficiency variance = 2000 F

$(SH - AH) \times SR = 2000$

$(0.5 - AH) \times 20 = 2000$

~~$10 - 20AH = 2000$~~

$\boxed{\cancel{20\,AH = -2000 + 10} \quad \cancel{AH = \frac{1990}{20}}}$

$[0.5 \times (5000\ FG] - AWHr) \times 20 = 2000$

$[2500 - AWH] \times 20 = 2000$

$50000 - 20\,AWH = 2000$

AH =

$20\,AWH = 48000$

$AWH = \frac{48000}{20} = 2400$ Hr

③ Material Usage Variance = 2500 A.

$(SQ - AQ) \times SR = -2500$

$[(1 \times 5000)A - AQ] \times 5 = -2500$

~~$125000 - 20AQ = -2500$~~ $\quad 25000 - 5AQ = -2500$

~~$AQ = \frac{100000+2500}{20}$~~ $\quad AQ = \frac{25000+2500}{5}$

Actual Qty used → AQ = 5500 kg

④ MPV = (SR − AR) × AQ = 3250 A

~~= (5 − AR) × 5500 = −3250~~

~~27500 − 5500 AR = = 3250~~

~~$AR = \frac{27500 + 3250}{5500} = 5.5909$~~

(5 − AR) × AQ = −3250

= 5 × AQ Pur − Actual Price/kg × A.Q. Purch = −3250

= 5 AQP − 68250 = −3250

5AQP = 68250 − 3250

$AQP = \frac{65000}{5} = 13000kg.$

⑧ Actual Material Price = $\frac{68250}{13000} = 5.25$/kg

SQ for Actual output = 1 kg × 5000 = 5000kg.

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Working Note

① $$\frac{\text{Budgeted labour Cost}}{\text{Budgeted labour Hour}} = \text{Standard Rate/Hr.}$$

$$\frac{152000}{80000} = ₹1.9/\text{Hr} \ [SR]$$

LRV = 0.20/Hr (A)

~~AR~~ Actual labour Rate = 1.9 + 0.2 = 2.1/Hr.

or (SR − AR) = LRV/Hr

(1.9 − AR) = −0.2

AR = 1.9 + 0.2 = 2.1/Hr.

② $$\frac{\text{Budgeted Variable Production O/H}}{\text{Budgeted Hours}} = \frac{96000}{80000} = 1.2/\text{Hr}$$

③ $$\text{Actual Labour Hr} = \frac{\text{Actual wage}}{\text{Actual Rate}} = \frac{163800}{2.1} = 78000\text{H}$$

④ MPV = 9000 (F).
(Purchase)

(SR − AR) × AQ = 9000.

~~0.75 − 0.75~~

[0.05] × AQ = 9000

$$AQ = \frac{9000}{0.05} = 180000 \text{ kg.}$$

∴ Actual Quantity Purchase = 180000kg.

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## Fixed overhead Variance (Absorption).

Fixed overhead Cost Variance.
- Fixed O/H Expenditure Variance
- Fixed O/H Volume Variance
  - Fixed O/H Capacity Variance (management)
    - [Management] want to take Remedial Action
    - Revised Capacity Variance
    - Calender Variance
  - Fixed O/H Efficiency Variance. (Production managers)

**Absorption** :- Management would like to know the Total Cost of regular product, at the time of production (Just before the production of each unit) for this purpose.

eg 1 Finished Goods

| | ₹/unit | |
|---|---|---|
| Material | 100 | Factory Rent = ₹ 100000. |
| Labour | 200 | |
| Variable O/H. | 50 | |
| Variable Cost | 350 | |

Fixed overhead ??/unit.

Management A/c suggest that It's not possible to calculate Total Cost of a product (Just before its production) Hence You should wait for the month at the end of the month actual fixed overhead divided by Actual production, It becomes Actual fixed overhead/unit, Total Cost could be arrived.

Management would not agree for this, Management would like to know strictly total Cost at the time of Production. For this purpose mgt Accountant estimates F.O. for the month as well as

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output i.e, budgeted overhead, & budgeted output & calculate f.o/unit (called Recovery Rate)

$$\frac{\text{Recovery Rate}}{\left[\begin{array}{l}\text{budgeted fixed}\\ \text{overhead/unit}\end{array}\right]} = \frac{\text{budgeted overhead}}{\text{budgeted output.}}$$

Applied Rate.

by doing so we can calculate Total Cost of the product at the time of Production like.

| | |
|---|---|
| Material | 100 |
| labour | 200 |
| V.O. | 50 |
| F.O | 500 |
| Total Cost | 850 |

(#) budgeted f.O = 500 000

" output = 1000

$$R.R = \frac{500\,000}{1000} = 500/\text{unit}.$$

If at the end of Monthe Production of only 800 unit.
then Fixed overhead Recovered = 500 × 800 = 400000.
∴ 100 000 under Recovered.

**Cost sheet**

| | |
|---|---|
| VC | xx |
| FC | 400000 |
| Profit | xxx |
| (-) Under Recovery | 100000 |
| Net Profit = | xxx |

**Situation (1)** if Actual output = budgeted output (end of the month)
then, Recovered = budgeted (overhead). No difference exist between Recovered & budgeted.

(2) if Actual output < budgeted output (end of month), Recovered O/H
then, Recovered overhead < budgeted overhead.

classmate Difference exist: Under Recovery.

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(3) If Actual output > budgeted output, Recovered > budgeted.

Differences exist: Over Recovery.

Differences between Recovered & budgeted exist only when Differences exist in output (volume). Hence we can say Recovery & Budgeted Differences occured due to volume of output. Hence Called "Fixed overhead Volume Variance"

(#) Fixed overhead Volume Variance = $\left[\begin{matrix}\text{Recovered} \\ \text{overhead}\end{matrix} - \begin{matrix}\text{budgeted} \\ \text{overhead}\end{matrix}\right]$

Recovered overhead is a part of fixed overhead Presented in Individual Cost sheet.

i.e [R.R/unit X Actual output]

(#) Fixed overhead Expenditure Variance.

Inflation →

| | Budgeted | Actual |
|---|---|---|
| eg Machine Rent = | 100 000 | 125000. |
| Salary | 200 000 | 250000 |
| Electricity Bill | 100 000 | 125000. |
| Depⁿ | 100 000 | 100000 |
| | 500 000 | 600000 |

If Actual overhead becomes different from budgeted overhead, such difference occured due to Inflation effects but not due to Volume effects. Hence Called "Fixed overhead Expenditure Variance" i.e,

(#) Fixed overhead Expenditure Variance = $\left[\begin{matrix}\text{budgeted} \\ \text{overhead}\end{matrix} - \begin{matrix}\text{Actual} \\ \text{overhead}\end{matrix}\right]$

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Data for fixed overhead Variance.

| Budgeted output | Recovery Rate/unit. | Budgeted Overhead. | Actual overhead |
|---|---|---|---|
| 1000 | 500 | 500 000. | 600 000. |

expenditure Variance

| Actual output | Recovered overhead |
|---|---|
| 800 | 500×800<br>= 400 000. |

Volume Variance

(1) fixed O/H volume Variance = [Recovered - budgeted]

(2) fixed O/H Expenditure Variance = [budgeted O/H - Actual overhead]

(23) [Page-222]

Data for fixed overhead Variance.

| Budget output | Recovery Rate | Budgeted fixed overhead |
|---|---|---|
| 10000 | 2. | 20,000 |

Actual - 30000.

| Actual output | Recovered |
|---|---|
| 8 000 | 16000 |

fixed overhead Volume Variance = Recovered O/H − budgeted O/H

= 16000 - 2000

= 4000 (A)

fixed overhead Expenditure Variance = Budgeted F/O/H − Actual F.O.

= 20000 - 30000

= 10 000 (A).

fixed O/H Cost Variance = Recovered for A/O − Actual O/H = 16000 - 3000 = 1400 (A).

↓

S.C.

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[Page-223]

Data for fixed overhead Variance

| Budgeted fixed ~~output~~ | Recovery Rate | Budgeted f.O |
|---|---|---|
| 5000 | 8 | 40000 |

Actual = 50000.

| Actual output | Recovered |
|---|---|
| 4500 | 36000. |

Fixed O/H Volume Variance = Recovered − Budgeted f.O
= 36000 − 40000 = 4000 A.

Fixed O/H Expenditure Variance = Budgeted f.O. − Actual f.O
= 40000 − 60000
= 20000 (A)

Fixed overhead Cost Variance = Recovered − Actual
= 36000 − 60000
= 24000 (A).

(#) Another Method to Recovered fixed overhead ~~is~~ fixed overhead is on the basis of budgeted Hours,

Budgeted Hour here mean budgeted Hours for budgeted production.

eg budgeted output = 1000 unit
budgeted Hour/unit = ₹10 Hr/unit.
budgeted overhead = ₹500000
∴ budgeted fixed O/H/Hr or Recovery Rate/Hr = budgeted fixed overhead / budgeted Hour = 500000 / 10000 Hr

. Recovery Rate = ₹50/Hr

## Data for fixed overhead variance

| Budgeted Hrs for budgeted Production | Recovery Rate / Hr. | Budgeted Fixed overhead |
|---|---|---|
| 10 000 Hr | 50 / Hr | 500000 |

Actual
600000.

Budgeted Hours for Actual output.

~~50~~ X 10X800

= 8000 Hr.

Recovered

800 X 50.

R.R / Hr X bud. Hrs for A.O.

= 50X8000= 400000.

(20)

## Data for fixed overhead Variance

| Budgeted Hours | Recovery Rate / Hr | Budgeted O/H. |
|---|---|---|
| 40 Hrs. | 35 / Hr | ₹ 1400 |

Actual fixed O/H
1,700

~~Budgeted Hours~~
Actual ~~output~~

~~32 Hrs.~~

$\frac{40}{1400} \times 1200 = 34.285$ Hr.

Recovered.

₹ 1,200

Fixed O/H Cost Variance = Recovered − Actual O/H

= 1200 − 1700 = 500 (A)

fixed O/H Volume Variance = Recovered overhead − Budgeted overhead

= 1200 − 1400 = 200 (A)

fixed O/H expenditure Variance = Budgeted O/H − Actual O/H

= 1400 − 1700

= 300 (A).

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| Budgeted Hours for budgeted output | Recovery Rate/Hr | Budgeted overhead |
|---|---|---|
| 60000 Hrs | 0.75 | 45000. |
| | | Actual |
| | | 50000 |
| Budgeted Hrs for Actual output | | |
| $\frac{60000}{30000} \times 32500 = 65000$ | 0.75 | Recoved<br>48750<br>(0.75×65000) |

Statement of Variances

| S.No | Particulars | Basis | Amt |
|---|---|---|---|
| 1) 1) | Fixed overhead Volume Variance | Recovered OH − Budgeted O/H<br>[48750 − 45000] | 3750 (F.) |
| 2) | Fixed O/H. exp. Variance | Budgeted O/H − Actual overhead<br>45000 − 50000 | 5000 (A) |
| (3) | Fixed O/H Cost Variance | Actual O/H − Recovered ~~OH~~<br>~~− Actual~~<br>[~~50000 − 48750~~<br>48750 − 50000] | 1250 (A) |
| 4) | Variable O/H Exp. Variance | (SR − AR) × AQ(Hr)<br>$\left(\frac{₹60000}{60000\,hr} - \frac{68000}{60000}\right) \times 60000$ | 8000 (A) |
| 3) | Variable O/H Efficiency Variance | (SH − AH) × SR<br>(2Hr × 32500 − 60000) × 1 | 5000 (F). |

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## Data for fixed O/H variance

| Budgeted Hours | Recovery Rate/Hr | Budgeted O/H |
|---|---|---|
| 5000 | 5/Hr. | 25000. |

| | | Applied/absorbed ~~Recovered~~ | ~~Actual~~ |
|---|---|---|---|
| Budgeted Hr for A.O | 5 | 26000 | 26500 |

~~Budgeted Hr for A.O~~

~~$\frac{26000}{5}$ = 5200 Hr~~

Fixed overhead Expenditure Variance = Budgeted O/H − ~~Recovered~~ Actual O/H
= 25000 - 26500 = 1500 A

Fixed overhead Volume Variance = Recovered - Budgeted
= 26000 - 25000 = 1000 F

Fixed O/H Cost Variance = Recovered - Actual = 500 (A)
26000 - 26500

(#) Fixed overhead Volume Variance (Adverse) :- It will occurs due to following two (2) Reasons.

(#) (a) Due to those Hours which have Not been utilized. eg. Power failure etc.
~~Eg~~ (i) Power failure (iii) Non Availability of Raw Material
(ii) Machine Break down (iv) Labour strike.

Such Variance Called Capacity Variance.

