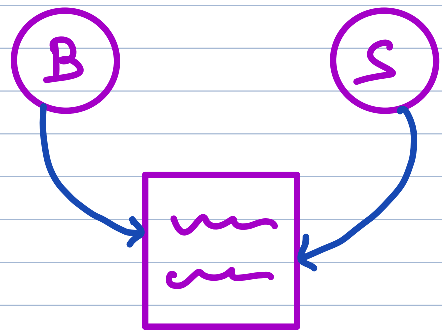
However, Forward Contract has one Drawback - Default Risk

Parties can default at the time of Loss

Solution? → YES



Futures Contract

Introduce A Guarantor

Intermediary

Exchange

- will take the risk of default
- Also it will ask for Initial Margin (Deposit) to protect itself from losses.

①

Spot Tata Motors	100 LONG	10% ↓	70	(30)
Futures Tata Motors	100 SHORT	10% ↓	LONG 70	30

$$\frac{30}{0}$$

②

Spot Portfolio ↳ β:1.2	100 LONG	12% ↓	88	= (12)
Futures Index	100 SHORT	10% ↓	LONG 90	= 10

$$\frac{10}{-2}$$

③

Spot Portfolio ↳ β:1.2	100 LONG	12% ↓	88	= (12)
Futures Index	120 SHORT	10% ↓	LONG 108	= 12

$$\frac{12}{0}$$

↓

position Required in Index Futures for Hedging

↓

Spot position × Risk to Be reduced [Old Beta − New Beta]

1.2 − 0

= 100 × 1.2

= 120

④

Spot Portfolio ↳ -1.2	100 LONG	12% ↓	88	= (12)
Futures Index	120 SHORT	10% ↑	LONG 132	= (12)

$$\frac{(12)}{(24)}$$

4

Spot Portfolio 100 LONG — 12% ↓ — 88 = (12)
↳ -1.2

Futures Index 120 LONG — 10% ↑ — SHORT 132 = 12
$\overline{\quad 0 \quad}$

Hedging
↓
β

+ve → Take opposite position (in Index futures)

-ve → Take same position (in Index futures) → Spot position × Risk to be Reduced → [Old Beta - New Beta]

	+ve		-ve	
Spot	Long	SHORT	LONG	SHORT
Futures	SHORT	LONG	LONG	SHORT

Purpose :- An investor wants to own shares of xyz co after 3 months

Spot Market | Forward Market

$r = 12\%$ p.a.

Spot	100	S = 108	140
Forward	120	F = 108	140

Amp

MAYANK | YOU

MAYANK

t=0

1. Borrow Money +100
2. Buy stock −100

0

t=3m

1. Repayment of Borrowing [100 x 1.03] −103

cost of carrying the stock from 0 to 3m or Holding cost carrying cost

& You own the stock

YOU

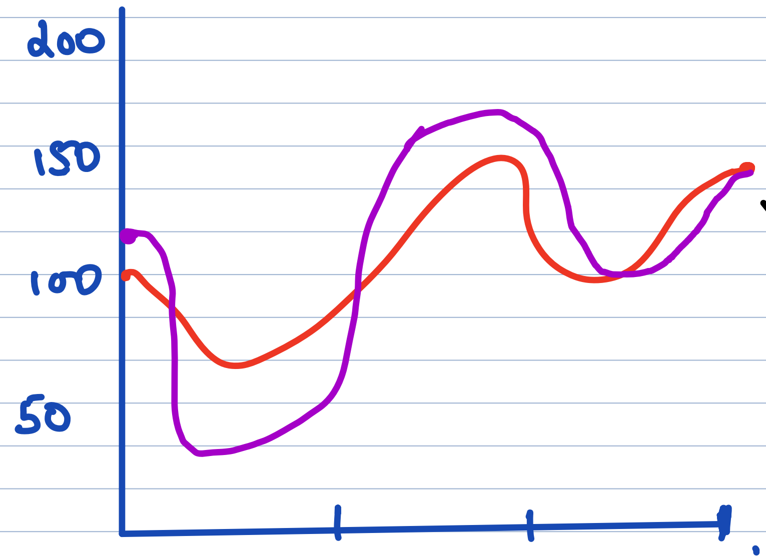
t=0 Long

1. Take Forward = fix price @ 120 —

0

t=3m	S	108	140
1. Settle Forward		−12	+20
2. Buy stock @ spot		−108	−140
		−120	−120

& You own the stock



spot price of xyz

Forward price of xyz
or Futures

convergence effect
it means
At expiry
spot price = futures price

At the end of 3m

1. Mayank own the stock with cof = 103
2. You own the stock with cof = 120

Mayank won, you lost

why?

you were overcharged for the futures price of xyz stock by Rs.17.

