NORMAL OR GAUSSIAN DISTRIBUTION

Basics:

- In case of continuous random variable probability distribution, if there exist a function that defines probability, that function is called as Probability Density Function.
- Various Mathematical experiments have proved that most of the continuous random variables will follow normal distribution. It is universally accepted distribution.
- Probability Density function is also defined on Normal Distribution and is given by below:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-(\frac{x-\mu}{\sigma})^2 \times \frac{1}{2}}$$

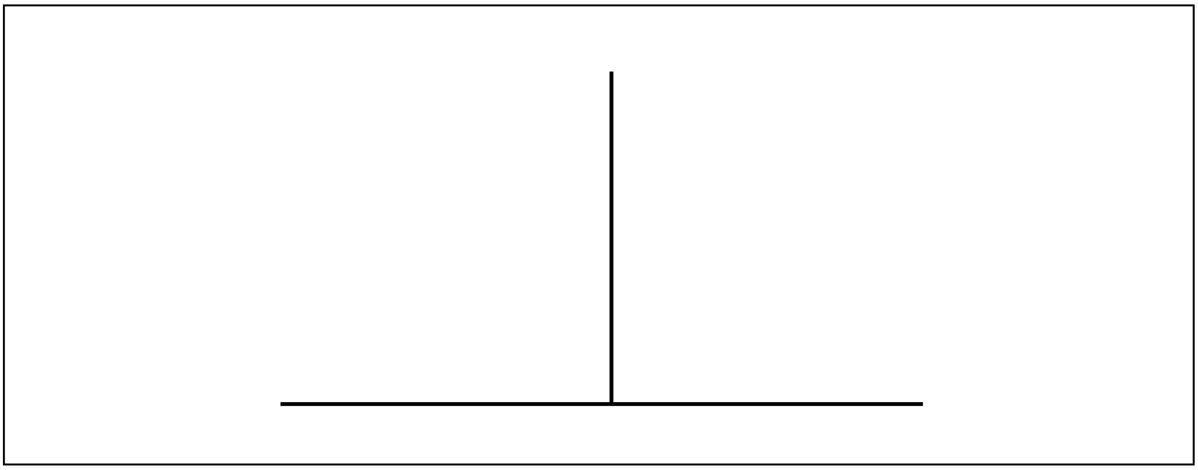
  It is defined for $-\infty < x < \infty$

- The function can be used if we have two values of 2 parameters x *and* σ^2, hence it is called as bi-parametric distribution.

Properties of Normal Distribution

1. Curve of Normal Distribution is Bell Shaped. (it shows less frequency/ probability at the extremes and max frequency/ probability at the center)
2. Here, Mean = Median = Mode = μ
3. Standard Deviation = σ, Mean Deviation = $\sigma \times \sqrt{2/\pi} = 0.8\,\sigma$
4. Quartile Deviation, $Q_3 = \mu - 0.675\sigma$ and $Q_1 = \mu + 0.675\sigma$
5. Normal Distribution is denoted by $X \sim N(\mu, \sigma^2)$ [X is a random variable]
6. Additive property – only applicable when two different random variables are independent. Assume we have two variables X and Y such that $\boldsymbol{X \sim N(\mu_1, \sigma_1{}^2)}$, $\boldsymbol{Y \sim N(\mu_2, \sigma_2{}^2)}$ then X+$\boldsymbol{Y \sim N(\mu_1 + \mu_2, \sigma_1{}^2 + \sigma_2{}^2)}$
7. Normal Curve is symmetrical at $x = \mu$, skewness is zero
8. Points of Inflexion (where convex becomes concave and concave becomes convex) are $\boldsymbol{\mu - \sigma}$ & $\mu + \sigma$