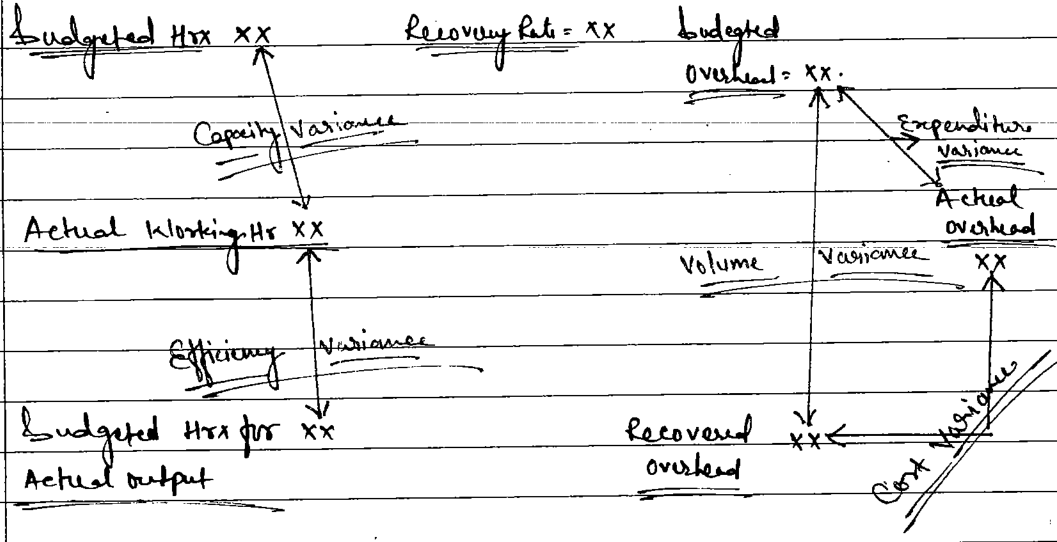
Difference of Budgeted Hrs & Actual Hrs., represents Capacity Variance.

= [Actual Working Hours − Budgeted Hours] × R.R/Hrs. = Capacity Variance.

(#) (b) Due to those Working Hours which have been utilized but Not so effectively & efficiently. i.e, Difference of Budgeted Hours for Actual output & Actual Hrs., Called Efficiency Variance.

[BH for AO − AWH] × R.R/Hrs.

(#) Data for fixed O/H Variances



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Data for fixed overhead Variance

Budgeted Hours = 24000 [120X8X25]

Recovery Rate = 5/Hr

Budgeted Overhead = 120000.

Actual Hours Worked = 21840 [840X26]

Actual Overhead = 150000

Budgeted Hrs for Actual output ~~24000~~ [5305X4] = 21220.

Recovered overhead = 106100 [21220X5]

Statement of Variance

1 Fixed overhead Expend. Variance = Budgeted O/H − Actual O/H = 120000 − 150000 = 30000 (A)

2) Fixed overhead Volume Variance = Recovered O/H − Budgeted O/H = 1061000 − 120000 = 13900 (A)

3) Fixed overhead Efficiency Variance = [Budgeted Hr for Actual output − Actual ~~budgeted~~ Hours] X R.R = [21220 − 21840] X5 = 3100 (A)

4) Fixed overhead Capacity Variance = [Actual Hr − Budgeted Hr] X R.R = [21840 − 24000] X5 = 10810 (A)

5) Fixed overhead Cost Variance = Recovered − Actual = 106100 − 150000 = 43900 (A).

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## Data for fixed overhead Variance.

budgeted Hours= 2400 Recovery Rate = 5 budgeted overhead = 12000

Actual Working Hour = 1600 Actual Overhead = 12800

budgeted Hr for Actual output = 2000. Recovered Overhead = 10000.

[#1] fixed overhead Cost Variance = fixed O/H Volume Variance + F.O Expenditure Var.

2800 (A) = 2000 (A) + fixed O/H Exp. Variance

∴ fixed overhead Exp Variance = −2800 + 2000 = 800 (A)

[#2] fixed overhead Expenditure Variance = budgeted O/H − Actual O/H

800 (A) = 12000 (F) − Actual O/H

∴ Actual O/H = 12000 (F) + 800 (A) = 12800 A.

[#3] Actual Overhead Recovery Rate = $\frac{\text{Actual overhead}}{\text{Actual Hours}} = 8$

$\frac{12800}{8} = 1600$ Hrs.

∴ Actual Hrs = 1600 Hrs.

[#4] fixed O/H Volume Variance = Recovered O/H − budgeted overhead

−2000 = Recovered O/H − 12000

Recovered O/H = 12000 − 2000 = 10000

[#5] ~~Fixed~~ Recovered O/H = budgeted O/H for Actual output × $\frac{\text{bud. O/H}}{\text{bud Hr}}$

10000 = bud O/H for A.O × $\frac{12000}{2400}$

classmate budgeted O/H for A.O = $\frac{10000}{5}$ = 2000.

## Statement Of Variance

| Particulas | Basis | Amt |
|---|---|---|
| 1) Overhead Exp. Variance | [#1] | 800 (A) |
| 2) Actual overhead. | [#2] | ₹12800 |
| 3) Actual Hours for Actual Production | [#3] | 1600 Hrs |
| 4) Overhead Capacity Variance = | (AWH - BH)×R.R.\|Hr<br>[1600 - 2400]×5. | 4000 (A) |
| 5) Overhead Efficiency Variance = | [SHr for AO − AWR]×R.R<br>[2000 − 1600]×5 | 2000 (F) |
| 6) Standard Hour for Actual output = | [#5] | 2000 Hrs. |

### Fixed overhead Capacity Variance

**fixed overhead Calender Variance**

⇓

If Differences of Cost occurs due to differences of days: Called F.O Calender Variance

⇓

$\boxed{[\text{Actual Days} - \text{Budgeted Days}] \times \text{RR}|\text{Day}}$

**fixed overhead Revised Capacity variance**

⇓

If Differences of Cost Hr occur Just due to loss of Hours Called Fixed O|H Revised Capacity variance

⇓

(B|F)

$\boxed{[\text{Total Capacity Variance} - \text{Calender Var}]}$

(eg)

| Budget | | Actual | |
|---|---|---|---|
| Days | Hours | Days | Hours |
| 25 | 200 Hrs. | 24 | 190. |

1 Day = 8 Hrs.
Budgeted overhead = 100000.

Fixed overhead Capacity variance = $(190 - 200) \times \frac{100000}{200 \text{ Hr}}$

$= 5000\ (A)$

Fixed overhead Calender Variance = $\left(\text{Actual Days} - \text{Budgeted Days}\right) \times \frac{100000}{25}$

$(24 - 25) \times 4000 = 4000\ (A)$

Fixed overhead Revised Capacity variance = Total Capacity Variance - Calender Variance

$= 5000(A) - 4000\ (A)$

$= 1000\ (A)$

(24) ~~Page 223~~ Fixed overhead Calendar Variance = $\left(\text{Actual Days} - \text{Budgeted Days}\right) \times R.R|D$

$(25 - 24) \times \frac{12000 \times 0.5}{24} = 250\ (F)$

(#) **Ratio Concepts :-** If Management Would like to know the Relation between efficiency, Capacity Instead of Amt of variance, We should apply Concept of Ratio, Instead of variance Concept.

(i) **Efficiency Ratio :-** How Workers are efficient, (Not in terms of Amt, but in terms of Ratio).

$$\boxed{\frac{\text{Budgeted Hours for Actual output}}{\text{Actual Working Hours}} \times 100}$$

(2) Capacity Ratio = How much part of our Capacity, we are going to utilize.

$$\boxed{\frac{\text{Actual Hours}}{\text{budgeted Hours}} \times 100}$$

It means we are utilizing % of our Capacity.

(3) Activity Ratio = or Volume Ratio = It is a Combined effects of efficiency & Capacity Ratio.

$$\boxed{\frac{\text{budgeted Hours for Actual output}}{\text{budgeted Hours for budgeted output}} \times 100}$$

Logic

(37) [Page-230] budgeted Hours for budgeted output $= \frac{240000}{12} =$ 20000 Labour Hours.

↑ Capacity variance

Volume Variance

Actual Hours = 22 days × 8 Hrs/day × 100 Workers = 17600 Hours.

↑ Efficiency variance

budgeted Hours for Actual output $= \left[(8\text{ Hrs} \times \overset{A}{1200}\text{ Unit}) + (12\text{ Hr} \times \overset{B}{800}\text{ unit})\right]$

$= 9600 + 9600 = 19200$ Hr.

Capacity Ratio $= \frac{\text{Actual Hours}}{\text{budgeted Hours}} \times 100 = \frac{17600}{20000} \times 100 = 88\%$ (A)

~~Efficiency Ratio~~ = Activity Ratio $\frac{\text{bud. Hrs for A O}}{\text{bud. Hr for B.O}} \times 100 = \frac{19200}{20000} \times 100 = 96\%$ (A)

Efficiency Ratio $= \frac{\text{bud. Hr for A O}}{\text{Actual Hours}} \times 100 = \frac{19200}{17600} \times 100 = 109.09\%$ (F).

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Standard Hours Produced i.e Standard Hour for Actual Production = 11000

Budgeted Hour for Budgeted Production = 8 Hr X 25 days X 50 labours
= 10,000 Hrs.

Downtime = 10% i.e 1000

Budgeted Hrs for Bud output = 9000 Hrs
(Productive / after Normal loss)

Data for Variance

Bud Hrs for Bud. Prod$^n$ = 9000

Recovery Rate = $7.\dot{7}$

Budgeted overhead = 70000

Actual Hours = 10000

Actual overhead = 80000

Bud. Hrs for Actual Prod$^n$ = 11000.

Recovered overhead = 85,555 $(11000 \times 7.\dot{7})$

Efficiency Ratio = $\dfrac{\text{Budgeted Hours for A.O.}}{\text{Actual Hours}} \times 100 = \dfrac{11000}{10000} = 110\%$ (F)

Activity Ratio = $\dfrac{\text{Budgeted Hours for A.O}}{\text{Budgeted Hours for B.O.}} = \dfrac{11000}{9000} \times 100 = 122.\dot{2}\%$ (F)

Capacity Ratio = $\dfrac{\text{Actual Hours}}{\text{Budgeted Hours}} \times 100 = \dfrac{10000}{9000} \times 100 = 111.\dot{1}\%$ (F)

Calendar Ratio = $\dfrac{\text{Actual Days}}{\text{Budgeted Days}} \times 100 = \dfrac{24}{25} \times 100 = 96\%$

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[Page-226]

Working Note

①

| Budgeted Cost | P.U |
|---|---|
| Material (20x2) | 40 |
| Labour (5Hrs x 6/Hr) | 30 |
| Fixed O/H | 30 |
| Budgeted Cost/unit | 100 |

② Material Price Variance = 8000 F

= (SP - AP) x AQ = 8000

= (2 - 1.95) x AQ = 8000

∴ AQ = $\frac{8000}{0.05}$ = 1,60,000 ~~units~~ mtr (iv)

③ ~~Material Usage Variance = 5000 A~~

~~(SQ - AQ) x SR = -5000~~

~~(SQ - AQ) x 2 = -50000~~

④ Direct Labour Rate Variance = 5700 A

Direct Labour Efficiency Variance = 2760 F.

Direct Labour Cost Variance = 3000 A.

Direct Labour Cost Variance = LC for Actual output - Actual Cost = 3000

30 x AO/unit - 156000 = -3000

30 x AO/~~unit~~ = 153000

AO ~~Output units~~ $\frac{153000}{30}$ = 5100

∴ AO = 5100 units. [put in W3] (i) =

③ Material Usage Variance = 5000 A

(SQ for AO - AQ) x SR = -5000

((20 x 5100) - AQ) x 2 = -5000

102000 - AQ = $\frac{-5000}{2}$

- AQ = -2500 - 102000

AQ = ~~1270~~ 1,04500.

⑤ Labour Efficiency Variance = 2760 F

$(SH - AH) \times SR = 2760$

for Ao.

$(5\,Hr \times 5100 - AH) \times 6 = 2760$

$25500 - AH = \frac{2760}{6}$

$-AH = 460 - 25500$

Actual Hours = 25040. (#2)

⑥ Actual wages = Actual Hour X AR/Hr.

158000 = 25040 X AR/Hr

$\therefore$ AR/Hr = $\frac{158000}{25040}$ =

AR/Hr = 6.23/Hr (vi)

⑦ fixed overhead Expenditure Variance = Bud - Actual.

-8000 = budgeted - 158000

$\therefore$ budgeted = 158000 - 8000

fixed budgeted O/H = 150000

$\therefore$ budgeted ~~Hours~~ Unit = $\frac{150000}{30}$ = 5000 ~~Hour~~ Unit

$\therefore$ budget unit = 5000 unit (iii)

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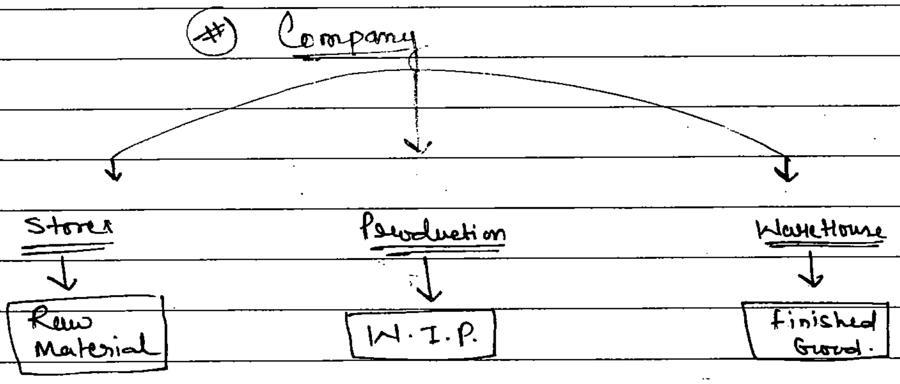
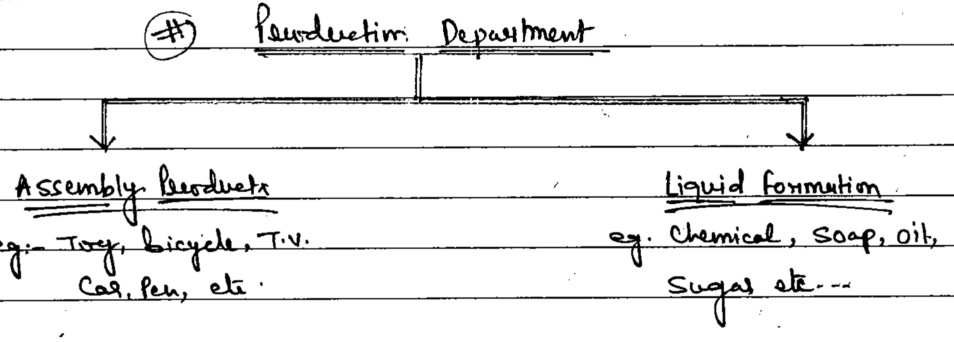
# ㊟ Concept of WIP [Equivalent Production].