How to know whether we are overcharged or undercharged in Forward / Futures Market

or

How to avoid loss of ₹17.

↓

Before Entering into Forward/Futures contract

↓

Calculate TMP

↓

Compare it with AMP

AMP < TMP	AMP > TMP
120 150	120 103
↓	↓
Forward is UNDERVALUED	Forward is OVERVALUED
↓	↓
Buy Forward	Sell Forward

How to calculate TMP

TMP of Forward/Futures is nothing But what we would have paid in the spot market for achieving the same purpose.

$$F = 103$$
$$= 100 + 3$$
$$= 100 + 100 \times 3\%$$

Forward price = spot + spot x rate of interest

$$= S + S \times r$$

$$F = S[1+r]$$

$$CA = P[1+r]$$
$$FV = PV[1+r]$$

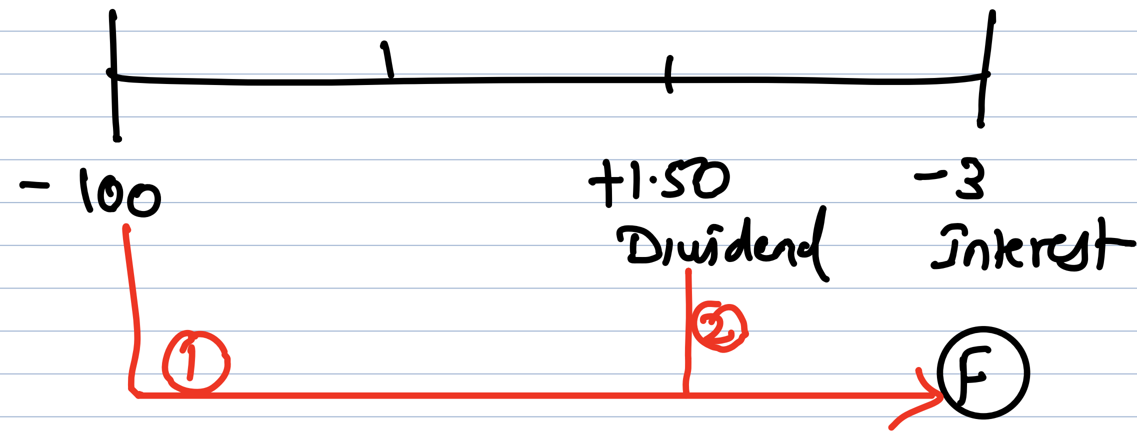
(a) Dividend $t = 3m$, ₹1.50

F = spot + carrying cost - Dividend ... cost of carry.

