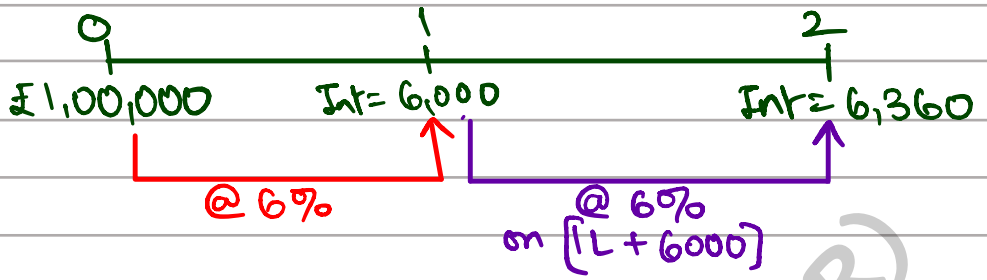
In previous eg of Alia, if ₹1,00,000 are deposited for 2 years at compound interest rate of 6%, then



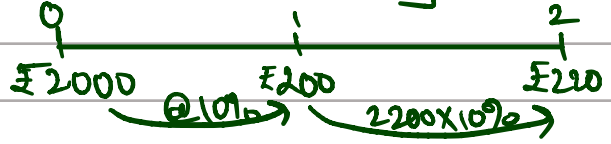
a) Compound Interest = 6,000 + 6,360
= ₹12,360.

b) Final value of Deposit = 1,00,000 + 12,360
= ₹1,12,360.

* **Example**
₹2,000 is invested at 10% p.a.
What is the amount after 2 years, if compounding is done
a) Annually
b) Semi-Annually
c) Quarterly
d) Monthly

Solution:

a) **Annual compounding**

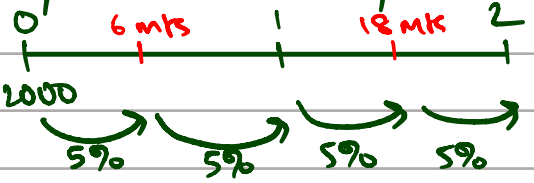


Value at end of 2 yrs = ₹ 2,420

OR

$FV_2 = PV \times (1+r)^2$
$= 2000 \times (1+10\%)^2 = 2420$

b) **Semi-Annually**

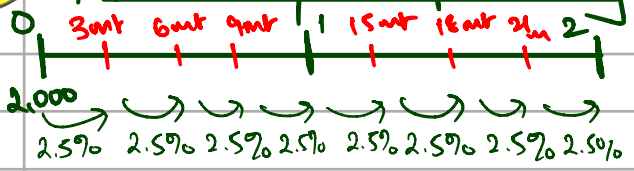


$FV_2 = 2000 + 5\% + 5\% + 5\% + 5\%$

OR

$FV_2 = 2000 \times (1+0.05)^4$
$= 2431.01$

c) Quarterly compounding



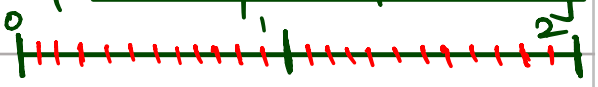
$FV_2 = 2000 + \underbrace{2.5\% + \dots + 2.5\%}_{8 \text{ times}}$

or

$FV_2 = 2000 \times (1+0.025)^8$

$= 2000 \times 1.218 = 2436.81$

d) Monthly compounding



$FV_2 = 2000 + \underbrace{0.83\% + \dots + 0.83\%}_{24 \text{ times}}$

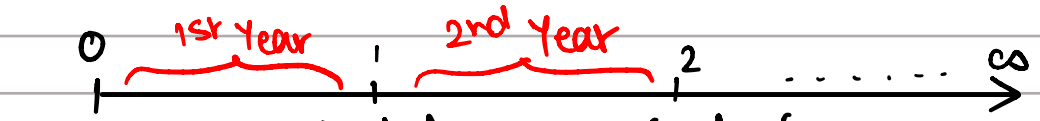
$= 2000 \times (1+0.0083)^{24}$

$= 2000 \times 1.219$

$= ₹ 2,438.85$

*	FUTURE VALUE (F.V.)	PRESENT VALUE (P.V.)
	F.V. is the cash value of an investment at some time in future. It is tomorrow's value of today's money compounded at a rate of interest. $FV = PV(1+r)^n$	P.V. is the sum of money to be invested today in order to achieve a specific amount in future. OR P.V. is the current (today's) value of future sum of money or stream of cashflows, at a specified rate of int. $PV = \frac{FV}{(1+r)^n}$