㊟ Work In Progress represents the stage of Production which indicates neither Raw material nor finished Goods. i.e, Work has been started but not Completed.

We have two methods - (A) FIFO
(B) Weighted Average.





(eg) 1) Assemble Product
bicycle

Ist Period : 1000 Bicycle Produced. Complete
200 " Incomplete (100%, 80%)
$\downarrow$ Material $\downarrow$ WIP.

IInd Period : Opening WIP.: 200 (100%, 80%)
Current stock: 1500 Bicycle. (Introduced)
: 1600 bicycle complete
100 bicycle incomplete
(100%, 40%)

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| | | | |
|---|---|---|---|
| Opening WIP (100, 80%) | 200 | Opening | 200 |
| Introduce | 1500 | Current | 1400 |
| | | Closing WIP (100%, 40%) | 100 |
| | 1700 | | 1700 |

| | Material | Labour O/H |
|---|---|---|
| Opening | — | 40 (20%) |
| Current | 1400 | 1400 |
| Closing | 100 | 40 |

$$\Downarrow$$

$$\frac{\text{Current Material Cost}}{\text{Current Qty}} = \text{Material Cost / unit} \qquad \frac{\text{Current Labour Cost}}{\text{Current Qty.}} = \text{Labour Cost / unit}$$

## (2) Liquid Formation

Always we should follow FIFO Method in production Department even in Liquid formation because WIP exist in pipeline will produce finished goods at first subsequent Input will produce Finished goods at later stage

However, some times management applies Weighted Average Method in production Department due to labour Negligence (Ineffective Working)

**For WIP in Weighted Average**

St of Equivalent Production

| | | | | Material | Labour |
|---|---|---|---|---|---|
| Opening WIP | 200 | Complete | 1600 | 1600 | 1600 |
| Current Introduce | 1500 | Closing WIP (100%, 40%) | 100 | 100 | 40 |
| | | | | 1700 | 1640 |

↓

It includes work done on WIP in last period.
(Opening)

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(#) In Calculation of Variances always we should follow FIFO Method. as its not possible to Calculate Correct variance from Weighted Average Method.

1) Statement of Equivalent Production (~~Actual~~ output in Current Period).
2) Data for Resource variance
3) Statement of Variance.

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Statement of Equivalent Production.

[FIFO]

| | | | |
|---|---|---|---|
| Opening WIP (100%, 60%) | 250 | Opening (b/d) | 250 |
| | | Current | 950 |
| | | Finished Goods. | 1200 |
| Introduce (b/f) | 1400 | Closing WIP (100%, 20%) | 450 |
| | 1650 | | 1650 |

| | Material | Labour |
|---|---|---|
| Opening | — | 100 [40% of 250] |
| Current | 950 | 950. |
| Closing | 450 | 90 [20% of 450] |
| | 1400 | 1140 |

Data for Resource variance

| Budgeted | | | Revised (1400 & 1140) | | | Actual | | |
|---|---|---|---|---|---|---|---|---|
| Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rate | Amt |
| 2 kg | 10 | 20 | 2800 kg | 10 | 28000 | 2900 kg | 11.0344 | 32000 |
| 3 Hr | 20 | 60 | 3420 Hr | 20 | 68400 | 3300 Hr | 20.60 | 68000 |

Data for fixed O/H Variance.

Budgeted Hrs for bud output = 3×1000 = 3000
Actual Hrs = 3300
bud Hr for A.O = 3 Hrs × 1140 = 3420

R.R = 30/Hr

Budgeted O/H = 1000×90 = 90000
Actual O/H = 99000.
Recovered = 3420 × 30 = 102600.

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Statement of Variance.

| SNo | Particulars | Basics | Amt |
|---|---|---|---|
| 1) | Material Price variance. | (S.P − AC)AQ.<br>(10 − 11.0344) X 2900 | 3000 A. |
| 2) | Labour Rate Variance | (SR − AR) X AH<br>(20 − 20.60) X 3300 | 2000 A |
| 3) | Material Usage variance | (SQ − AQ) X SR<br>(2800 − 2900) X 10 | 1000 . A. |
| 4) | Labour Efficiency Variance | (SH − AH) X SR<br>(3420 − 3300) X 20 | 2400 F. |
| 5) | Fixed O/H Volume Variance = | Recovered − Budgeted<br>102600 − 90000 | 12600 F. |
| 6) | Fixed O/H Exp. Variance = | ~~Actual~~ Budgeted OH − Actual O/H<br>90000 − 88000 | 2000 F |
| 7) | Fixed O/H Efficiency Variance. = | (LH for AO − AH) X R.R<br>(3420 − 3300) X 30 | 3600 F |
| 8) | Fixed OH Capacity variance = | (~~[illegible]~~ −<br>(AH − LH for BO) X R.R<br>(3300 − 3000) X 30 | 9000 F |

(50) [Page-238] Variable overhead per unit = $\frac{92400 - 81600}{7500 - 6000}$ = ₹ 7.20/unit

Fixed = 92400 - 7.2 × 7500

= 33400.

Budget output = 8000 unit

Factory overhead = ₹ 12/unit

Variable = 7.20/unit

Fixed = 4.80/unit

Statement of Equivalent Production

| | | Fifo | | Material | Labour |
|---|---|---|---|---|---|
| Opening [100% / 60%] | 300 | Opening | 300 | ~~739~~ | 120 (40% of 300) |
| Current Introduce (b/f) | 7520 | Current | 7320 | 7320 | 7320 |
| | | Actual output | 7620 | | |
| | | Closing (50%, 40%) | 200 | 100 (50%) | 80 (40%) |
| | 7820 | | 7820 | 7420 | 7520 |

Data for Resource Variance

| Revised budget [7420 : 7520] | | | Actual [7420 : 7520] | | |
|---|---|---|---|---|---|
| Qty | Rate | Amt | Qty | Rate | Amt |
| 1.5 kg × 7420 = 1,11,30 | 24 | 267120. | 11224 | 23.75 | 266570 |
| 3 Hr × 7520 = 22560 Hr | 4 | 90240 | 22400 | 4.3 | 96320. |
| | | | 22400 | 2.6 | 58240 |
| 22560 | $\frac{7.2}{3}$ = 2.4. | 54144 | | | |

DATE

## Data for Fixed overhead variance

BH for BO = 24000. Recoveryrate = 1.6/Hr B. O/H = 38400
(3x8000)

AH = 22400. Actual O/H = 96440.-58240 = 38200.

BH for AO = 22560. Recovered = 36096.

## Statement of variance.

| SN | Particulars | Basic. | Amt |
|---|---|---|---|

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Statement of Equivalent Unit.
(Weighted Average)

| | | | | | | |
|---|---|---|---|---|---|---|
| Opening | WIP (25%) | 200 | | | Labour | Overhead |
| Current | | 825 | Completed | 900 | 900 | 900 |
| ~~Closing~~ | | | Closing WIP. (80%) | 125 | 100 | 100. |
| | | 1025 | | 1025 | 1000 | 1000 |

Statement of Cost

| | Labour | Overhead | Closing WIP (125 units) |
|---|---|---|---|
| Opening | 12000. | 5000 | |
| Current | 178000. | 70000 | Labour = 190×125×80%<br>= 19000 |
| Cost Per unit | $\frac{12000+178000}{100+900.}$ = | $\frac{95000}{1000}$ | Overhead = 95×125×80%<br>= 9500 |
| | $\frac{190000}{1000}$ = 190. | = 95. | 28500 |

(#) Actual output as per Weighted Avg Method always include work done on opening Qty in last year.

Statement of Equivalent Production (FIFO).

| | | | | | Labour (75%) | Overhead (75%) |
|---|---|---|---|---|---|---|
| Opening | WIP (25%) | 200 | Opening | 200 | 150 | 150 |
| | | | Current | 700 | 700 | 700 |
| Current | | 825 | Complete | 900. | | |
| Closing WIP | | – | Closing WIP = (80%) | 125 | 100 | 100 |
| | | 1025 | | 1025 | 950 | 950 |

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Labour Rate Variance = (SR - AR) × AH

= (40 - 44.5) × 4000

= - 4.5 × 4000

= 18000 A.

Labour Efficiency Variance = (SH - AH) × SR

[(950×5) - 4000] × 40

= 30000 F.

Overhead Volume Variance = Recovered - budgeted

= ~~4000X~~ [(20×950×5) - 98000]

[illegible]

= 95000 - 98000

= 3000 A.

Overhead expenditure Variance = budgeted overhead - Actual overhead.

↓

budget

= 98000 - 90000

= 8000 F.

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## Statement of Variance

| | Material | | | Labour | | |
|---|---|---|---|---|---|---|
| Lot No | Actual Qnty | SQ for Actual Prodn | variation | Actual Lab. Hr | SH for Actual Prodn | Variation |
| 245 | 40440 | 24×1700<br>=40800 | 360 F | 5130 | 3Hr×1700 | 30 Hr A. |
| 246 (WIP) | 28825 | 9.24×1200 (100%)<br>=28800 | 25 A | 2890 | 3Hr×80%×1200 | 10 Hr A |
| 247 | 24100 | 24×1000<br>=24000 | 100 A | 2980 | 3×1000 | 20 F |
| | | | | 11000 | 10980 Hr | |

㊉ Material Price Variance = (SR- AR) AQ

$$= \left[22 - \frac{2128000}{95000}\right] \times 95000.$$

$$= (22 - 22.40) \times 95000 = 38000 A.$$

㊉ Labour Rate Variance = (SR-AR) AH.

$$(98-100)\frac{1100000}{100}$$

$$= -2 \times 11000 = 22000 A$$

㊉ Variable O/H Efficiency Variance = (SHr - AWH) × SR

$$= (10980 - 11000) \times 80 \times 60\%$$

$$= 20 \times 48 = 960 A.$$

Data for fixed O/H Variance

Budgeted Hours for Budgeted output 1200

Recovery Rate 80×40% = 32

Budgeted O/H. $\frac{1,15,20,000}{12 month} \times 40$

= 384000

Actual Hours = 11000.

Actual = 912000×40% = 364800.

Budgeted Hours for A.O = 10980

Recovered. O/H

= 351360.

㊉ fixed overhead Volume Variance = Recovered - Budgeted

= 351360 - 384000

= 32640 (A)

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In Marginal Cost, there is no fixed overhead volume Variance. There is only fixed overhead Expenditure variance because we are spreading budgeted fixed O/H over Actual output not as per Absorption basis.

In this approach it is not possible to prepare Individual Cost sheet for every product.

Ⓗ **Sales Variance** = (1) Turnover based
(2) Margin based.

## Margin based

Ⓗ **Sales Margin Value Variance.**

Sales Margin Price Variance | Sales Margin Qty/volume Variance

Ⓗ Actual Profit maybe different from budgeted Profit due to following:-

a) Change in Cost
(b) Change in sales → Price Effects
→ Qty Effects

If we had no change in S.P. & cost, change in profit would be Qty variance.

(eg)

| | Budgeted | Actual |
|---|---|---|
| Qty | 100 | 120 |
| S.P. | 10 | 12 |
| Cost | 5 | 4 |
| Profit | 500 | 960. |

$\boxed{460}$

Cost factor [Cost Mgr] | Sales factor (Sales Mgr)

Overall Cost Variance = SC for AQ − AC for AQ
$= [(120 \times 5) - (120 \times 4)]$
$= 120\ F.$

Ⓗ Sales Margin Price Variance =
$(B.S.P - A.S.P) \times AQ.$
$= (10 - 12) \times 120$
$= 240\ F.$

[ज्यादा बेचने की वजह से कितना Profit बढ़ा है] Ⓗ Sales Margin Volume (Qty) Variance =
$= (BQ - AQ) \times$ budgeted Margin/unit
$(150 - 120) \times (10-5)$
$= 150\ F.$

(eg)

| | Budgeted | Actual |
|---|---|---|
| Qty | 100 | 150 |
| SP | 10 | 14 |
| Cost | 7 | 5 |
| Profit | 300 | 1350 |

(#) ~~Sales Margin Price~~ Overall Cost Variance = Standard Cost for Actual Output − Actual Cost for Actual output

$= [(150 \times 7) - (150 \times 5)]$

$= 300$ F.

(#) Sales Margin Price Variance = (bud S.P − A.SP) × AQS.

$= (10 - 14) \times 150$

$= 600$ F.

(#) Sales Margin ~~Cos~~ Volume Variance = (BQ − AQ) × (BSP − BCP)

$= [(100 - 150) \times (10 - 7)]$

$= 150$ F.

[⇒] In Sales Margin Variance (Profit Reconciliation) We have two (2) approach.

1) Marginal Approach.

a) Overall Cost Variance should be analysed into two parts

(i) Overall Variable Cost Variance

(ii) Fixed overhead :- (A) Fixed O/H Exp. Variance

(B) Fixed O/H Volume Variance (Not Applicable)

(b) Sales Variance

Budgeted Margin Means.

