Chapter 3 – Linear Inequalities

Introduction

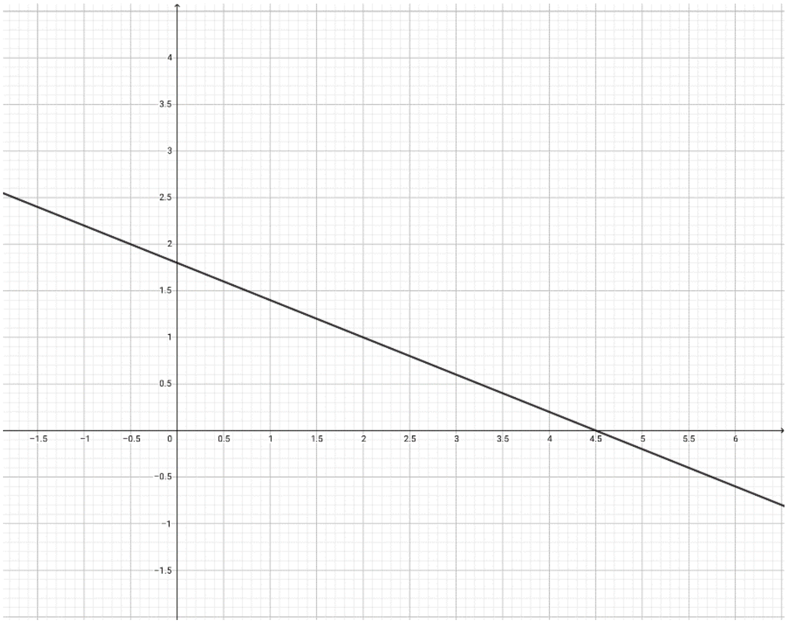
We have already studied Equations in the previous chapter. An example of a linear equation in one variable is $5x = 10$. Notice the "=" sign between the terms "$5x$" and "10". This is the equality sign which signifies that the term "$5x$" is **equal** to "10". This would give the value of x to be 2. This implies that the equation $5x = 10$ holds true only for $x = 2$. For all the other values of x, this equation won't hold true. Therefore, there's only 1 solution to the equation.

An **Inequality** on the other hand is of the type $5x < 10$. Notice the "<" sign between the terms "$5x$" and "10". This is the inequality sign which signifies that the term "$5x$" is always **less** than "10". On solving this, we'll get $x < 2$. This means that the inequality $5x < 10$ holds true for all the values of x which are less than 2. Thus, it is clear that while an equation has only 1 solution, an inequality has infinite solutions. These infinite solutions that an inequality has is called **Solution Space**. Again, since the highest power of the variables is 1, it is said to be a **Linear Inequality**.

Solving Linear Equations in Two Variables Graphically

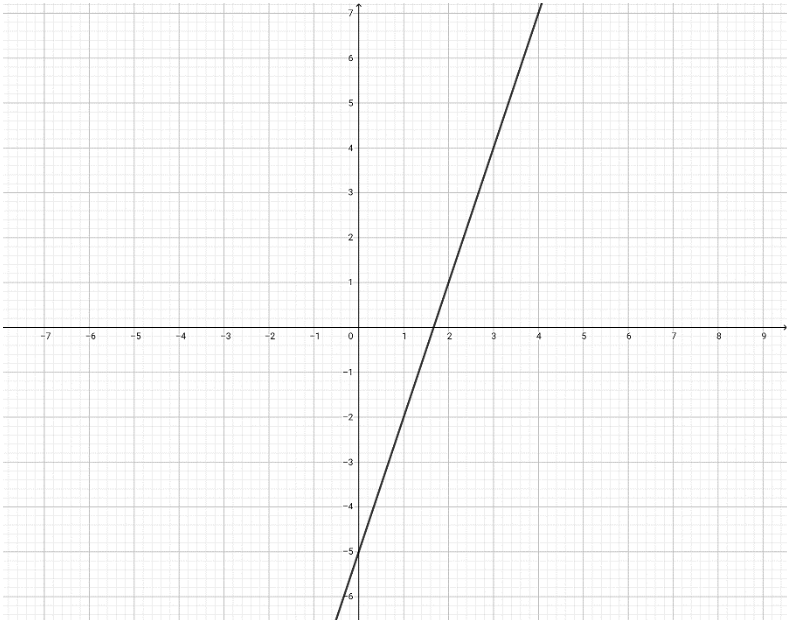
Any linear equation in two variables can be plotted on a graph paper. The graph of a linear equation in two variables is always a straight line.