* Timeline

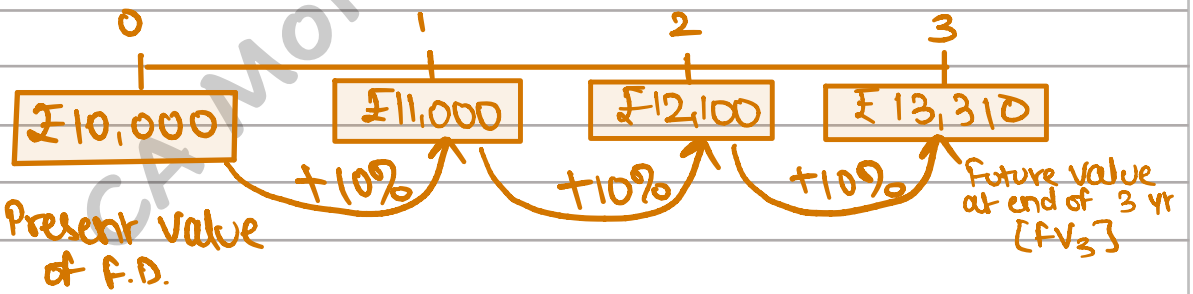


- Today (Present) or Beginning of 1st year
- End of 1st year or Beginning of 2nd year
- End of 2nd year. or Beginning of 3rd year.

Please note, while solving questions of FM, if que does not specify that cashflow are of beginning or end of the year, we ASSUME them to be at the END OF THE YEAR only.

IY FV of a Single Amount

* Suppose you invest ₹10,000 in a bank's fixed deposit. Interest rate is 10% p.a. What will be the F.V. at the end of 3 years?



Thus, the value of FD [Today at ₹10,000] becomes ₹13,310 [FV_3] at the end of 3rd year if interest rate is 10% p.a.

$$10,000 + 10\% + 10\% + 10\% = 13,310$$

10,000 + (10% × 10,000) = 11,000

11,000 + (10% × 11,000) = 12,100

12,100 + (10% × 12,100) = 13,310

10,000 + 10% + 10% + 10%

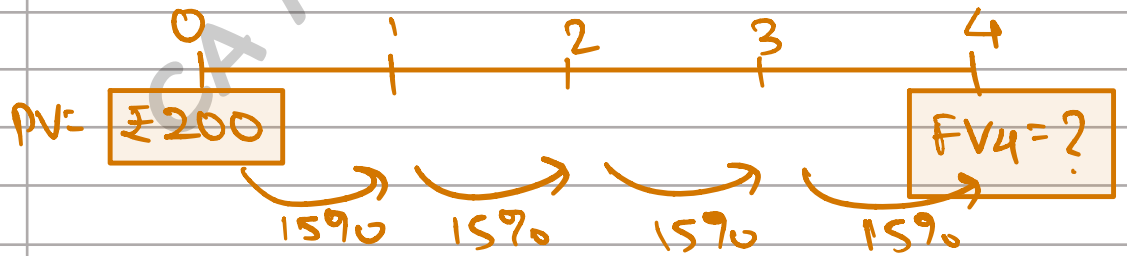
$10,000 \times (1+10\%) \times (1+10\%) \times (1+10\%)$

$\Rightarrow 10,000 \times (1+0.10)^3 = 13310$

$PV \times (1+r)^n = FV_n$

$\Rightarrow PV \times (1+r)^3 = FV_3$

* Suppose CMP of a share is ₹200. The shares value increases by 15% every year. What will be its value at the end of 4th yr?



$FV_4 = PV(1+r)^4$

$= 200(1+0.15)^4$

$= 200(1.15)^4$

$= 200 \times 1.749$

$= ₹349.80$

How to do on calculator?