Budgeted Contribution / unit i.e

| | |
|---|---|
| bud. S.P | xx |
| (−) b.v.c | xx |
| | xx |

2) Absorption Approach

| | | |
|---|---|---|
| a) (i) Overall Variable Cost Variance | | xxx |
| (ii) Fixed OH Variance (Exp) | | xxx |
| " | (Volume) | xxx |

(b) Budgeted Margin means

Budgeted Profit / unit

| | |
|---|---|
| B.S.P | xx |
| (−) b.v.c | x |
| (−) b.f.c / unit | x |
| | xx |

DATE

(eg) Calculate Cost & Sales Variance and Reconcile the Profit as per Marginal & ~~Absorption~~ Approach from the following data.

| | Budget | Actual |
|---|---|---|
| Qty | 100 | 150 |
| S.P | 10 | 14 |
| Cost | 4 | 3 |
| fixed O/H | 500 | 600 |
| Profit | 100 | 1050 |

Profit Variance

Budgeted profit − Actual Profit

= 100 − 1050

= 950 F

Cost variance | Sales Margin Variance

① Overall variable Cost Variance.

= [(150×4) − (150×3)]

= 600 − 450

= 150 F

① Sales Margin Price Variance.

= (B.S.P − A.S.P) × AQ

= (10 − 14) × 150

= 600 F

② fixed O/H Exp. Variance.

Bud O/H − Actual O/H.

= 500 − 600

= 100 A.

② Sales Margin Qty Variance

= (BQ − AQ) × Bud. Contribution/unit

= (100 − 150) × 6

= 300 F

Absorption में [illegible]
Contribution की [illegible]
Profit आएगा

∴ Total Cost Variance = 50 F.

∴ Total Sales Margin Variance = 900 F.

In above situation if we Consider Absorption Approach then Calculate Cost & Sales Variance. See below

Absorption P-7-①

③ Fixed O/H Volume Variance =

Addition = Recovered − Budgeted

= [5×150 − 500]

= 250 F. (3ⁿ F.

[illegible] Sales Margin Qty Variance

= [illegible] − AQ) × Bud Profit/unit

[illegible]

[illegible] F.

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## (#) Reconciliation Statement

| Marginal Approach | | Absorption Approach | |
|---|---|---|---|
| Budgeted Profit | XXX. | Budgeted Profit. | XXX |
| Sales Margin Variance | | Sales Margin Variance | |
| Sales Margin Price Variance<br>(BSP - ASP) X AQ. | XX | Sales Margin Price Variance | XX |
| Sales Margin Qty Variance<br>[(BQ - AQ) X B. Cont/unit] | XX | Sales Margin Qty Variance<br>{[BQ - AQ] X Budget Profit/unit.<br>Budgeted Profit/unit. | XX |
| Cost Variance | | Cost Variance | |
| Material Price Variance [SR-AR)XAQ] | XX | Material Price Variance | XX |
| Material Usage Variance([SQ-AQ]XSR) | XX | Material Usage Variance | XX |
| Labour Rate Variance[(SR-AR)XSH] | XX | Labour Rate Variance | XX |
| Labour Efficiency Variance[(SH-AH)XSR] | XX | Labour Efficiency Variance | XX |
| Variable O/H Exp. Variance | XX | Variable O/H Exp Variance | XX |
| Variable O/H. Efficiency Variance | XX | Variable O/H Efficiency Variance | XX |
| Fixed O/H Exp. Variance | XX | Fixed O/H Exp. Variance | XX |
| Fixed O/H Volume Variance | Not Applicable | Fixed O/H Volume Variance | XX- |
| Actual Profit | XXX | Actual Profit | XXX |

[#] In Marginal Approach Fixed overhead Volume Variance automatically Covered in Sales Margin Qty Variance. Hence No need to Calculate seperately.

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[Page-298]

## Statement of Budget

| | 5000 unit | Perunit |
|---|---|---|
| Material [#1] | 30000 | 6 |
| Labour [#2] | 45000. | 9 |
| Variable O/H [#3] | 75000 | 15 |
| fixed O/H. [#4] | 37500 | 7.5. |
| | | |
| Budgeted Cost | 1,87,500 | 375 |
| Profit | 37500 | 75 |
| | | |
| Sales. [#5] | 2,25000 | 45 |

## Statement of Reconciliation.
(Marginal)

| | |
|---|---|
| Budgeted Profit | 37500 |
| | |
| Sales Margin Variance | |
| Sales Margin Price Variance | 6750 F. |
| Sales Margin Qty Variance | |
| [(BQ- AQ) x Bud Cont/unit] | |
| [(5000-4800) x (7.5+7.5)] | 3000 A. |
| | |
| Cost Variance | |
| Material Price Variance | 300 A |
| Material Usage Variance | 600 A. |
| Labour Rate Variance | 750 F |
| Labour Efficiency Variance | 2250 A |
| Variable O/H Expenditure Variance | 3000 F |
| Variable O/H Efficiency Variance | 3750 A |
| Fixed O/H expenditure Variance | 1500 A. |
| | |
| Actual Profit. | 36,600 |

Working

(1) Material Cost Variance = Material Price Variance + Material Usage Variance.

= -300 + -600

= 900 A.

∴ Standard Cost - Actual Cost = -900.

Standard Cost - 29,700 = -900

Standard Cost = 29700-900

= 28800

Budgeted Cost X Actual output = 28800

Budgeted Cost/unit = $\frac{28800}{4800}$

[#1] = ₹6./unit

(2) Labour Cost Variance = Labour Rate Variance + Labour Efficiency Variance

= 750 + (-2250)

= -1500.

Standard Cost for Actual output - Actual Cost = -1500.

Std Cost/unit X AO - 44700 = -1500

Std Cost/unit = $\frac{44700-1500}{4800}$

[#2] = ₹ 9/unit

(3) Variable OIH Cost Variance = Variable OIH Exp. Variance + Variable OIH Efficiency-Variance

= 3000 + (3750)

= -750.

Std Cost for AO - AC = -750

Std Cost/unit = $\frac{-750+72750}{4800}$

[#3] = 15/unit

(4) Fixed overhead Expenditure Variance = Budgeted O/H − Actual O/H. Exp

$-1500 = \text{Budgeted O/H} - 39000$

$\therefore$ Budgeted O/H = 37500 [#4]

(5) Sales Price Variance = 6750 (F)

(Budgeted selling Price − Actual selling Price) X AQ sold = − 6750

$\left(Bsp - \frac{222,750}{4800}\right) \times 4800 = -6750$

$Bsp = \frac{-6750}{4800} + 46.40625$

Bsp/unit = ₹ 45 /unit [#5]

## (#) Identification of Approach

1) If We are given fixed O/H Volume Variance / fixed O/H Volume Variance is required, Always we should follow Absorption Approach.

(2) If all Variances are given except Fixed O/H Volume Variance then We should follow Marginal Approach.

(3) If all details are given then We have an option either to follow Marginal / ~~Ap~~ Absorption Approach.

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(66) [Page 255] **Working**

Let original selling Price be = 100

Revised Price = 115

Last Year 31-3-2013 (budgeted)

Sales ₹3000 Lakh. i.e. 30 lakh unit.

Current Year: 31.3.2014 (Actual)

Sales ₹3277.5 Lakh i.e 28.5 lakh unit.

## Statement of Budget

### Data for Resource Variance

| Budget | | | | Revised [28.5 lakh unit] | | | Actual | | |
|---|---|---|---|---|---|---|---|---|---|
| Qty | Rate | Unit Amt | Total Amt | Qty | Rate | Amt | Qty | Rate | Amt |
| (kg) | [let] | ₹ in Lakh [30 lakh unit] | | | | | | | |
| Material | | | | 190kg | ~~100~~ | 1900 | 205kg # L | 11.5 | 2357.5 |
| 6.66 kg | 10 | ~~2000~~ 66.66 | 2000 | (6.66×28.5 L) | | | | | |
| Variable O/H ~~Labour~~ | | | | 47.5 | 10 | 475. | 50 Hr. # L | 10.5 | 525 |
| 1.66 Hr. | 10 | 16.66 | 500 | (1.66×28.5 L) | | | | | |

## Statement of Reconciliation statement

### Marginal Approach

₹ in lakh

**Budgeted Profit** 200.

Sales Margin Variance → (BSP − ASP) AQ.

Sales Margin Price Variance (100 − 115) 28.5 lakh 427.5 F

Sales Margin Volume Variance (BQ − AQ) × B. Cont/unit

(30 − 28.5) × (100 − 66.6 − 16.6) 25 A

**Cost Variance**

Material Price Variance (SR − AR) AQ = (10 − 11.5) × 205 = 307.5 A

Material Usage Variance (SQ − AQ) × SR = (190 − 205) × 10 = 150 A

Variable OH Exp. Variance [SR − AQ) × AQ = [(10 − 10.5) × 50] 25 A

Variable OH Eff. Variance [(SH − AH) × SR] = [(47.5 − 50) × 10] 25 A

Fixed OH Exp Variance [Bud − Actual] = [300 − 367.5] 67.5 A.

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Actual Profit 27.5 Lakh

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As we know that Fixed overhead Remain same in totality. It does not change due to change in output level change. and Variable overhead per unit remain same.

| At level of 30,000 | | At level 36000 | |
|---|---|---|---|
| overhead = 12/unit | | overhead = | |
| ∴ Total overhead: 30000 × 12 = 360000 | | Total overhead: | 403200. |
| (–) Fixed | 1,44,000 | Fixed | 1,44000 |
| Variable | 2,16 000. | Variable $\left[\frac{216000}{30000} \times 36000\right]$ | 259200 |

However they have taken 432000 as overhead just due to following the relation of 50% of wages

Statement of Profit

| | Budgeted | Actual |
|---|---|---|
| Production | 36000 | 25000 |
| Sales. | 18 00 000<br>[₹50 × 36000 unit] | 12,50 000<br>[₹50 × 25000 unit] |
| Material | 2,88,000 | 192000.<br>[96000 kg × ₹2] |
| Direct wages. | 864000 | 640000<br>[160000 Hr × ₹4] |
| overhead. | 4,03,200 | 310 000<br>[160000 + 150000] |
| Profit | 2,44,800 | 1,08,000 |

Data for Resource Variance

| | Budgeted Per unit (~~36000~~) | | | Revised (25000) | | | Actual (25000) | | |
|---|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Qty | Rat | Am | Qty | Rate | Amt |
| Material | 4kg | 2/kg | 8 | 100000 | 2 | 200000 | 96000 | 2 | 192000 |
| Wages | 6 Hr | 4/Hr | 24 | 150000 | 4 | 600000. | 154000 | 4 | 616000. |
| Variable O/H. | 6 Hr | 1.2/H | 7.2 | 150000 | 1.2 | 180000 | 6000 | 4 | 24000. |
| | | | | | | V.O/H | 154000 | 1.03896. | 160000 |

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## Statement of Reconciliation statement
## Marginal Approach

Budgeted Profit 2,44,800.

Sales Margin Variance

Sales Margin Price Variance (BSP - ASP) X AQ

Sales Margin Volume Variance (BQ - AQ) X B. Cont/unit

(36000 - 25000) X (50 - 8 - 24 - 7.2) 1,18,800 A.

Cost Variance

Material Price Variance [(SR - AR) X AQ] [(2 - 2) X 25000] Nill

Material Usage Variance [(SQ - AQ) X SR] [(100000 - 96000) X2] 8000 F.

Labour Rate Variance [(SR - AR) X AH] [(4 - 4) X 154000] Nill.

Labour Idle Variance [Idle Hr X SR] [6000 X 4] 24000 A.

Labour Efficiency Variance [(SH - AH) X SR] [(150000 - 154000) X4] 16000 A.

Variable OH Exp Variance [(SR - AR) X AH] [(1.2 - 1.03896] X 154000] 24800 F

Variable OH Effe Variance [(SH - AH) X SR] [(150000 - 154000) X 1.2] 4800 A.

Fixed OH Exp. Variance [Bud - Actual] [144000 - 150000] = 6000 A

Actual Profit ⇒ 1,08000.

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## ㊟ Concept of Mix & Yield Variance.

Material Cost Variance

Material Price Variance | Material Usage Variance

Material Mix Variance | Material Yield Variance

**Material Mix Variance**

SR X (Standard Ratio for Actual Input − Actual Ratio for Actual Input)

OR

$$\left[ SR \times \frac{\text{Total Actual Input}}{\text{Total Stand. Input}} - \text{Actual Input} \times SR \right]$$

**Material Yield Variance**

[SR (Standard Ratio for Total Standard Qty − Standard Ratio for Total Actual Qty)

(or) [SC for AO − SC X $\frac{TAI}{TSI}$]

eg 1kg biscuits

| | Qty | Rate | Cost | Actual Qty | Rate | Cost |
|---|---|---|---|---|---|---|
| Atta | 1kg | 5 | 5 | 1.5kg | 5 | 7.5 |
| Maida | 1kg | 10 | 15 | 0.5 kg | 10 | 5 |
| | 2kg | | 15 | 2kg. | | 12.5 |

∴ Material usage Variance =

Atta = (1kg − 1.5 kg) X 5 = 2.5 A

Maida = (1kg − 0.5kg) X10 = 5 F

2.5 F.

Usage Variance or Yield Variance.

This is called Manipulation Parts i.e Mix Variance.