$$= 100 + 3 - 1.5$$
$$= 101.50$$

$$F = S[1+r] - \text{Dividend}$$

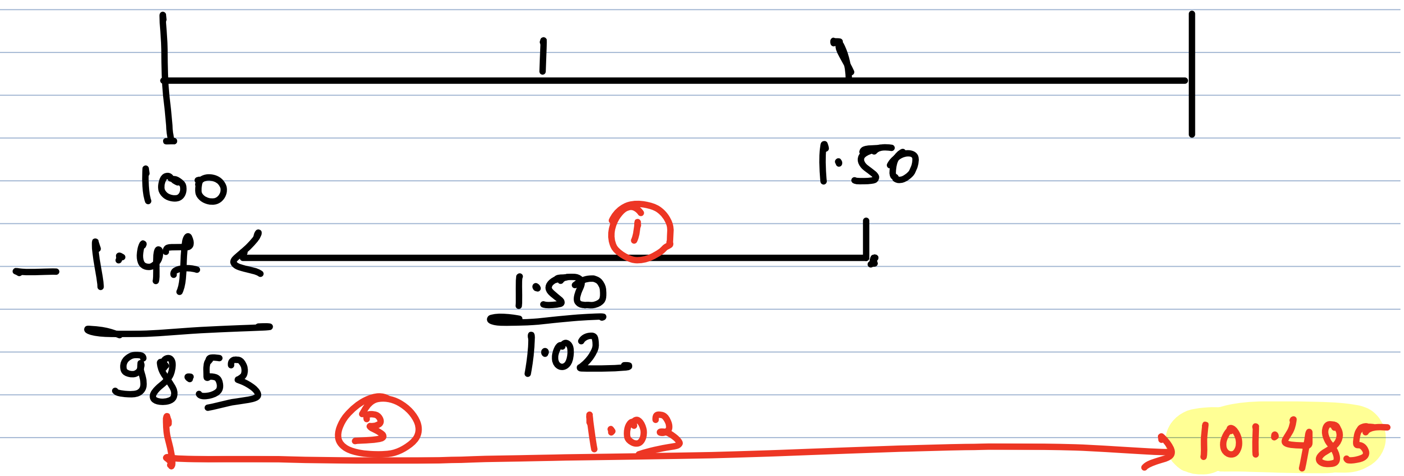
(b) Dividend, $t = 2m$, ₹1.50



$$F = S(1+r) - \text{Dividend}(1+r)$$
$$= 100(1.03) - 1.50 \times (1.01)$$
$$= 103 - 1.515$$
$$= 101.485$$

or

(c)



$$F = [S - \text{pv of } I][1+r]$$

Summary

Dividend

₹, % Rate

↓

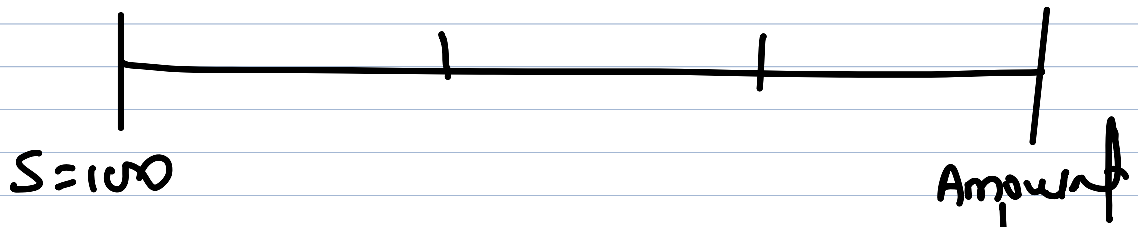
FV

$$F = S[1+r] \qquad t = 3m$$
$$F = S[1+r] - \text{Dividend} \qquad t = \text{before } 3m$$
$$F = S[1+r] - D[1+r] \qquad t = \text{before } 3m$$
$$F = [S - \text{pv of } D][1+r]$$

② Dividend, $t = 3m$, yield = 24% p.a. ✓

↓

Mp = 100



	24% p.a. yield ✗	24% p.a. Rate (Company) [Face value = 10]	₹24 p.a. (Company) 6 p.a.
Dividend to be considered in formula	$= 6$ ($100 \times 24\% \times \frac{3}{12}$)	2.4	24 6 p.a.
	→	60% p.a. Rate	₹6 p.a.

$\frac{6}{100} \times \frac{12}{3} \times 100$

24%

$f = S(1+r) - \text{Dividend}$

$= S(1+r) - S \times \text{yield}$

$= 100(1+0.03) - 100 \times 6\%$ → $24\% \times \frac{3}{12}$

$= 100\left[1 + r - y\right]$

$= 100\left[1 + 0.03 - 0.06\right]$

$= 100\left[0.97\right]$

$= 97$

	r	y
Applied calculated	spot 3m	spot 3m

Summary

Normal Compounding

1. Without dividend $F = S[1+r]$

2. With Dividend

a. ₹
b. % Rate

i] At Expiry $F = S[1+r] - \text{Dividend}$

ii] Before Expiry $F = [S - PV \text{ of } D][1+r]$

c. % yield p.a.

i] At Expiry $F = S[1+r-y]$

ii] Before Expiry $F = [S - PV \text{ of } D][1+r]$

Per Annum → Do not Convert

Per Annum → Convert

For the Expiry period

Continuous Compounding = e^{rt} ~~$(1+r)^t$~~

In Forward contract cash Flows are settled at the expiry only.