Consider the equation $2x + 5y = 9$. Following is its graph:



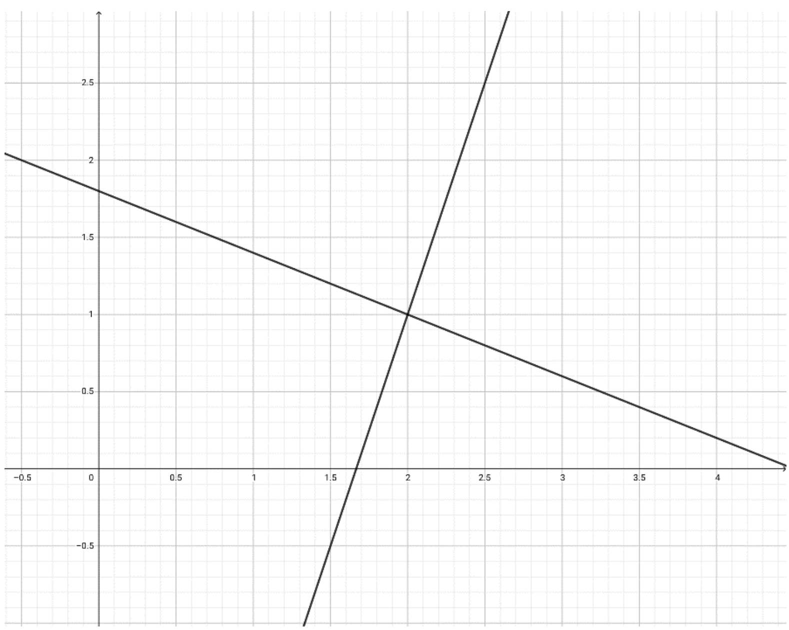
The above is the graph of the equation $2x + 5y = 9$. The straight line that you see is the line of the various solutions of this equation. This means that all the points falling on this straight line will solve the equation $2x + 5y = 9$. Take, for example, the point (4.5, 0). Putting 4.5 for x and 0 for y, we get the LHS $\rightarrow 2 \times 4.5 + 5 \times 0 = 9 + 0 = 9 =$ RHS. Therefore, we can see that the point (4.5, 0) is the solution of the equation $2x + 5y = 9$. Similarly, any such point falling on this line is the solution of this equation.

Now consider another equation: $3x - y = 5$. Following is its graph:



The above is the graph of the equation $3x - y = 5$. The straight line that you see is the line of the various solutions of this equation. This means that all the points falling on this straight line will solve the equation $3x - y = 5$. Take, for example, the point (0, –5). Putting 0 for x and –5 for y, we get the LHS $\rightarrow 3 \times 0 - (-5) = 0 + 5 = 5 =$ RHS. Therefore, we can see that the point (0, –5) is the solution of the equation $3x - y = 5$. Similarly, any such point falling on this line is the solution of this equation.

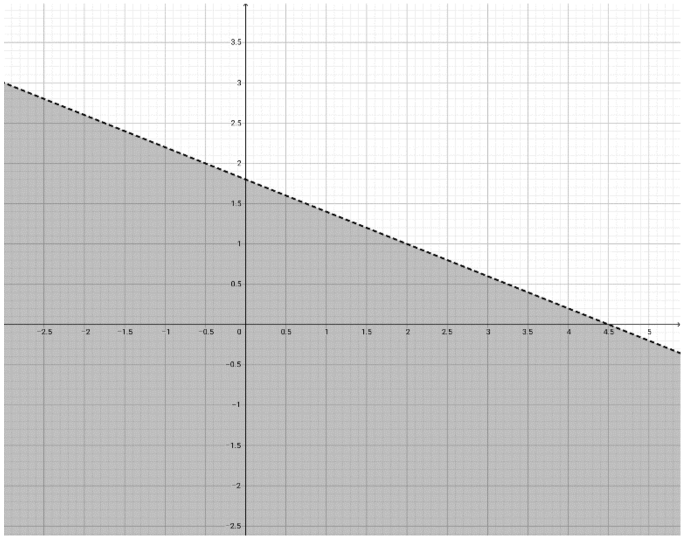
If we superimpose these two graphs, the lines will intersect at a point. This intersection point will give us the solution of these two equations when solved simultaneously.