Now Again Actual (1kg biscuits)

| | Stad | Qty | Rate | Amt |
|---|---|---|---|---|
| Atta | ½ | 0.75 kg | 5 | 3.75 |
| Maida | ½ | 0.25 kg | 10 | 2.5 |
| | 1kg | 1kg. | | 6.25 |

Mix variance by Comparing Actual Ratio with standard Ratio.

[Material usage variance = (SQ − AQ) X SR.]

Atta = (1kg − 0.75) X5 = 1.25

Maida = (1kg − 0.25) X10 = 7.50

8.75 F.

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**Concept of Mix Variance :-** Sometimes Management declare Bonus on Favourable Part of Material Usage Variance i.e, Saving in Qty with SR.

In such circumstances, Production Manager may manipulate the resources by increasing the Qty of having low Rate correspondingly decreasing the Qty of Material having High rate, Just to increase the Amount of Favourable Variance (Material Usage). Management Accountant identified such part & seperate from Material Usage Variance & called Material Mix Variance.

Bonus should not be provided on such Manipulated parts. i.e Bonus should be given only on Effective Working i.e Actual Qty saved with maintaining our standard Ratio.

Material Usage Variance

↓ Material Mix Variance ↓ Material Yield Variance

Material Mix Variance = [Std Ratio for Actual Input − Actual Ratio for Actual Input] X SR

Material Yield Variance = SR X [Standard Ratio for Total Std Qty − Std Ratio for Total Actual Qty]

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## Data for Material Variances

| | Budget (90 Finished kg.) | | | Revised (620 kg) | | | Std Ratio ⇓ | Actual (620kg) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Qty kg | Rate | Amt (₹) | Qty | Rate | Amt | | Qty | Rate | Amt |
| A | 40 | 30 | 1200 | 275.55<br>$\left[40 \times \frac{620}{90}\right]$ | 30 | 8267 | 300 | 350 | 25 | 8750 |
| B. | 60 | 40 | 2400 | 413.33<br>$\left[60 \times \frac{620}{90}\right]$ | 40 | 16533 | 450 | 400. | 45 | 18000 |
| | 100kg | | 3600 | 688.88<br>$\left[100 \times \frac{620}{90}\right]$ | | 24800 | 750 | 750 | | 26750 |

## Statement of Variances

Material Cost Variances. = Standard Cost - Actual Cost

| | | |
|---|---|---|
| A | 8267 - 8750 | 483 A |
| B. | 16533 - 18000 | 1467 A |
| | | 1950 A |

Material Price Variance $(SR - AR) \times AQ$.

| | | |
|---|---|---|
| A | $(30 - 25) \times 350$ | 1750 F |
| B | $(40 - 45) \times 400$ | 2000 A. |
| | | 250 A |

Material Usage Variance. $(SQ - AQ) \times SR$

| | | |
|---|---|---|
| A | $[(275.55 - 350) \times 30]$ | 2233 A |
| B | $[(413.33 - 400) \times 40]$ | 533 F. |
| | | 1700 A |

Material Mix Variance $\left(\begin{matrix}\text{Std Ratio} \\ \text{for AI}\end{matrix} - \begin{matrix}\text{Actual Ratio} \\ \text{for AI}\end{matrix}\right) \times SR$

| | | |
|---|---|---|
| A | $(300 - 350) \times 30$ | 1500 A |
| B | $(450 - 400) \times 40$ | 2000 F. |
| | | 500 F |

Material Yield Variance $\left(\begin{matrix}\text{Std Ratio} \\ \text{for Total} \\ \text{Std. Output}\end{matrix} - \begin{matrix}\text{Std Ratio} \\ \text{for Total} \\ \text{Actual output}\end{matrix}\right) \times SR$.

| | | |
|---|---|---|
| A | $(275.55 - 300) \times 30$ | 733 A |
| B | $(413.33 - 450) \times 40$ | 1467 A |
| | | 2200 A |

(4) Data for Material Variance.

| | Budgeted (90 kg) | | | Revised | | | Actual (90 kg) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Qty | Rate | Amt | SR | Qty | Rate | Amt. |
| P | 50 | 15 | 750 | | | | 55 | 40kg. | 25 | 1000 |
| Q | 50 | 30 | 1500 | | | | 55 | 70kg. | 32.14 | 2250 |
| | 100 kg | | 2250 | | | | 110kg | 110 kg [#1] | | 3250 [#2] |

Working Material Yield Variance = 225 A.

$$\begin{pmatrix} \text{Std Ratio for} & - & \text{Std Ratio for} \\ \text{Total Std Qty} & & \text{Total Actual Qty} \end{pmatrix} \times SR = -225.$$

P (50 − 0.5x) 15 = 750-75x

Q (50 − 0.5~~x~~) 30 = ~~600~~ 1500-15x

$\underline{\underline{-225}}$

∴ ~~750-15x+600=-225~~ = 750 − 7.5x + (1500 − 15x) = −225

~~15x = 750-600+225~~ 7.5x+15x = 750+1500-225

~~x = 375/15 = 25~~ = 22.5x = ~~2475~~

x = 2475/22.5 = ~~[illegible]~~ 110

∴ ~~x = 25~~

∴ x = 110 [#1]

Material Cost Variance = 1000 A.

= Std Cost − Actual Cost

= 2250 − Actual Cost = −1000

Actual Cost = 2250+1000

= 3250. [#2]

Material Mix Variance = $\left[\text{Std Ratio for Actual Input} - \text{Actual Ratio for Actual Input}\right] \times SR$

P (55 − 40)15 = ~~60 F~~ 225 F

Q (55 − 70) × 30 ~~120 A~~ 450 A

~~60 A~~ 225 A

Material Usage Variance = (SQ − AQ)SR.

P = (50 − 40) × 15 = 150 F

Q (50 − 70) × 30 = 600 A

450 A

Material Price Variance = (SR − AR) × AQ

P = (15 − 25) × 40 = 400 A

Q = (30 − 32.14) × 70 = 150 A

550 A

Actual loss of Actual Input.

(100 − 90) = 20 kg.

Actual Input of Chemical 'P' = 40 kg

Actual Price per kg of chemical Q = $\frac{2250}{70}$ = ₹32.14/kg

(11) ~~page-217~~

Data for Material Variance

| | Budgeted (1Eq) | | | Std Ratio | Actual | | |
|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | | Qty | Rate | Amt |
| S | 50 | 12 | 600 | 0.5x 55 | x-70.<br>40. | 15 | 600 |
| T | [#1]<br>50kg | 15 | 750 | 0.5x 55 | 70 | 20 | 1400 |
| | 100 | | 1350 | x. | x i.e 110<br>[#2] | | 2000. |

Material Usage Variance = $(SQ - AQ) \times SR$

$T = (SQ - 70) \times 15 = -300$

$SQ = \frac{-300 + 1050}{15}$

$SQ = 50 \text{ kg}$ [#1]

Material Mix Variance = (S. Ratio for A.I - ~~Actu~~ A. Ratio) XSR

~~0~~ S = $[0.5x - (x - 70)] \times 12 =$

$- T = [0.5x - 70] \times 15 =$

$(0.5x - x + 70) \times 12 + (0.5x - 70) \times 15 = -45.$

~~[illegible]~~ $-6x + 840 + 7.5x - 1050 = -45$

$1.5x = -45 + 1050 - 840$

$x = \frac{165}{1.5} = 110.$ [#2]

Material Yied Variance = $\left( \begin{array}{c} \text{Std Ratio for} \\ \text{Total Std Qty} \end{array} - \begin{array}{c} \text{Std Ratio for} \\ \text{Total Actual Qty} \end{array} \right) \times SR$

1st Approach

S = $(50\text{kg} - \overset{55}{\cancel{45}}) \times 12 =$ ~~[illegible]~~ 60 A

T $(50\text{kg} - \underset{55}{\cancel{35}}) \times 15 =$ ~~[illegible]~~ 75 A

~~[illegible]~~ 135 A

2nd Approach $(TSQ - TAQ) \times$ ~~L~~ W.A.R.

$(100 - 110) \times \frac{1350}{100} =$

$-10 \times \frac{1350}{100} = 135\ A.$

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Material Cost Variance = Std - Actual

S = ~~100~~ 600 - 600 = Nill

T = 750 - 1400 = 650 A

Material Price Variance = (SR - AR) x AQ

S = (12 - 15) x 40 = 120 A

T = (15 - 20) x 70 = 350 A.

Material Usage Variance = (SQ - AQ) x SR

S = = (50 - 40) x 12

= ~~300A~~ . 120 F

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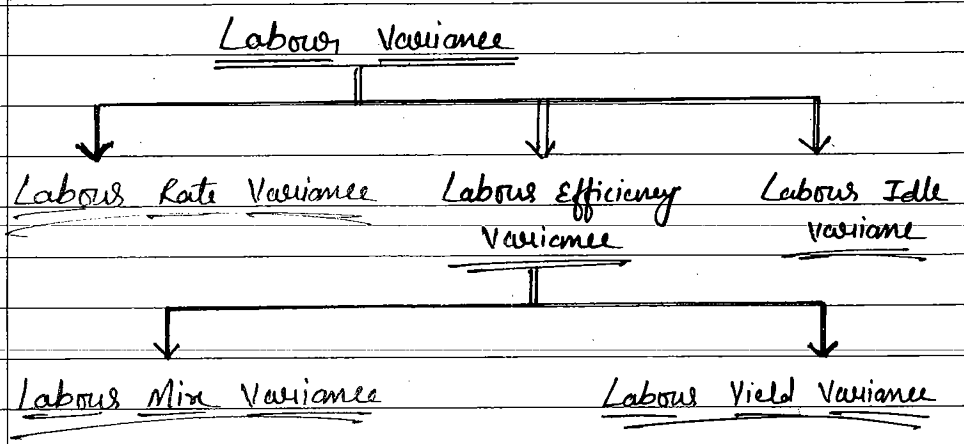
## Labour Variance



$SR\left[\text{Standard Ratio for Actual Input Hrs} - \text{Actual Ratio for Actual Input Hrs}\right]$

$SR\left[\text{Standard Ratio for Total std Labour Hrs} - \text{Standard Ratio for Total Actual Labour Hrs}\right]$

OR

$[TSLH - TALH] \times WASLR$

(12) [Page-218]

Labour Rate Variance = 160 F.

$(SR-AR)\times\underset{AH}{\cancel{\ }} = 160\ 1$

$(3-2)\ AH = 160$

$AH = 160.$

∴ Actual Labour Hours (semi skilled) = 160

Semi Skilled Labour = $\frac{160}{40}$ = 4 Labour. #

∴ Unskilled Labour = ½ of semiskilled

= Unskilled Labour = ½ × 4 = 2 Labour #

∴ Skilled Labour = 9 − 4 − 2 = 3 Labour. #

**Budgeted Hours**

| | | |
|---|---|---|
| Skilled | 4 labour X 40 Hr = | 160 |
| Semi skilled | 3 labour X 40 Hr = | 120 |
| Unskilled | 2 labour X 40 Hr = | 80 |
| | | 360 LHr |

## Data for Labour Variance

| | Revised budget | | | Std Ratio for Actual labour Hrs | Actual | | |
|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | | Qty | Rate | Amt. |
| Skilled | 120 | 6 | 720 | 160 | 3X40=120. | 7 | 840 |
| Semi skilled | 90 | 3 | 270 | 120 | 4X40=160 | 2 | 320 |
| Unskilled | 60. | 1 | 60 | 80 | 2X40=80 | 2 | 160 |
| | 270 LHr<br>[16: 12: 8] | | 1050 | 360<br>[16:12:8] | 360 | | 1320 |

### # Labour Rate Variance (SR- AR)AWH.

Skilled = (6-7) X 120 = 120 A

Semi skilled = (3-2) X 160 = 160 f

Unskilled = (1-2) X 80 = 80 A

40 A

### # Labour Efficiency Variance = (SH-AH) X SR

Skilled = (~~160~~ 120 - 120) X 6 = ~~240 f~~ Nill

Semi Skilled = (~~120~~ 90 - 160) X 3 = ~~120 A~~ 210 A

Un Skilled = (~~80~~ 60 - 80) X 1 = ~~Nill~~ 20 A

~~120 f~~ 230 A

### # Labour Mix Variance = [Standard Ratio for Actual Hours - Actual Ratio for Actual Hrs] X SR

Skilled = (160 - 120) X 6 = 240 F

Semi skilled = (120 - 160) X 3 = 120 A

Un skilled = (80 - 80) X 1 = Nill

120 F

### # Labour Yield Variance = [Standard Ratio for Total Std Labour Hr - Standard Ratio for Total Actual Labour Hrs] X SR

Skilled = (120-160) X 6 = 240 A

Semi skilled = (90-120) X 3 = 90 A

Un skilled = (60-80) X 1 = 20 A

350 A.

## # Sales Variance Turnover based.

# This is to be analysed only when it is required in Question.

# It has no Impact on profit

# This is to be analysed to evaluate the performance of sales manager i.e, to provide bonus / commission his effective Working.

# Effective Working :- (a) If Selling Price reduced and Quantity sold increased then its a Market Rule, then No. Effective Working from sales Department.