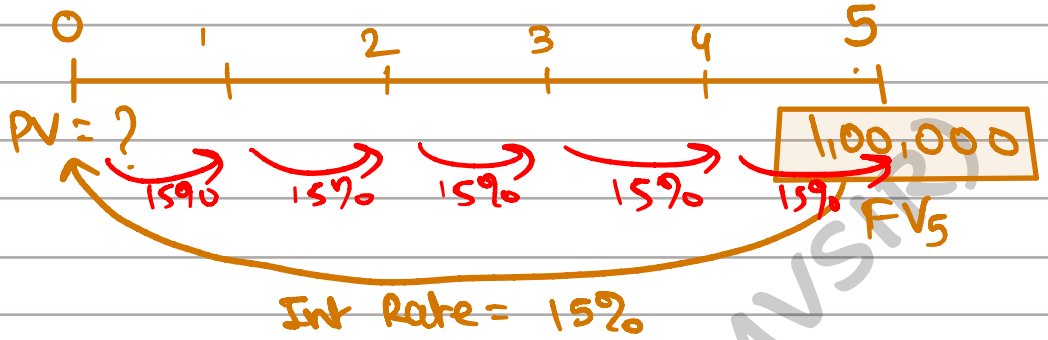
Step1: 1.15 "x ="

Step2: =

Step3: = 4 Ans.

II) PV of a Single Amount

Suppose you are going to receive ₹1,00,000 after 5 yrs from now. Then what will be the P.V., if interest rate is 15% p.a.?



[FV5]

$PV + 15\% + 15\% + 15\% + 15\% + 15\% = 1,00,000$

$PV \times (1+0.15)^5 = 1,00,000$

$\Rightarrow PV = \frac{1,00,000}{(1+0.15)^5} = 1,00,000 \times \frac{1}{(1+0.15)^5}$

$= 1,00,000 \times \boxed{\frac{1}{(1.15)^5}}$ → How to do on calculator?

Step1: 1 ÷ 1.15

Step2: "=" 4 times

Ans: 0.497

$= 1,00,000 \times 0.497$

$= ₹49,700$

$PVIF_{(15\%,5)} = 0.497$

---

* Discounting factors @ 15%

[Present value Interest factors → $PVIF_{(15\%,n)}$]

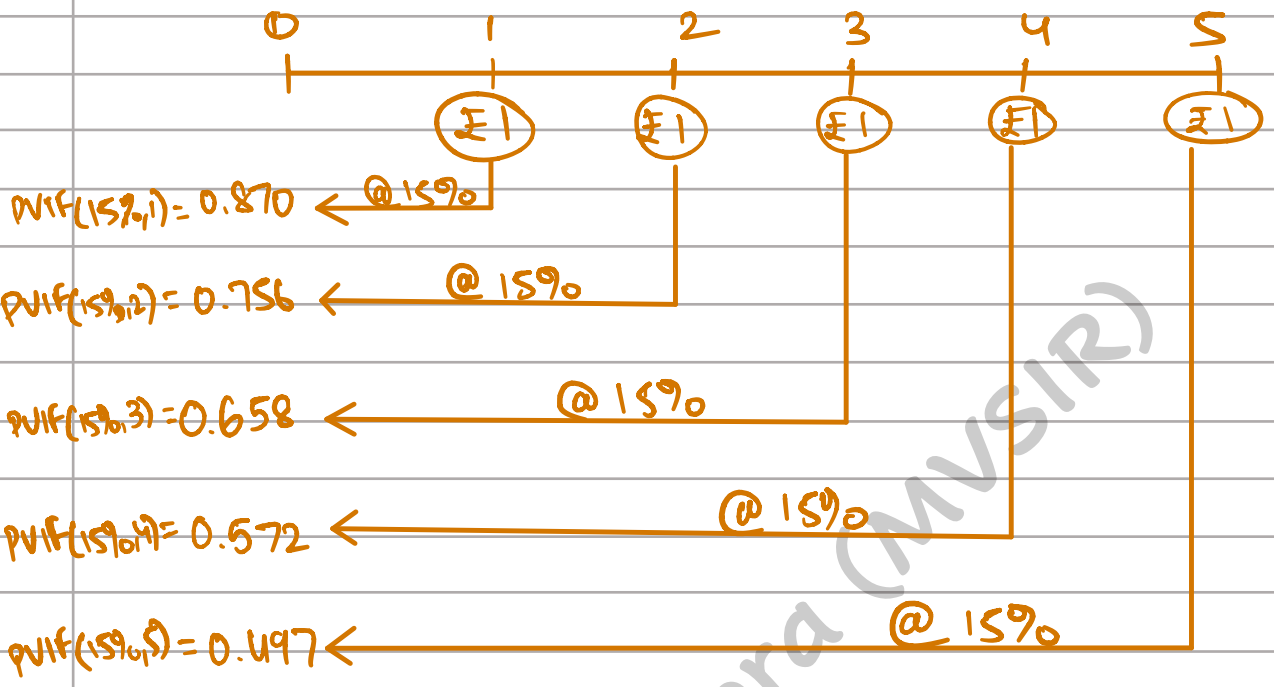
1st yr ke end mei ₹1 chahiye → aaj 15% → 0.870