But in Futures contract [Exchange Traded] cash Flows occurs continuously

$= S[1+r]$

$= 100[1.03]$

$= 103$

Se^{rt}

$= S \times e^{0.12 \times 3/12}$

$= 100 \times e^{0.03}$

$= 100 \times 1.0305$

$= 103.05$

$2.7183^{0.03}$

2.7183

$\sqrt{}$ 12

$- 1$

$\times 0.03$

$+ 1$

$x = 12$

1.0305

Question

↓

Forward / futures pricing

value of e

is given ↓ Continuous Compounding e^{rt}

is not given ↓ Normal compounding $(1+r)$

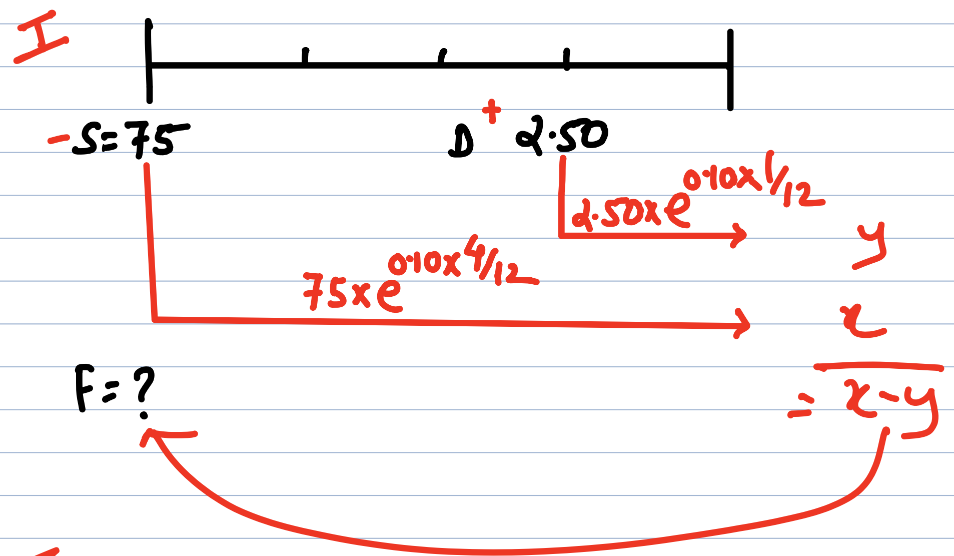
Continuous Compounding :

a. No Dividend $\qquad F = Se^{rt}$

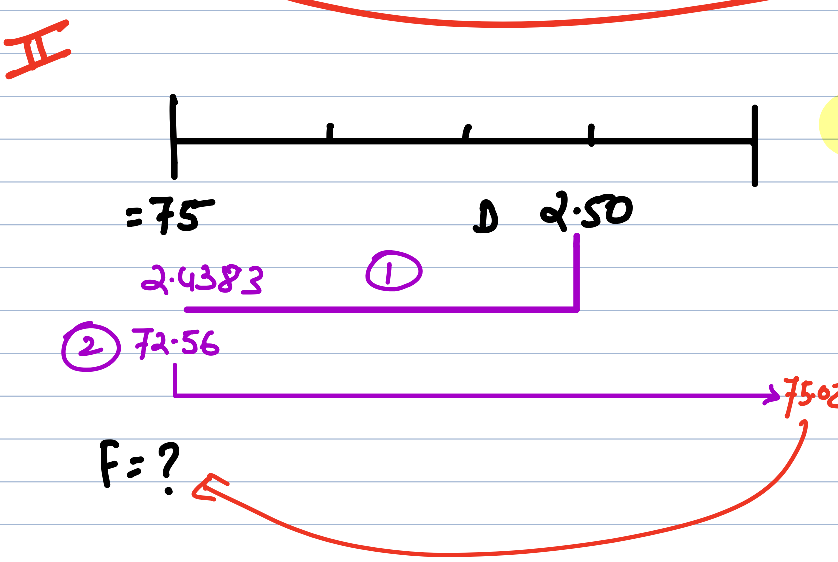
b. Dividend ₹ , % Rate $\qquad F = [S - PV\ of\ D]e^{rt}$