(b) Selling Price increased or Constant and Quantity sold increased then it is known as Effective Working.
For this purpose we analyse selling Price variance & sales Volume Variance (Turnover based). —

### Sales Variance (Turnover based).

**Sales Price Variance**

$[BSP - ASP] \times AQ.$

**Sales Volume Variance.**

$[BQ - AQ] \times BSP$

**Sales Mix Variance**

$BSP \begin{bmatrix} \text{Std Ratio for Actual Qty sold} - \text{Actual Ratio for Actual Qty sold} \end{bmatrix}$

**Sales Yield Variance**

$\begin{bmatrix} \text{Std Ratio for Total Bud. Qty sold} - \text{Std Ratio for Total Actual Qty Sold} \end{bmatrix} \times BSP.$

OR

$[TSQ - TAQ] \times W.A.B.S.P.$

**Sales Market Size Variance**

(Change in size X Bud. share) X W.A.B.S.P

**Sales Market share variance**

[Change in share X Industry Sale X W.A.B.S.P]

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| | Turnover Based. | Margin Based |
|---|---|---|
| 1) | It has no Impact on Profit | 1) It has impact on Profit |
| 2) | To fix the Commission/Bonus to analyse the efficiency of Sales Department. | 2) To find out the factors which are responsible for change in Profit. |

Sales Variance (Margin Based) = (BP - AP with Bud. Cost)

- Sales Margin Price Variance.
  [(BSP - ASP) x AQS]
- Sales Margin Volume Variance.
  [(BQ - AQ) x Bud Margin/unit]
  - Sales Margin Mix Variance
    Budgeted Margin/unit (Std Ratio for Actual Qty sold - Actual Ratio for Actual Qty sold)
  - Sales Margin Yeild Variance.
    (Std Ratio for ~~Actual~~ Total std Qty ~~sold~~ - Std Ratio for for Total Actual Qty) x Bud Margin/unit
    OR
    (TSQ - TAQ) W.A.B.M/unit
    - Sales Margin Market Size Variance
    - Sales Margin Market share Variance

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Data for Sales Variance.
[Turnover based]

| | Budgeted sales | | | Std Ratio for | Actual sales | | |
|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Actual Qty sold | Qty | Rate | Amt |
| x | 70 | 10 | 700 | 105 | 80 | 20 | 1600 |
| Y | 30 | 20 | 600 | 45. | 70 | 25 | 1750 |
| | 100 | | 1300 | 150<br>7:3 | 150 | | 3350 |

Sales Value Variance = Budgeted sale - Actual sales.

x = 700 - 1600 = 900 F
Y = 600 - 1750 = 1150 F.
2050 F.

Sales Price Variance = (BSP - ASP) X AQ

x = (10 - 20) X 80 = 800 F
Y = (20 - 25) X 70 = 350 F
1150 F.

Sales Volume Variance = (BQ - AQ) X BSP.

x = (70 - 80) X 10 = 100 F
Y = (30 - 70) X 20 = 800 F
900 F

Sales Mix Variance = (Std Ratio for Actual Qty sold - Actual Ratio for Actual Qty sold) X BSP

x = (105 - 80) X 10 = 250 A
Y = (45 - 70) X 20 = 500 F
250 F

Sales Yield Variance = (Std Ratio for Total Std Qty sold - Std Ratio for Total Actual Qty sold) X BSP.

OR

(TBQ - TAQ) X BSP.

x = (70 - 105) X 10 = 350 F
Y = (30 - 45) X 20 = 300 F
650 F

classmate

Data for sales Variance
[Margin basis.]

|  | Budgeted Profit |  |  | Std Ratio for Actual Qty sold. | Actual Profit |  |  |
|---|---|---|---|---|---|---|---|
|  | Qty | Rate Profit | Amt |  | Qty | Rate Profit | Amt |
| x | 70 | 5 | 350 | 105 | 80 | 15 | 1200 |
| y | 30 | 16. | 480 | 45. | 70 | 21 | 1470 |
|  | 100 |  | 830 | 150 | 150 |  | 2670 |
|  |  |  |  | 7:3 |  |  |  |

Sales Margin Value Variance. = (Bud. Profit - Actual Profit)

x = 350 - 1200 = 850 F
y = 480 - 1470 = 990 F
1840 F

Sales Margin Price Variance = (BSP - ASP) X AQS.
x = (10 - 20) X 80 = 800 F
y = (20 - 25) X 70 = 350 F
1150 F

Sales Margin Volume Variance = (BQ - AQ) X B Profit/unit
x = (70 - 80) X 5 = 50 F
y = (30 - 70) X 16 = 640 F
690 F

Sales Margin Mix Variance = (Std Ratio for Actual Qty sold - Actual Ratio for Actual Qty sold) X bud Profit

x = (105 - 80) X 5 = 125 A
y = (45 - 70) X 16 = 400 F
275 F

Sales Margin Yield Variance = (TSQ - TAQ) X W.A B.P/Unit =
= (100 - 150) X 8.3 = 415 F.

= (Std Ratio for Total Std Qty - A Std Ratio for Total Actual Qty) X B P/unit

X = (70 - 105) X 5 = 175 F
y = (30 - 45) X 16 = 240 F   415 F.

(#) **Industry** = A Group of Company Producing/selling same types of Product e.g. Car Industry, Steel Industry etc.

(#) Market Size = Total Demand of Product in Industry

(#) **Market share** :- Demand of Product for a particular Company in Industry ie Increase in Qty of our product Correspondingly Decrease the Qty of others.

(eg)

| | Industry | Our share | Qty |
|---|---|---|---|
| budgeted | 10000 | 40% | 4000 |
| Actual | 15000 | 50% | 7500. |
| | | | 3500 Qty↑ |

Weighted Avg. bud. Selling Price = ₹10.

**Sales Market Size Variance**

5000 X 40% = 2000 X 10 = 20000.

$$\left\{\left[(\text{Change in size}) \times \underset{\text{Share}}{\text{budget}}\right] \times \text{WABSP}\right\}$$

**Sales Market Share Variance**

15000 X 10% X 10 = 15000.

$$\left[(\text{Change in share}) \times \underset{\text{sale}}{\text{Actual Industry}} \times \text{WABSP}\right]$$

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[Page-233]

## Data for Sales Variance [Margin based]

| | Budgeted Contribution | | | Std Ratio for | Actual Contribution | | |
|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Actual Qty | Qty | Rate<br>[ASP-BC] | Amt |
| x | 1000 | 6 | 6000 | 1100 | 8000 | [22-14]<br>8 | 64000 |
| Y | 9000 | 2 | 18000 | 9900 | 3000 | [13-8]<br>5 | 15000 |
| | 10000 | 2.4 | 24000 | 11000 | 11000 | | 79000 |

Sale Margin Value Variance = Budget Contribution - Actual Contribution.

x = 6000 - 64000 = 58000 F
Y = 18000 - 15000 = 3000 A
55000 F.

Sales Margin Price Variance = (Bud Contribution Per unit - ~~Actual~~ Contribution Per unit) X AQ

x = (6-8) X 8000 = 16000 F
Y = (2-5) X 3000 = 9000 F
25000 F

Sales Margin Volume Variance = (BQ - AQ) X Bud Contribution/unit

x = (1000 - 8000) X 6 = 42000 F
Y = (9000 - 3000) X 2 = 12000 A
30000 F.

Sales Margin Mix Variance. = (Std Ratio for Actual Qty - Actual Ratio for Actual Qty) X B. Cont/unit

x = (1100 - 8000) X 6 = 41400 F
Y = (9900 - 3000) X 2 = 13800 A
27600 F.

Sales Margin Yield Variance = (Std Ratio for Std Qty - Std Ratio for Actual Qty) X B. Cont/unit.

x = (1000 - 1100) X 6 = 600 F
Y = (9000 - 9900) X 2 = 1800 F
2400 F.

Sales Market Size Variance = (Change in Market size X Bud. Share) X W. A. B. Cont.

= $\left[\{(68750 - 50000) \times 20\%\} \times 2.4\right] =$

= $[18750 \times 20\% \times 2.4] = 9000$ F

Sales Market Share Variance = [(Change in share X Actual Industry Sale) X W. A. B. Cont]

= $[(20-16)\% \times 68750 \times 2.4] =$ ~~6600 A~~

$[(68750 \times 4\%) \times 2.4] = 6600$ A

Working

| | Industry | Share | Qty |
|---|---|---|---|
| Budgeted | 50000 | 20% | 10000 |
| Actual | 68750 | 16% | 11000 |
| | 18750 | | 1000 |

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| | Budgeted ~~Combination~~ | | | Std Ratio for | Actual | | |
|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Actual Qty | Qty | Rate | Amt |
| Bravo Toy | 4000 | 100 | 400000 | 7200 | 9000 | 150 | 1350000 |
| Champim Toy | 6000 | 200 | 1200000 | 10800 | 9000. | 300. | 2700000 |
| | 10000 | 160 | 1600000 | 18000 | 18000 | | 40,50000 |

4:6.

| | Industry | Share | Qty |
|---|---|---|---|
| Budgeted | 50000 | 20% | 10000 |
| Actual | 60000 | 30% | 18000. |
| | 10000 | | 8000 |

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Sales Market Size Variance = (Change in size X bud share) X W.A.b.S.P

= [(50000 - 60000) X 20% X 160] = 320000 F.

Sales Market share Variance = [Change in share X $\overset{\text{Actual}}{\text{Industry}}$ Sale X W.A.b.S.P]

= [(20 - 30)% X 60000 X 160]

= (60000 X 10%) X 160 = 960000 F.

Sales ~~Price~~ Value Variance = bud. Sale - Actual Sales

Bravo = 400000 - 1350000 = 950000 F

Champion = 1200000 - 2700000 = 1500000 F    2450000 F

Sale Price Variance.

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~~(Page 231)~~

Data for Sales Variance.

|  | Budgeted sale |  |  | Std Ratio for |  Actual Sales |  |  |
|---|---|---|---|---|---|---|---|
|  | Qty | Rate | Amt | Actual Qty | Qty | Rate | Amt |
| R | 400 | 12 | 4800 | $0.5x$ 650 | 500 | 15 | 7500 |
| S. | 400 | 15 | 6000 | $0.5x$. 650 | $\overset{800.}{x-500.}$ | 20. | 16000. |
|  |  |  | 10800 | $x=1300$ | $x=1300$ |  | 23500 |

Sales Volume Variance = $(SQ-AQ)\times SR$ = ~~1200~~

$R = (SQ-500)\times 12 = -1200 \quad \therefore SQ = 400.$

$S = (400-AQ)\times 15 = (400-800)\times 15 = 6000$ F.

Sales Mix Variance = $\left(\text{Std Ratio for } \cancel{\text{Std}} \text{ Actual Qty} - \text{Actual Ratio for Actual Qty}\right)$ BSP. = -450

$R = (0.5x - 500)\times 12 =$

$S = (0.5x - x + 500)\times 15 =$

$(0.5x - 500)\times 12 + (0.5x - x + 500)\times 15 = -450$

$6x - 6000 + 7.5x - 15x + 7500 = -450$

~~$2x = -450 - 6000 + 7500$~~

$-1.5x = 6000 - 7500 - 450$

$$x = \frac{1950}{1.5}$$

$x = 1300.$

Sales Mix Variance $\overset{(R)}{=} (6500 - 500)\times 12 = 1800$ A

$(S) = (650 - 800)\times 15 = 2250$ F.

Sales Price Variance = $(BSP \; ASP)\times AQ$

R = $(12-15)\,500 = 1500$ F

S = $(15-20)\times 800 = 4000$ F.

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Sales value Variance = Budgeted sale - Actual Sales

R = 4800 - 7500 = 2700 F

S. = 6000 - 16000 = 10,000 F.

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## (#) Planning & Operating Variances.

Traditional Variance. = [ORiginal Std - Actual Std]

Planning Variance
~~[Revised Standard - Actual Standard]~~
[Original Std - Revised Std]

Operating Variance.
[Revised Std - Actual Std]

(eg)

| | Original standard | | | Revised standard | | | Actual standard | | |
|---|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rate | Amt |
| x | 5kg | 10 | 50 | Y= 6.5kg | 11 | 71.5 | Y= 8kg | 13 | 104 |

| | MCV | MPV | MUV |
|---|---|---|---|
| Traditional = Variance<br>[original std _ Actual std] | [ORg std - Act std]<br>= 50 - 104<br>= 54 A | (10-13)×8<br>= 24 A | (5-8)×10<br>= 30 A. |
| Operating = Variance<br>[Reviced std _ Actual std] | 71.50-104<br>= 32.5 A | (11-13)×8<br>= 16 A | (6.5-8)×11<br>= 16.5 A |
| Planning Variance<br>[ORg. Stand - Revised std] | 50-71.5<br>= 21.5 A | (10-11)×6.5<br>= 6.5 A | (5-6.5)×10<br>= 15 A. |

Concept = If the resources (Material | Labour) which are being utilized at the time of Actual working, had not been existed in our original budget. (Non availabity of resoure in market due to market Condition | Govt Policy), We should prepare revised budget for the resource which are being utilized. In such circumstances we should analysed Traditional Variance into Planning & operating variance

Some times Rates are being revised due to change in govt Policy, then ~~also~~ also original budget should be revixed.

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| | Original budget [1164 F.G] | | | Revised Budget | | | Actual [1164 F.G.] | | |
|---|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rate | Amt |
| Fruit Extract | 400 | 1 | 400 | 400 | 2 | 800 | 428 | 4 | 1712 |
| G. Syrup | 700 | 2 | 1400 | 700 | 2.4 | 1680 | 742 | 5 | 3710 |
| Peeting | 100 | 3 | 300 | 100 | 3 | 300 | 125 | 5 | 625 |
| | 1200 | | 2100 | 1200 | | 2780 | 1295 | | 6047 |
| Labour | 18 | 2 | 36 | 18 | 2 | 36 | 20 Hrs | 4 | 80 |
| | | | 2136 | | | 2816 | | | 6127 |

(#) Traditional Variance = Original budget - Actual budget
= 2136 - 6127 = 3991 A.