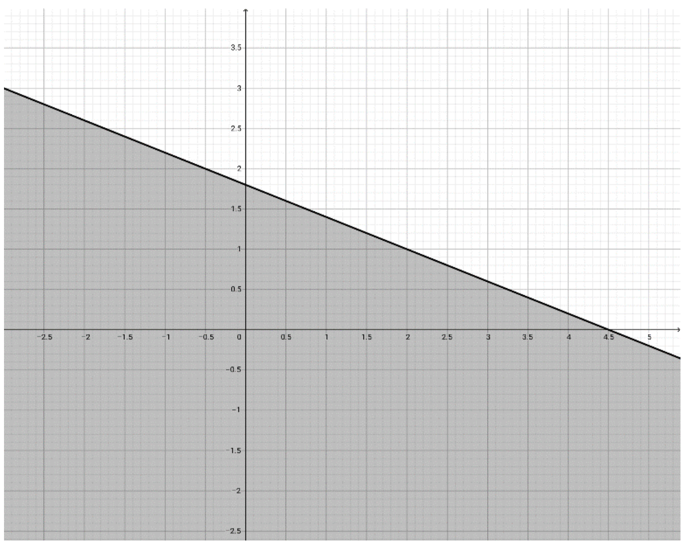
As can be seen from the above graph, the lines intersect at the point (2, 1). Hence, this is the solution of the set of the equations: $2x + 5y = 9$; $3x - y = 5$.

Graphing Linear Inequalities in Two Variables

Consider the inequality $2x + 5y < 9$. Following is its graph:

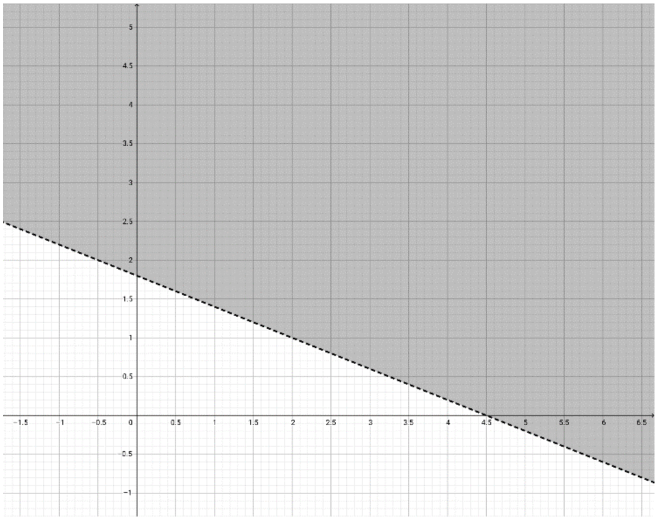


You can see that the line of the inequality is a dotted line and the area below it is shaded. The shaded area is the solution space of the inequality $2x + 5y < 9$. Any point lying in the shaded area will satisfy this inequality $2x + 5y < 9$. The dotted line indicates that any point lying on this line will not satisfy the inequality. However, if the inequality was $2x + 5y \leq 9$, then the line would not have been dotted:

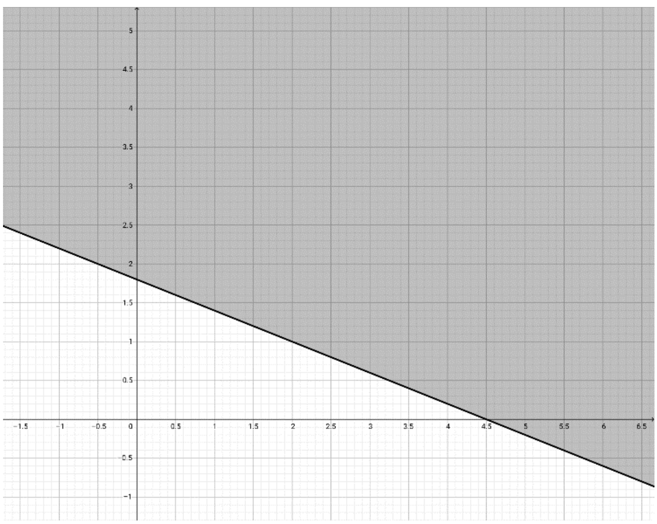


This means that any point lying on the fixed line, as well as in the shaded area will satisfy the inequality $2x + 5y \leq 9$.