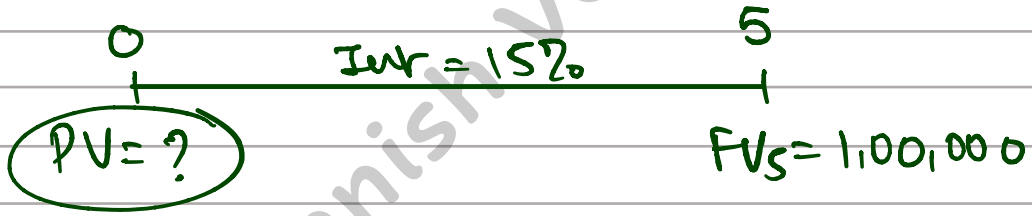
2nd yr ke end mei ₹1 chahiye → aaj 15% → 0.756

3rd yr ke end mei ₹1 chahiye → aaj 15% → 0.658

4th yr ke end mei ₹1 chahiye → aaj 15% → 0.572

5th yr ke end mei ₹1 chahiye → aaj 15% → 0.497





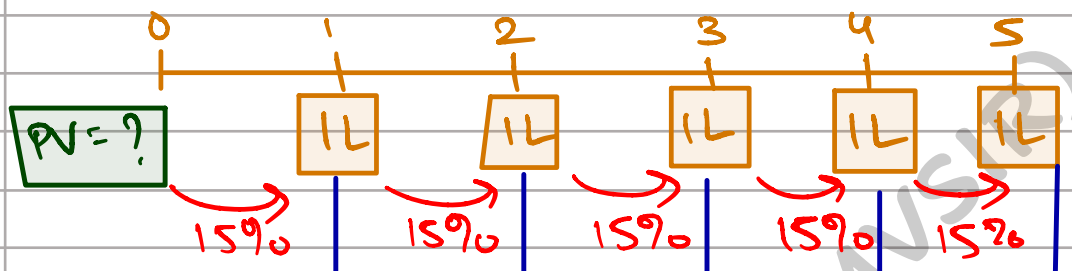
Amt to be invested today	If at end of 5th yr
₹0.497	₹1

PV = ? ₹1,00,000

PV = 1,00,000 × 0.497 = ₹49,700

III) PV of Annuity [Uniform cashflows each year for finite period]

Eg: Suppose as per a contract, you are going to receive ₹1,00,000 at the end of every year upto 5 yrs. Then what is the P.V., if rate is 15%.



$$PV_1 = \frac{1L}{(1+0.15)^1}$$

$$PV_2 = \frac{1L}{(1+0.15)^2}$$

$$PV_3 = \frac{1L}{(1+0.15)^3}$$

$$PV_4 = \frac{1L}{(1+0.15)^4}$$

$$PV_5 = \frac{1L}{(1+0.15)^5}$$

$$PV = \frac{1L}{(1.15)^1} + \frac{1L}{(1.15)^2} + \frac{1L}{(1.15)^3} + \frac{1L}{(1.15)^4} + \frac{1L}{(1.15)^5}$$

$$= 1L\left(\frac{1}{1.15}\right)^1 + 1L\left(\frac{1}{1.15}\right)^2 + 1L\left(\frac{1}{1.15}\right)^3 + 1L\left(\frac{1}{1.15}\right)^4 + 1L\left(\frac{1}{1.15}\right)^5$$