Traditional Price Variance = (Original Rate - ~~Revised~~ Actual Rate) x AQ

Fruit Extract = (1-4) x 428 = 1284 A

Glucose Syrup = (2-5) x 742 = 2226 A

Peeting = (3-5) x 125 = 250 A ~~3760~~.

Labour = (2-4) x 20 = 40 A 3800 A

Traditional Usage Variance = (Original Qty - Actual Qty) x Original Rate

Fruit Extract = (400-428) x 1 = 28A

Glucose Syrup = (700-742) x 2 = 84A

Pectin = = (100-125) x 3 = 75A

Labour (Eff) = (18-20) x 2 = 40A ~~281 A~~ 191 A

(#) <u>Operating Variance</u> = Revised - Actual.
= 2816 - 6127 = 3311 A

~~Operating [illegible]~~ =
Material Price Variance = (Revised Rate - Actual Rate) X AQ.
= Fruit Extract = (2 - 4) X 428 = 856 A
Glucose syrup = (2.4 - 5) X 742 = 1929.2 A
Pectin = (3-5) X 125 250 A

Labour Rate Variance = (2-4)X20 = 40 A 3075.2 A

Material Usage Variance = (Revised Qty - Actual Qty) X Revised Rate
Fruit Extract = (400 - 428)X2 = 56 A
Glucose Syrup = (700 - 742)X2.4 = 100.8 A
Pectin (100 - 125)X 3 = 75 A

Labour Efficiency Variance = (18-20) X 2 = 4 A 235.8 A

(#) <u>Planning Variance</u> [Original - Standard]
= 2136 - 2816 = 680 A.

Material Price Variance = (Original Rate - Revised Rate) X Revised Qty
Fruit Extract = (1-2)X400 = 400A ~~400A~~
Glucose Syrup = (2 - 2.4)X700 = 280 A 680A.

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Most Important Always Askx in Exam

## (#) Balance ScoreCard :-

Balance scorecard is a statement Presented to Present Shareholder, Investor, Creditor, Bankerx, Public, financial Institution, to invest their money in our company. This statement presented along with Annual balance sheet.
Itx not a statutory obligation to Present Balance Score Card.
In this statement we place equal Rank for financial area as well as Non financial Area.
We should follow Marginal Approach in preparing Balance Score card to maintain Uniform Costing Principal.

Balance score care are of two types
1) Financial Area
2) Non financial Area.

[★] 1) Financial Area Covers the folloing point.

i) Growth factor
ii) Price factor
iii) Productivity factor

} Change in Profit for the Year.

(#) All these factors are responsible for the change in Profit i.e, "These are equal to sales & Cost Variance."

(1) Growth :-

Revenue Effects of Growth.

a) Excess amt received due to excess Qty sold. XXX
[(Bud Qty - Actual Qty) X bud. selling Price]

Cost Effect of Growth.

(b) Excess Cost incurred due to excess Qty sold. XXX
[(Bud Qty sold - Actual Qty sold) X Variable Cost/unit budget]

Sales Margin Volume Variance XXX

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(2) **Price factor**

c) **Revenue Effects of Price** = change in Revenue due to change in Price.

(BSP - ASP) AQ Sold.

d) **Cost Effects of Price** change in Price for Resources.

Inflation Effect > Price. Material = (SR - AR) X AI Used.
Per kg

Labour = (SR/Hr - AR/Hr) X Actual Hours.

Overhead (Fixed) = (Bud. Overhead - Actual overhead)

(3) **Productivity**

How effectively we utilized our Resources.

e)

Material Usage Variance = $\left(\begin{array}{c}\text{Std Input} \\ \text{for A O}\end{array} - \begin{array}{c}\text{Actual Input} \\ \text{for A.O}\end{array}\right) \times SR.$

Labour Efficiency Variance = $(SH - AH) \times SR$

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## Statement of change in Profit ₹ in lakh

| | 2013-14 | 2014-15 |
|---|---|---|
| Revenue | 16700 | 18,597.5 |
| | [4L X 4175] | [4.3L X 4325] |
| Less:- Cost | | |
| Material | 11280 | 12173.5 |
| | [2 4L x 470] | [2 51L X 485] |
| Labour. | 960 | 1078.8. |
| | [32 LHr X 30] | [34.8 L Hr X 31] |
| Fixed Overhead | 160 | 176. |
| Profit | 4300L | 5169.2 L. |

## Balance Score Card. (₹ in lakh)

| | | Growth | Price | Productivity |
|---|---|---|---|---|
| A | Revenue | SMV<br>Revenue Effect of Growth.<br>(bQ - AQ) X bSP.<br>[(4L - 4.3L) X 4175] = 1252.5 L F | SMP<br>Revenue Effect of Price<br>(bSP - ASP) X AQS.<br>645 F.<br>[(4175 - 4325) X 4.3L] | |
| b) | Material | Cost Effect of Growth.<br>(bQ - AQ) X b.v.c./unit<br>846 A<br>$\left[(4L - 4.3L) \times \frac{24L \times 470}{4 = 1.12820}\right]$ | MPV<br>Cost effects of Price<br>(bR - AR) X AI.<br>376.5 A.<br>[(470 - 485) X 25.1L] | MUV<br>[(SI for A.O - AI for AO) SR]<br>329 F.<br>$\left[\left(\frac{24}{4} \times 4.3\right) - \overset{25.1}{\cancel{240}}\right] \times 470]$ |
| | Labour. | [(4L - 4.3L) X 240] = 72 A. | LRV<br>[(30 - 31) X 34.8] = 34.8 A. | LEV.<br>$\left[\left(\frac{32}{4} \times 4.3\right) - 34.8\right] \times 30$ = 12 A. |
| | | | Fixed Overhead exp vari.<br>Bud - Actu = 16 A<br>[160 - 176]. | |
| | | 334.5 Lakh. F | 217.7 F | 317. F. |

Reconciliation statement.

| | ₹ in lakh |
|---|---|
| Growth | 334.5 F |
| Price | 217.7 F |
| Productivity | 317 F |
| Change in Profit | 869.2 |

(#) Growth factor should be divided into two Parts

1) Market size factor (Growth occured due to size factor)

2) Market share factor (Growth occured due to share factor)

Market share reason (factor of Growth) should be Incorporated with Price factor : Called Product Differentiation Policy

Statement of Reconciliation

| | | |
|---|---|---|
| Growth (Size factor only) | | XX |
| Product Differentiation | | |
| Growth (Share factor) | XX | |
| Pricing | XX | XX |
| Productivity | | XX |
| Change in Profit | | XX |

(#) Change in Price (∵ P, Resources) will reflect our excess Qty sold (Share ⇒ Product Differentiation Policy.

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Working Notes

Last Year Qty = 400000 unit
Current Year Qty = 420000 unit
Growth Qty 20000 unit

Market Size = 12000 unit [400000 X 3%]
Market share :- 8000 unit (20000-12000)

Growth factor ₹ 140000 F.

| | Qty | Income | |
|---|---|---|---|
| Market Size | 12000 | 84000 | $\left[140000 \times \frac{12}{20}\right]$ |
| Market Share | 8000 | 56000 | $\left[140000 \times \frac{8}{20}\right]$ |
| | 20000 | 1,40,000 | |

Statement of Reconciliation.

| | | |
|---|---|---|
| Growth (size) | | ₹ 84000 F |
| Product Differentiation | | |
| Growth (share) | ₹ 56000 | ₹ |
| Pricing | ₹ 164000 | ₹ 2,20,000 F |
| Productivity | | ₹ 58000 F |
| change in Income | | 3,62,000 F |

Check: [1442000 - 1080000]

[★] [2] Non financial Area. Covers the following points :-

i) Financial Perspective
ii) Customer Perspective
iii) Internal Business Perspective
iv) Innovation / Learning / Growth Perspective.

i) Financial Perspective

a) Sales.
b) Cost of sales
c) Return on Investment
d) Different Rate of Interest charged on different loan / Deposit / Services.

e) Economic Value Added.
f) How shareholder look our Company.

(ii) Customer Perspective

a) On time Delivery
b) Time Taken to process Loan
c) No. of Customer Complaint
d) Defective Goods Percentage decreased.
e) Discount Voucher Redeemed.
f) What we have done in respect of Customer Satisfaction.
g) Price / Quantity

(iii) Internal Business Perspective

a) Opening New shops, New Product, New sales outlet (Existing/future)
b) Performance of Sales Department (budget/Actual)
c) Complaint Raised by Employee
d) % of Cost incurred on Maintainance.

(iv) Innovation / Learning / Growth Perspective

a) Introduction of New Product with New Features [Insurance Policy mandatory with Loan]
b) Computer Training to Customer
c) Continious Improvement (Backup facility for student)
d) Research & Developement.
e) Cost Leadership.

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1) Innovation | Learning | Growth Perspective

2) Financial Perspective.

3) ~~Customer Perspective.~~

(#) Single Plan Vs Partial Plan.

| Single Plan | Partial Plan |
|---|---|
| (#) All Variances to be analysed at the time of event Occurred<br>i.e, MPV: at the time of Purchase<br>MUV: at the time of Production (within the year) | (#) All variances of one Period should be analysed on Yearly basis i.e, along with the b/s. |
| (#) Cost can be Controlled in very effective manner because of Remedial Action could be taken against adverse Variances immidiately. | (#) Remedial Action against the 'A' variances should be analysed on yearly basis. Hence it is not so much effective to Control the Cost as Compare to 'Single Plan |
| (#) Extra Cost to be incurred to implement a system of single Plan i.e, a seperate Cost Account Department have to installed. | (#) No extra Cost will be incurred to analyse such variances.<br>eg = [01/04/16 ⟷ 31/3/17] — 31/7/17<br>F.Y.<br>Variance of these period.<br>Founded / Pointed out on 31/7/17. |

## # How to Analyse the Variances under single Plan.

## # For Material

SR × AQ (1) Material Control A/c
(b/f) Material Price Variance (b/f)
AR × AQ. To Supplier.

- # Entry already feeded in software
- # SR ⇒ fixed by Mgt A/c on the basis of Research
  Feature :- Blinkage. [till data has been entered]
- # AR for AQ :- Feeded by Purchase Mngr on the basis of Invoice.
- # AQ automatically Placed in Material Controlled A/c.

(2) Supplier — Dr [AR × AQ]
To Bank.

(3) Material issued to Production Department
Work in Progress Control A/c [SQ for 1 F.G X Actual Production X SR]
Material Usage Variance (b/f)
To Material Control A/c. [SR X AQ]

(#) WIP Controlled A/c is always debited by standard Cost for Actual Production.

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## # For Labour

1) Wage Control A/c [SR X AH]
Labour Rate Variance (b/f)
To Labour/wage Payable A/c [ARX AH]

2) Wages Payable A/c [ARXAH]
To bank.

3) Labour charged to Production.
WIP Control A/c ——— Dr. [SR X SH for 1 FG X AO]
Labour Efficiency Variance. (b/f)
To Wage Control A/c [SRX AH]

[#] WIP Control always debited with Standard Cost for Actual Production.

## # For Overhead

1) WIP Control A/c [SC for AO]
Overhead Cost Variance. (b/f)
To overhead Payable. [AC]

Overhead Exp Variance & overhead Volume Variance will be Calculated as per Partial Plan Method & entered in Journal Entry.

2) Overhead Volume Variance
Overhead Exp Variance
To overhead Cost Variance

[Debit = Adverse
Credit = favourable]

classmate

Department 'A'

(51) [Page-238]

1) Material Control A/c [₹2.15 X 8600 Pcs] 18490
Material Price Variance (b/f) 3010 (b/f)
To Supplier. [₹2.5 X 8600 Pcs 21500

2) Supplier A/c —— Dr 21500
To Bank 21500.

3) WIP Control A/c [₹2.15 X 3 pcs X 2800 unit] 18060
Material Usage Variance (b/f) 430 (b/f)
To Material Control 18490.

4) Wage Control A/c [₹1.75 X $\overset{5200}{2}$ Hrs] 9100

To Wage Payable [₹1.75 X 5200] 9100

5) Wage Payable A/c : 9100
To Bank 9100

6) WIP Control A/c [₹1.75 X 2 Hrs X 2800 unit] 9800
To Wages Efficiency Variance (b/f) 700 (b/f).
To Wage Control A/c. 9100.

7) WIP Control A/c [₹1 X 2800 unit] 2800
Overhead Cost Variance (b/f) 200 (b/f)
To Overhead Payable 3000.

8) Overhead Payable A/c 3000
To Bank 3000.

9) Department 'B' [18060 + 9800 + 2800] 30660

To WIP Control A/c 30660.

Department 'B'

1) WIP ~~Material~~ Control A/c 30,660
To Department 'A' 30,660.

2) Wage Control A/c. [1.5 X 12000] 18000
To Wage payable [₹1.5 X 12000 Hrs] 18000

3) Labour Payable A/c 18000
To bank 18000

4) WIP Control A/c [₹1.5 X 4 X 2800] 16,800
Labour Efficiency variance (b/f) 1200 (b/f)
To Wage Control A/c 18000.

