EQUATIONS - BASICS

Equation Means	mathematical statement of equality
Identity Equation	If equality is true for all the values of variable, ex. $2x + 3 = x + x + 3$
Conditional Equation	If the equality is true for certain value of the variable ex. $2x + 1 = 3$
Solution or Root	It is the value of variable that satisfies the equation
Degree	Highest power of variable in equation

SIMPLE EQUATION

Type	Linear equation with one unknown	Linear equation with two unknowns	Quadratic Equation	Cubic Equation
Form	$ax + b = 0$, where a and b are constants	$ax + by + c = 0$ a,b,c are constants	$ax^2 + bx + c = 0$ a,b,c are constants with a≠0	$ax^3 + bx^2 + cx + d = 0$
Degree	1 (One)	1	2	3
Roots	1 (One)	1 each for both	2 (α, β)	3
Remarks	NA	Need minimum two equations to get roots	Trial Error/ Formula based	Trial and Error
Methods for solution	NA	1. Elimination 2. Cross Multiplication	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	NA

LINEAR EQUATIONS WITH TWO UNKNOWNS

Elimination	Eliminate one variable by algebraic operations on given equations, and then calculate the value of variable that remains. Using this value, find out the value of other root.
Cross Multiplication	$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ Solution is given by: $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

QUADRATIC EQUATION

Formula	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Sum of Roots	$\alpha + \beta = -\frac{b}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^2}$
Product of Roots	$\alpha \times \beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$
How to construct a quadratic equation	$x^2 -$ (sum of roots: $\alpha + \beta$)x + Product of Roots: $\alpha \times \beta = 0$
Nature of Roots	see table below
Irrational Roots	If one root is $(m + \sqrt{n})$, then other root will be $(m - \sqrt{n})$

Nature of Roots:

Condition	Nature of Roots
$b^2 - ac = 0$	Real and Equal (α=β)
$b^2 - ac > 0$	Real and Unequal
$b^2 - ac < 0$	Imaginary
$b^2 - ac$ is a perfect square	Real, Unequal and Rational
$b^2 - ac > 0$ but not perfect square	Real, Unequal and Irrational

MATRICES

Matrix	A rectangular array of numbers (real/complex) with m rows and n columns
Order of Matrix	Order is m × n where m= no. of rows and n = no. of columns
Row Matrix	Matrix having only one row $\begin{bmatrix}1 & 4 & 2\end{bmatrix}$
Column Matrix	Matrix having only one column $\begin{bmatrix}1\\4\\2\end{bmatrix}$
Zero/ Null Matrix	If all the elements of matrix (any order) are zero $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$
Square Matrix	If in a matrix, no. of columns = no. of rows $\begin{bmatrix}1 & 3\\9 & 2\end{bmatrix}$
Rectangular Matrix	If in a matrix, no. of columns ≠ no. of rows $\begin{bmatrix}1 & 3 & 2\\9 & 2 & 5\end{bmatrix}$
Leading Diagonal	Diagonal elements starting from top left to bottom right
Diagonal Matrix	A square matrix where all the elements except leading diagonal elements are zero. $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}$
Scalar Matrix	A diagonal square matrix where all the leading elements are equal $\begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}$
Unit Matrix	A scalar matrix whose leading diagonal elements are equal to 1 $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
Upper Triangle Matrix	A matrix whose all the elements below the leading diagonal are zero $\begin{bmatrix}3 & 4 & 5\\0 & 1 & 9\\0 & 0 & 5\end{bmatrix}$
Lower Triangle Matrix	A matrix whose all the elements above the leading diagonal are zero $\begin{bmatrix}3 & 0 & 0\\5 & 1 & 0\\2 & 8 & 5\end{bmatrix}$
Sub Matrix	The matrix obtained by deleting one or more rows or columns or both of a matrix is called its sub matrix.
Equal Matrices	Two matrices are are equal matrices if order of both is same and corresponding elements are same
Addition/ Subtraction	All the corresponding elements will be added/ subtracted to make a new matrix. (only possible when both matrix are of same order)
Properties of Addition/ Subtraction	**a**. A+B = B+A [Commutative], **b**. (A+B)+C = A+(B+C) [Associative], **c**. k(A+B) = kA + kB, k is constant
Multiplication	Multiplication of two matrices is possible only when no. of columns of first matrix = no. of rows of second matrix. *[To understand how to do multiplication – refer page 2.40 Example 3]*
Properties of Multiplication	**a.** In general, A×B ≠ B×A, **b**. (A×B)×C = A×(B×C) if defined, **c.** A(B+C) = AB + AC also, (A+B)C= AC+BC, **d.** if AB = AC then B≠C in general, **e.** A×O = O [O means null matrix], **f.** A×I = IA = O [I means Unit Matrix],

Transpose of a Matrix	A matrix obtained by changing rows and columns of a matrix **A** is called as Transpose Matrix of **A**. It is denoted by **- A^T or A'**
Properties of Transpose	a. A = (A')' ; b. (A+B)' = A' + B' ; c. (KA)'=K.A' ; d. (AB)'=B'×A'
Symmetric Matrix	If after transposing also there is no change in matrix. A'=A
Skew Symmetric	If after transposing a matrix, it becomes its negative. A'=−A

DETERMINANTS

Determinants	It is a valuation of a matrix using some rules. It only applies for square matrix
Denote	It is denoted by **det A** or **| A |** or **Δ**
2 × 2 Matrix	$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$
3 × 3 Matrix	$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2c_3-b_3c_2)-a_2(b_1c_3-b_3c_1)+a_3(b_1c_2-b_2c_1)$
Minor	M_{ij}=Minor of the element located in i^{th} row and j^{th} column. It is equal to determinant of sub matrix obtained after i^{th} row and j^{th} column
Cofactor	$C_{ij} = (-1)^{i+j} M_{ij}$
3 × 3 Formula using Cofactors	$a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$
Properties	a. Δ remains unaltered if its rows or columns are interchanged. b. Δ change its sign if two rows or columns interchanges c. If any two rows or columns of a determinant are identical, then Δ =0 d. If each element of matrix is multiplied by constant k , Δ will also get multiplied by k e. If some or all of the elements of a row or column of a determinant are expressed as sum, then Δ is expressed as sum of Δs f. Δ will remain same if equi-multiple of any row or column is added to each element of any row or column
Singular Matrix	if det A = 0, then singular matrix otherwise non-singular matrix
Adjoint Matrix	Adjoint of A Matrix is the transpose of the Cofactor Matrix
Inverse Matrix	If A is a square matrix, and det A ≠ 0 (non-singular), then $\mathbf{A^{-1}} = \frac{\mathbf{1}}{\mathbf{\|A\|}} \times$ **Adj. A**
Cramer's rule to find solution of linear eq. in 3 variables	$x = \frac{\Delta x}{\Delta}$, $y = \frac{\Delta y}{\Delta}$, $z = \frac{\Delta z}{\Delta}$, provided Δ≠0 [$\Delta x$ means determinant of matrix by replacing first column of matrix with RHS values of equations] See Example
Properties of Cramer's	a. If Δ≠0, the system has unique solution b. If Δ=0 and atleast one of $\Delta x, \Delta y, \Delta z$ ≠0, then system has no solution and it is inconsistent c. If Δ=0 and all of $\Delta x, \Delta y, \Delta z$ ≠0, then system may or may not have solution,. If it has solution, equations are dependent and there will be infinite no. of solutions. If it doesn't have solution, equations are inconsistent.