$= 1L(0.870) + 1L(0.756) + 1L(0.658) + 1L(0.572) + 1L(0.497)$

$= 1L(0.870 + 0.756 + 0.658 + 0.572 + 0.497)$

$= 1L(3.353)$

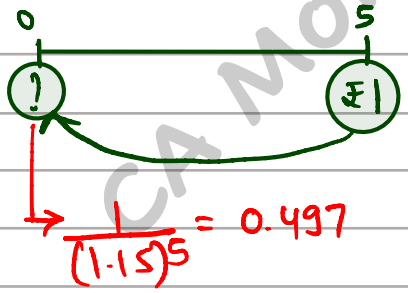
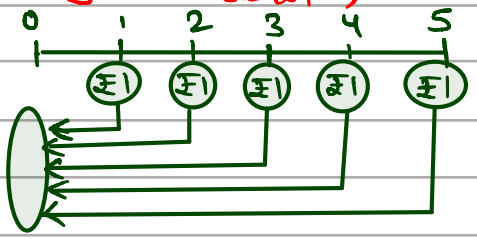
$= ₹3,35,300$

Here, 3.353 is PVAF (15%, 5)

PV = Annuity × PVAF (15%, 5)

= 1,00,000 × (3.353)

= ₹3,35,300

Present Value Interest Factor [PVIF]	Present Value Interest Factor Annuity or Cumulative Factor or PV Annuity factor [PVIFA or PVAF]
Eg: PVIF (15%, 5)  $\frac{1}{(1.15)^5} = 0.497$	Eg: PVAF (15%, 5)  $PVAF_{(15\%,5)} = \frac{1}{(1.15)^1} + \frac{1}{(1.15)^2} + \frac{1}{(1.15)^3} + \frac{1}{(1.15)^4} + \frac{1}{(1.15)^5} = 3.353$
Calculator Steps ① 1 ÷ 1.15 ② "=" 5 time Ans. 0.497	Calculator Steps ① 1 ÷ 1.15 ② "=" 5 times ③ Press "GT" Ans. 3.352

Example

PV = ?

Int Rate = 12.50%

12.50%

$$PV = \frac{5L}{(1+0.125)^1} + \frac{5L}{(1+0.125)^2} + \dots + \frac{5L}{(1+0.125)^8}$$

$$= 5L \times PVAF(12.50\%, 8)$$

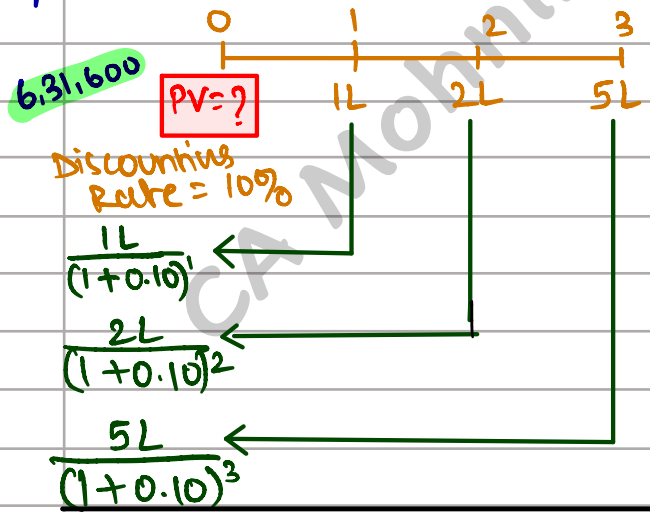
$$= 5,00,000 \times 4.882$$

$$= ₹ 24,41,000$$

PV of Annuity $= A \times PVAF(r, n)$ OR $A \times \left[\frac{1-\left(\frac{1}{1+r}\right)^n}{r}\right]$

Not to be used ever

IV) P.V. of uneven cashflow



6,31,600

$$PV = \frac{1L}{(1.10)^1} + \frac{2L}{(1.10)^2} + \frac{5L}{(1.10)^3}$$

$$= 1L \times \left(\frac{1}{1.10}\right)^1 + 2L\left(\frac{1}{1.10}\right)^2 \dots$$

$$= 1L \times (0.909) + 2L(0.826) + 5L \times (0.751)$$

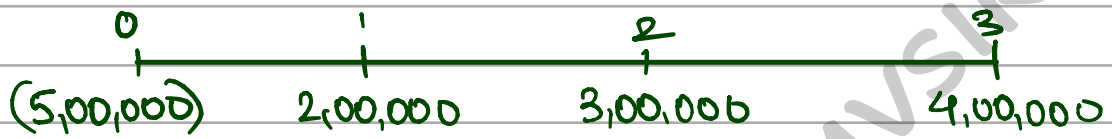
$$=$$

Tabular form

Yr	Particulars	CF	DF (10%)	DCF
1	CF	1,00,000	0.909	90,900
2	CF	2,00,000	0.826	1,65,200
3	CF	5,00,000	0.751	3,75,500
				6,31,600

Example

Suppose a machinery costs ₹ 5,00,000 today. We can receive cashflows of ₹ 2,00,000, ₹ 3,00,000 & ₹ 4,00,000 at the end of Yr 1, 2 & 3 respectively.
Our companies fund providers expect a return of 12% on their funds. (Cost of capital)
Whether should we purchase this M/C or not?