5) WIP Control A/c. [4 X 2800 unit] 11,200
Overhead Cost Variance (b/f) 1300 (b/f)
To overhead payable 12,500

6). Finished Goods A/c 58,660 [30660 + 16800 + 11200]
To WIP Control. 58,660

WIP Control A/c

| | | | |
|---|---|---|---|
| To Department 'A' | 30,660 | By Finished Good. | 58,660 |
| To Wage Control A/C | 16800 | | |
| To Overhead Payable A/c | 11200 | | |
| | 58660 | | 58,660 |

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[W.N#1] Data for Resources.

| | Budgeted [1FG] | | | Revised budget | | | Actual | | |
|---|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rat | Amt |
| RM1 | 2 | 24 | 48 | 160 | 24 | 3840 | 150 | 21.6 | 3240. |
| RM2 | 5 | 36 | 180. | 400 | 36 | 14400. | 420 | 38.4 | 16128. |
| Labour. | | 24 | | 400 | 24 | 9600 | 460 | 22.5 | 10350. |

[W.N #2] Raw Material-2 (Single Plan: fortnight: at Pur

Material Price Variance = 1320 A.

(SP- AP) X AQ = -1320

(36- AP) x 550 = -1320

- 550 AP = - 1320 - 36x550

AP = 38.4 $\left(\frac{21120}{550}\right)$

Raw Material Control A/c (RM2)

| | | | |
|---|---|---|---|
| Opening balance (100 x36) | 3600 | by Issued $(\frac{420\times36}{b/f})$ | 15120 (b/f) |
| | | by closing balance $\frac{(230\times36)}{b/f}$ | 8280 |
| Purchase (550x36) | 19800 | | |
| | 23,400 | | 23400 |

Budgeted Cost for Actual Production = 14400. [WIP RM2]

BC for 1 FG X A.O = 14400.

5X36 XAO : 14400

AO = $\frac{14400}{180}$ = 80.

Actual Production = 80 unit

[W.N#3] Raw Material 1.

Material Usage variance = 240 F

(SQ- AQ) X SR = = 240.

(SQ- 150) X 24 = 240

SQ - 150 = $\frac{240}{24}$ ∴ SQ = 10+150

= 110

Raw Material Control A/c (RM1)

| | | | |
|---|---|---|---|
| Opening Balance | Nill. | Issued $(\frac{3600}{24})$=(150×24) | 3600 |
| Purchase (b/f)<br>[200×24]<br>(b/f) | 4800 | Closing balance.<br>(50×24)<br>(b/f) | 1200. |
| | 4800 | | 4800 |

[W.N #4]

Actual Payment for

Raw Material 1 (200 X 21.6) 4320 (b/f)

Raw Material 2. (550 X 38.4) 21,120

25440

[W.N. #5]

Labour Efficiency Variance = 1440 A.

(SH - AH) X SR = -1440

(SH - 460) X 24 = - 1440

$SH = \frac{-1440}{24} + 460$

SH = 400.

Now calculate all Variance

$MPV_1 = (24 - 21.6) \times 150 = 360$

$MPV_2 = (36 - 33.4) \times 420 =$

$MUV_1 = (160 - 150) \times 24$

$MUV_2 = (400 - 420) \times 36$

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(1) Data for FO Variances

Traditional

|  |  |  |  |  |
|---|---|---|---|---|
| Budgeted output | 10,000 | 0.2<br>(b/f) | Budgeted overhead | 2000. |
|  |  |  | Actual overhead | 1760 |
| Actual output | 8800. |  | Recovered overhead | 1760. |

Fixed overhead Cost Variance = Recovered - Actual

= 1760 - 1760

= Nill.

(2) Data for Fixed overhead Variance (ABC)

|  |  |  |  |  |
|---|---|---|---|---|
| Budgeted Activity = 10 lot | 200.<br>(b/f) | Budgeted overhead | 2000. |
| Actual lot = | 11 |  | Actual overhead | 1760 |
| ~~Actual~~ budgeted lot Activity for budgeted = | 9 lot |  | ~~Ach~~ Recovered O/H | 1800. |

Fixed overhead Cost Variance = 1800 - 1760

= 40 F

Fixed overhead Volume Variance = Recovered - Budgeted.

= 1800 - 2000

= 200 A.

Fixed O/H Exp Variance = Bud - Actual

= 2000 - 1760

= 240 F.

Fixed O/H Eff Variance = (9 lot - 11 lot) x 200

= 400 A

Fixed O/H Capacity Variance = (11 lot - 10 lot) x 200

= 200 F.

40th class.

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Data for fixed ~~Bast~~ overhead variance.

| Budgeted output | Recovery Rate | Budgeted overhead |
|---|---|---|
| 1800 unit | 20 | 360.000 |

Actual

| Actual output | Recovered |
|---|---|
| 18080 unit | 3,61,600 |

Recovered = Recovery Rate / unit X Actual output

$361600 = 20 \times A\cdot O$

$\therefore A\cdot O = 18080$ unit

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## Reconciliation Statment

### [As Per Marginal]

| | | |
|---|---|---|
| Budgeted Profit | | 5,19,000 |
| Sales Margin volume variance. | $(200000-165000) \times 4.17$ [#1] | 1,45,950 A |
| Sales Margin Price variance | $\left[\left(\frac{2100000}{200000} - \frac{1692900}{16500}\right) \times 165000\right]$ | 39600 A |
| Variable Cost Variance (SC - AC) | $\left[\left(\frac{1266000}{200000} \times 165000\right) - 1074150\right]$ | 29700 A |
| Fixed overhead expenditure variance (Bud - Act) | | 15000 A. |
| Actual Profit | | 288750 |

$\left[\text{Contribution per unit} = \frac{2100000 - 1266000}{200000}\right] = 4.17$ (#1)

| | Industry sales | Share | Qty |
|---|---|---|---|
| Budgeted. | 400000 | 50% | 200000 |
| Actual | 375000 | 44% | 165000 |
| | | | 35000 |

Market size variance = $25000 \times 50\% \times 4.17$ = 52125

Market Share variance = $(50-44)\% \times 375000 \times 4.17$ = 93825 A

(70)
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**Working Note #1**

MCV = MPV + MUV

= 585(A) + 375(F)

[SC-AC] = 210 A.

SC for Actual Production − 6435 = 210 A

SC for Actual Production = 6435 − 210

15 X AO = 6225

(i) A.O = $\frac{6225}{15}$ = 415 Unit ——— ①

**Working -2**

MPV = 585 A.

(SP-AP) X AQ = - 585

(1.5 - AP) X $\overset{AQ}{}$ = -585

AQ X 1.5 − $\overset{\nearrow Input}{AQ}$ X AP = -585

1.5AQ − 6435 = -585

AQ = $\frac{6435 - 585}{1.5}$

AQ (Input) = 3900

(ii) Actual Price = $\frac{6435}{3900}$ = ₹1.65 ——— ②

**Working - 3**

Labour Rate Variance = 636 F.

(SR-AR) X AH = 636.

(8 X AH) − AR X AH = 636.

8AH − 10324 = 636

AH = $\frac{10324 + 636}{8}$ = 1370 Hrs.

(iii) AR = $\frac{10324}{1370}$ = ₹7.53 | Hr

Working-4

Overhead Exp. Variance = 400 F.

Budgeted - Actual = 400

Budgeted ÷ Actual = 400

20000 - Actual = 400

Actual = 20000 - 400

(iv) Actual = 19600.

Working-5

Fixed overhead Volume variance = 750 F

Recovered - budgeted = 750 F.

5D X 415 - budgeted = 750

Budgeted = 20750 - 750

budgeted = 20000.

(v) Fixed overhead Efficiency variance = (SH - AH) × SR

= [(5 Hr X 415) - 1370] × 10

= (2075 - 1370) × 10

= 7050 F.

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Data for fixed overhead Variance.

Budgeted Delivery = 20 R.R 200 Budgeted O/H = 4000

Actual Delivery = 19 Actual = 3910

Budgeted Delivery for Actual output Recovered = 200 × 21 = 4200.

$\frac{20}{2000} \times 2100 = \underline{21}$

Fixed overhead Exp Variance = [Bud - Actual] = 4000 - 3900 = 100 F.

Fixed overhead Efficiency Variance = [[B. Activity for A.O - Actual Activity]] × R.R.

If Absorption then

= (21 - 19) × 200 = 400 F.

Fixed overhead Volume Variance = [Recovered - budget]

Capity Variance = (4200 - 4000) = 200 F.

= (Actua - Bud) × R.R = (19 - 20) × 200 = 200 A.

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Data for Resource Variance

| | Budgeted | | | Revised | | | Actual (500) | | |
|---|---|---|---|---|---|---|---|---|---|
| | Q | R | A | Q | R | A | Q | R | A |
| Material (x) | 3kg | 9 | 27 | 1500 | 9 | 13500 | 1400 | 10 | 14000 |
| Self | 4kg | 5 | 20 | 2000 | 5 | 10000 | 1900 | 9.7368 | 18500 |
| Labour. | 14 | 2 | 28 | 7000 | 2 | 14000 | 6500 | 2.1538 | 14000 |
| Overhead | 14. | 1. | 14 | 7000 | 1 | 7000 | 6500 | 1.2307 | 8000. |
| | | | | | | 44500 | | | 54500 |

Data for fixed overhead Variance

| Budgted Hrs. | R.R | Budgeted overhead |
|---|---|---|
| 400 × 14 Hr = | 11/14 = 0.7857. | 11 × 400 = 4400 |

Actual Hr = 6500 Hr Actual = 4500.

Recovered

B.H for A.O = 7000 Hr. 5500

Statement of Performance [Dep-Y]

Budgeted Cost for Actual output. [500 × 100] 50,000

Responsibility variance (Controllable)

Material Usage Variance

| | | |
|---|---|---|
| x | (1500 * 1400) × 9 | 900 |
| Self | (2000 - 1900) × 5 | 500 |
| Labour Efficiency Variance | (7000 - 6500) × 2 | 1000 |
| Variable overhead Efficiency | (7000 - 6500) × 1 | 500 |
| Fixed overhead Efficiency. | (7000 - 6500) × 11/14 .393. | 3293 F |

Non Responsible

| | | |
|---|---|---|
| MPV x | (9 - 10) × 1400 | (1400) |
| Self | (5 - 9.7368) × 1900 | (9000) |
| Labour Rate variance | (2 - 2.1538) × 6500 | (1000) |
| Variable overhead Volume | (1 - 1.2307) × 6500 | (1500) |
| Fixed O/H Exp & ~~Budgt~~ - (4400 - 4500) | | (100) |
| Fixed O/H Capacity Variance | (6500 - 5500) × 11/14 | 707 F. |

12293 A

5900 Actual lost

CLASSMATE

Comment On the basis of above analysis we can say Standard costing should be applied to each & every department of Company, not only one department 'y', although Department 'y' has positive performance. In other words we can say all usage variance are ~~being held~~ occurred due to production department or production department held responsible for all usage var.

As Per Marginal

Trading and Profit & loss A/c (3500 unit)

Dr. Cr.

| Particulars | Standard | Actual | Variance | Particulars | Standard | Actual | Variance |
|---|---|---|---|---|---|---|---|
| Material | 175000 | 189000 | 14000 A | Sales @ 140. | 490000 | 490000. | — |
| ~~Labour.~~ | (50x3500) | | | | | | |
| Labour | 21000 | 22100 | 1100 A | | | | |
| | (6x3500) | | | | | | |
| Var. overhead | 70000 | 62000 | 8000 F | | | | |
| | (20x3500) | | | | | | |
| fixed overhead. | 192000 | 188000 | 4000 F. | | | | |
| | [40x4800] | | | | | | |
| | 458000 | 461100 | 3100 A. | | | | |
| Profit b/f) | 32000 | 28900 | 3100 A | | | | |
| Sales | 490000 | 490000 | — | | | | |

Data for Resource Variance. (As Per Marginal)

| | Budgeted | | | Revised | | | Actual | | |
|---|---|---|---|---|---|---|---|---|---|
| | Q | R | Amt | Q | R | Amt | Q | R | Amt |
| Material | 1 | 50 | 50 | 3500 | 50 | 175000 | 3600 | 52.50 | 189000. |
| Labour. | 2 | 3 | 6 | 7000 | 3 | 21000. | 6400 | 3.25. | 20800 |
| | | | | | | | 400 | 3.25 | 1300 |
| V.O. | 2 | 10 | 20. | 7000 | 10 | 70000. | 6400 | 9.6875. | 62000 |

Data for fixed overhead variance (if Absorption)

Budgeted. Hr = 9600 R.R = 20 Budgeted fixed O/H = 192000

Actual Hr = 6400 Actual = 188000

Budgeted Hr -
for Actual output = 7000 Recovered = 140000.

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No. of bricks to be produced from 20000 square feet Compound $= \frac{\text{Area of Compound}}{\text{Area of 1 bricks.}}$

$$= \frac{20000 \text{ square feet}}{6'' \times 6'' \text{ sq. inch}} = 20000 \times \frac{4}{1} = 80000 \text{ bricks.}$$

= 80000 brick

[ ∵ 12" × 12" = 1/8

∴ 6" × 6" = 1/4.

Data for Material Variance.

| | Budgeted(8000) | | | Revised (10000 ) | | | Actual (10000) | | |
|---|---|---|---|---|---|---|---|---|---|
| | Qty | Rate | Amt | Qty | Rate | Amt | Qty | Rate | Amt |
| A | 400 | 1 | 400 | 500 | 1 | 500 | 500 | 2 | 1000 |
| b | 500 | 2 | 1000 | 625 | 2 | 1250 | 300 | 4 | 1200 |
| c | 100 | 3 | 300 | 125 | 3 | 375 | 200 | 5 | 1000 |
| | 1000kg | | 1700 | 1250 | | 2125 | 1000 | | 2000 |