Consider the inequality $2x + 5y > 9$. Its graph is:



Any point in the shaded area will satisfy the inequality $2x + 5y > 9$. The dotted line indicates that any point lying on this line will not satisfy the inequality $2x + 5y > 9$. However, this line would not have been dotted if the inequality was $2x + 5y \geq 9$. The graph in such a case would have been:



This means that all the points lying on the line as well as in the shaded area will satisfy the inequality $2x + 5y \geq 9$.

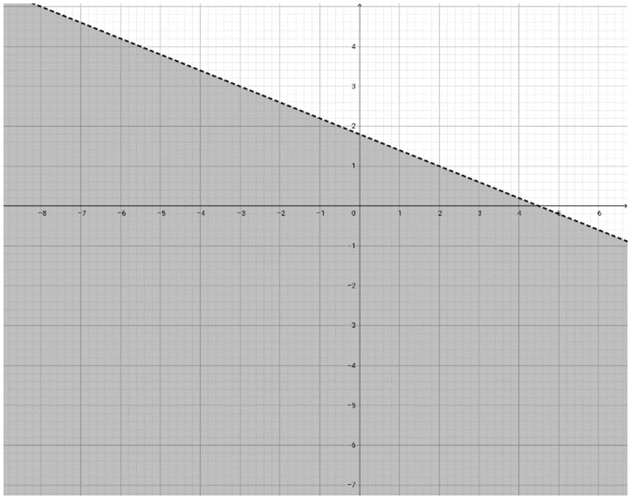
Points to be Noted

1. If the inequality sign is $>$ or $<$, a dotted line is drawn.
2. If the inequality sign is $\geq$ or $\leq$, a fixed line is drawn.
3. If the sign is $<$ or $\leq$, the area towards 0 is shaded.
4. If the sign is $>$ or $\geq$, the area away from 0 is shaded.

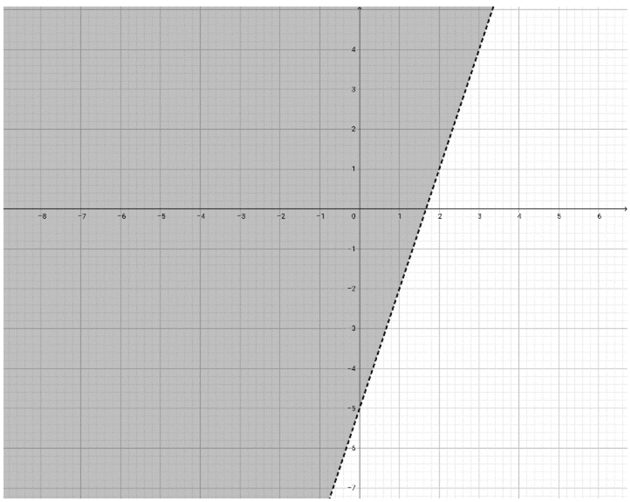
Solving a System of Linear Inequalities in Two Variables Graphically

Consider the following system of Linear Inequalities in Two Variables: $2x + 5y < 9$; $3x – y < 5$.

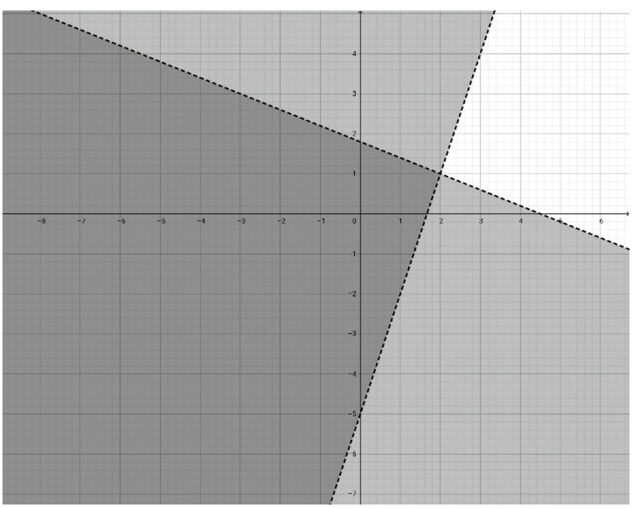
The graph of $2x + 5y < 9$ is:



The graph of $3x - y < 5$ is:



On superimposing these two graphs, we get:



The dark shaded portion in the above graph is the solution space of both the inequalities simultaneously, i.e. any point lying in the dark shaded portion will satisfy both the inequalities $2x + 3y < 9$, as well as $3x - y < 5$.