PV of M/C @ 12% = $\frac{2L}{(1.12)^1} + \frac{3L}{(1.12)^2} + \frac{4L}{(1.12)^3}$

= 2L(0.893) + 3L(0.797) + 4L(0.712)
= ₹ 7,02,500

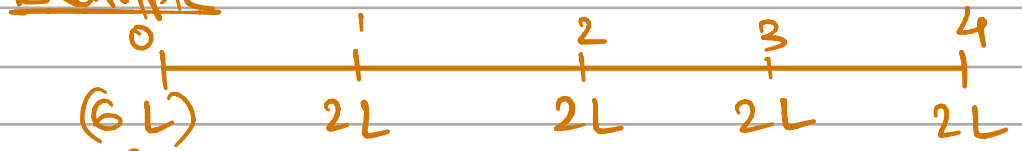
If we want earn a return of 12%, then we should purchase the M/C at ₹ 7,02,500.

However, cost of M/C is only ₹ 5,00,000.
Thus, we should purchase the M/C.

Now, we will be able to earn 12% on the asset along with additional ₹ 2,02,500 [NPV]
[7,02,500 – 5,00,000]

Net Present Value = Sum of PV of CF @ cost of capital (–) Initial Investment
= 7,02,500 – 5,00,000
⇒ NPV = ₹ 2,02,500

Example



Cost of M/c

If cost of capital = 10%, then whether should we purchase M/c using NPV method?

Sum of PV of cf from M/c discounted @ K_o = 10% = 2,00,000 × PVAf(10%,4)
= 2,00,000 × 3.170
= ₹ 6,34,000

NPV = Sum of PV of cf − Initial Invt
= 6,34,000 − 6,00,000 = ₹ 34,000

Since, NPV is +ve, we should purchase the M/c.

Example

Cost of M/c

If cost of capital = 12%, then whether should we purchase M/c using NPV method?

- Sum of PV of cf discounted @ K_o = 12% $= \frac{2L}{(1+0.12)^1} + \frac{1L}{(1+0.12)^2} + \frac{1L}{(1+0.12)^3}$

$= 2L(0.893) + 1L(0.797) + 1L(0.712)$
$= ₹ 3,29,500.$

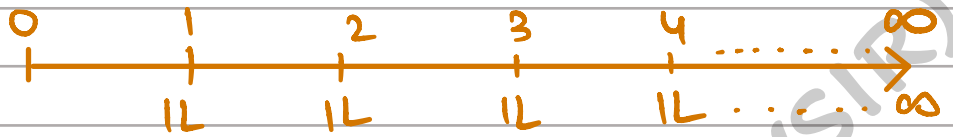
- NPV = 3,29,500 − 4,00,000 = (70,500)

Thus, M/c should not be purchased as NPV is -ve.

IV PV of Perpetuity [Uniform Cashflows for Infinite Period]

Perpetual Annuity

Eg: Suppose as per a contract, you are going to receive £1L at the end of each year for infinite period. Find PV, if interest rate is 12%.



PV of Perpetuity = $\frac{1L}{(1+0.12)^1} + \frac{1L}{(1+0.12)^2} + \frac{1L}{(1+0.12)^3} \ldots \ldots \infty$

Sum of Infinite GP (when $r<1$) $= \frac{a}{1-r} = \frac{\text{First Term}}{1 - \text{Common Ratio}}$

First Term $(a) = \frac{1L}{(1+0.12)}$

Common Ratio $= \frac{1}{(1+0.12)}$

$\therefore$ Sum of Infinite GP $= \frac{a}{1-r}$

$= \left[\frac{1L}{1+0.12}\right] \div \left[1 - \left(\frac{1}{1+0.12}\right)\right]$