Draw a Normal Curve

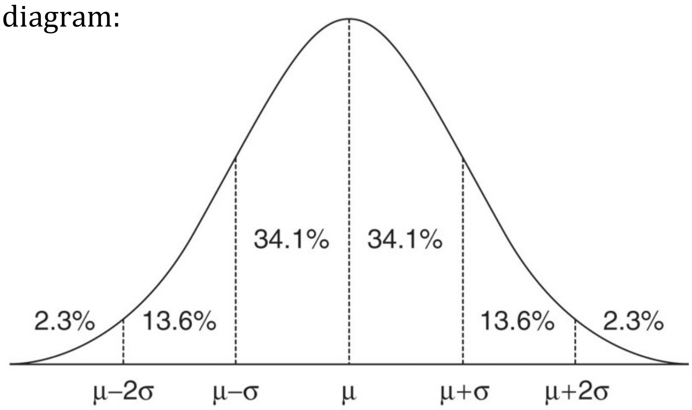


Area of a Normal Curve (Probability Curve)

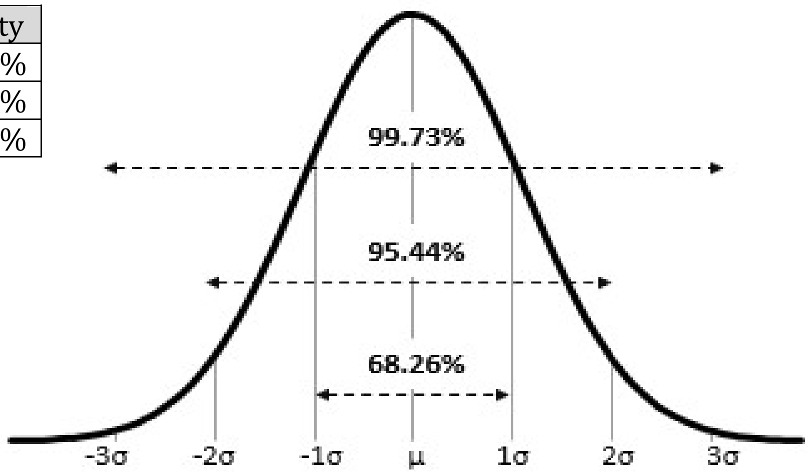
Total Area of Normal Probability Curve = 1 (because total probability is 1)

Total Area can be further composed as shown in diagram:

From	To	Area/Probability
μ	+σ	34.135%
+σ	+2σ	13.59%
+2σ	+3σ	2.14%
+3σ	∞	0.135%



From	To	Area/Probability
-σ	+σ	68.3%
-2σ	+2σ	95.5%
-3σ	+3σ	99.7%



Conclusion: 99.7% values of normal distribution lies within -3σ to +3σ

Other important Terms

- **Standard Normal Distribution** – A normal distribution with following conditions is said to be a Standard Normal Distribution.

Parameter	Value
Mean μ	0
Standard Deviation σ	1

The variable used in this distribution is called as Standard Normal Variate and is denoted by ~~Z~~ *[Striked Z]*

As we know that **99.73%** values are lying under **X=-3σ to X=+3σ** under a normal distribution, so for standard normal distribution 99.73% values will be lie between X=-3 to X=+3 (as σ is 1).

- **Z Table / Biometrika Table** - For a standard normal distribution, it is possible to calculate probability for interval of random variable (which mainly ranges from -3 to +3). This can be done using Z Table. This table gives us the probability of values from *X=μ=0* to *X=any value up to 3*

- **Z score** – Probability for normal variable (other than Standard Normal Variate) cannot be calculated directly using Z table. In that case, we need to convert normal variable X into standard normal variable Z.
  Formula: $Z = \frac{x-\mu}{\sigma}$

- **Cumulative Distribution Function** - Probability of Standard Normal Variate from negative -∞ to a particular value within distribution or in others words probability that variable takes value less than or equal to a particular value. It is denoted by $\phi(x) = P(X \leq x)$

  Example: $\phi(1.5) =$ The probability from *-∞ to Z=1.5* in a standard normal distribution.

- **Formulas for Standard Normal Distribution**

Probability Function	$f(z) = \frac{1}{\sqrt{2\pi}} e^{-(z)^2 \times \frac{1}{2}}$ for $-\infty < z < \infty$
Mean, Median, Mode	μ=0
SD, Variance	σ=1, σ^2=1
Points of Inflexion	-1, 1
Mean Deviation	0.8
Quartile Deviation	0.675