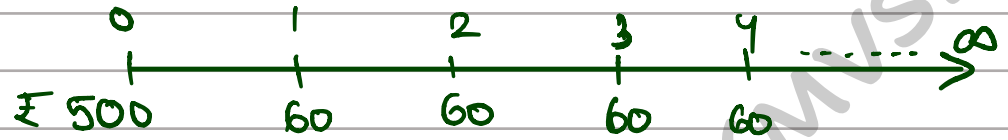
$= \frac{1L}{1.12} \div \frac{1.12-1}{1.12}$

$= \frac{1L}{1.12} \times \frac{1.12}{0.12} = \boxed{\frac{1L}{0.12}} = £8,33,333.33$

$$\text{PV of Perpetuity} = \frac{A}{r}$$

Example

Suppose today MV foods Ltd's share price is of ₹500. It is expected to earn ₹60 as dividend perpetually from this share. Whether should we purchase this share or not, if we expect a return of 10% [Ke] ?



PV of share discounted @ 10% $= \frac{A}{r} = \frac{60}{10\%} = ₹600.$

NPV = Sum of PV of CF − Initial Invt

= 600 − 500 = ₹100

Since NPV is +ve, we should purchase shares.

VI) PV of Growing Perpetuity [Constant Growth]

Example

Suppose today MV foods Ltd's share price is of ₹500. Last year's dividend was ₹80. It is expected that dividend will grow by 5% every year till infinity.

Whether should we purchase this share or not, if we expect a return of 12% [Ke] ?

$P_0 = 500$ $D_1 = 84$ $D_2 = 88.20$ $D_3 = 92.61$

$D_0 = £80$

- $D_1 = 80 + 5\% = 80 \times (1+0.05) = 84$
- $D_2 = \underbrace{84 + 5\%}_{[80+5\%+5\%]} = 80 \times (1+0.05)^2 = 88.20$
- $D_3 = \underbrace{88.2 + 5\%}_{[80+5\%+5\%+5\%]} = 80 \times (1+0.05)^3 = 92.61$

Now, PV of Growing Perpetuity $= \frac{84}{(1+0.12)^1} + \frac{88.20}{(1+0.12)^2} + \frac{92.61}{(1+0.12)^3} + \dots\dots \infty$

$= \frac{80(1+0.05)}{(1+0.12)^1} + \frac{80(1+0.05)^2}{(1+0.12)^2} + \frac{80(1+0.05)^3}{(1+0.12)^3} + \dots\dots \infty$

The above is also like an infinite G.P.

- First Term $(a) = \frac{80(1+0.05)}{(1+0.12)}$
- Common Ratio $(r) = \frac{1+0.05}{1+0.12}$

Sum of Infinite GP (when $r<1$) $= \frac{a}{1-r}$

$$\therefore \text{Sum of Infinite GP} = \frac{80(1+0.05)}{(1+0.12)} \div \left[1 - \frac{1+0.05}{1+0.12}\right]$$

$$= \frac{80(1+0.05)}{(1+0.12)} \div \frac{1+0.12-1-0.05}{1+0.12}$$

$$= \frac{80(1+0.05)}{(1+0.12)} \times \frac{(1+0.12)}{0.12-0.05}$$

$$= \frac{80(1+0.05)}{0.12-0.05} = \frac{84}{0.07} = ₹1200$$

[Here, $D_0 = 80$; $K_e = 0.12$ (12%); $g = 0.05$ (5%)]

$$\therefore \text{PV of Growing Perpetuity} = \frac{D_0(1+g)}{K_e - g} \text{ or } \frac{D_1}{K_e - g}$$

The value of share should have been ₹1,200 in the above example, but in market it is only of ₹500, which means it is "under-priced", thus we should buy the shares.

Example

Suppose today IG Rani Pvt Ltd's share price is of ₹1,200. The expected dividend at end of 1st year is ₹90. It is expected that dividend will grow by 6% every year till infinity. Whether should we purchase this share or not, if we expect a return of 15% [K_e] ?

PV of Share = $\frac{D_1}{Ke - g}$

= $\frac{90}{0.15 - 0.06}$ = $\frac{90}{0.09}$

= ₹ 1,000.

Theoritical MP = ₹1,000 (Kya price honi chahiye formula ke hisab se)

Actual MP = ₹1,200

The share is over-priced, we should not buy the share.