% yield $\qquad F = Se^{(r-y)t}$

Q4 I



II



PV of dividend

$2.50 \times e^{-rt}$

$2.50 \times \frac{1}{e^{rt}}$

$2.50 \times \frac{1}{e^{0.10 \times 3/12}}$

$2.50 \times \frac{1}{e^{0.025}}$

$2.50 \times \frac{1}{1.0253}$

2.4383
$\times 500$
1219.15

Future value

$F = (S - PV\ of\ D)e^{rt}$

$= (75 - 2.4383)e^{0.10 \times 4/12}$

$= 72.56\ e^{0.0333}$

$= 72.56 \times 1.03386$

$= 75.02$

3) value of contract

= price x lot size

= 75.02 x 500

= 37509

Q7

0   30 (1)   60 (2)   90 (3)

S= 1000   750   8·50   9·00

$e^{-0.01}$ $e^{-0.02}$ $e^{-0.03}$

24·49

975·51 $e^{rt} = e^{0.03} = 1.03045$ 1005·21

F=

$F = (S - \text{PV of Dividend})\,e^{rt}$

$= (1000 - 24.49)\,e^{0.03}$

$= 975.51 \times 1.03045$

$= 1005.21$

$F = Se^{rt} - \text{PV of dividend}$

$= 1000 \times 1.03045 - 24.49$

$= 1030.45 - 24.49$

$= 1005.96$

0·75

1000
− 24·49
975·51

− 30·45
+ 25·24
5·21

1000 → 1005·96

24·49

$= 24.49 \times 0.03045$

$= 0.75$

1000 → 1005·21

Q8

S= 125   117·50   4

F= 131   122·90

Long   SHORT

x100   x100

$F = S(1+r) - D$
$= 117.50(1.08) - 4$
$= 122.90$

Q9

Index

S : 2376.24 ——————————————— 2100

F =	2400
lot size =	75
Contract value =	180000 ①
× N :	5
Total value	900000
	SHORT

F = ? 2100
× 75
157500
5
787500 = 112500
LONG
P

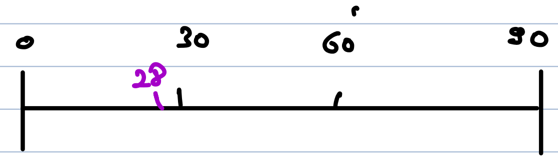
②
$F = S(1+r-y)$
$2400 = S\left[1 + (0.09 - 0.06) \times \frac{4}{12}\right]$
$2400 = S\left[1 + 0.01\right]$

$S = 2376.24$

Basis = Spot − Futures
$= 2376.24 - 2400$
$= -23.76$
↓
Negative
↓
S < F
↓
Growth
↓
Contango

$2376.24 - 2300$
$= 76.24$
↓
positive
↓
S > F
↓
Decline
↓
Backwardation

Q10



Portfolio Index	2290	2450	2470
Futures Index	2303.65 Long	2460.05 SHORT	2470 SHORT

time remaining till expiry

① $F = S e^{(r-y)t}$

$= 2290 e^{(0.0416 - 0.0175)\frac{90}{365}}$

$= 2290 \times e^{0.005942}$

$= 2290 \times 1.005960$

$= 2303.65$

② $F = S e^{(r-y)t}$

$= 2450 \, e^{(0.0416 - 0.0175) \times \frac{62}{365}}$

$= 2450 \, e^{0.00409}$

$= 2450 \times 1.00410$

$= 2460.05$

③ Due to convergence effect of expiry $S = F$

$\therefore$ gain $= 2470 - 2303.65$

$= 166.35$

At expiry $F = S e^{(r-y)t}$

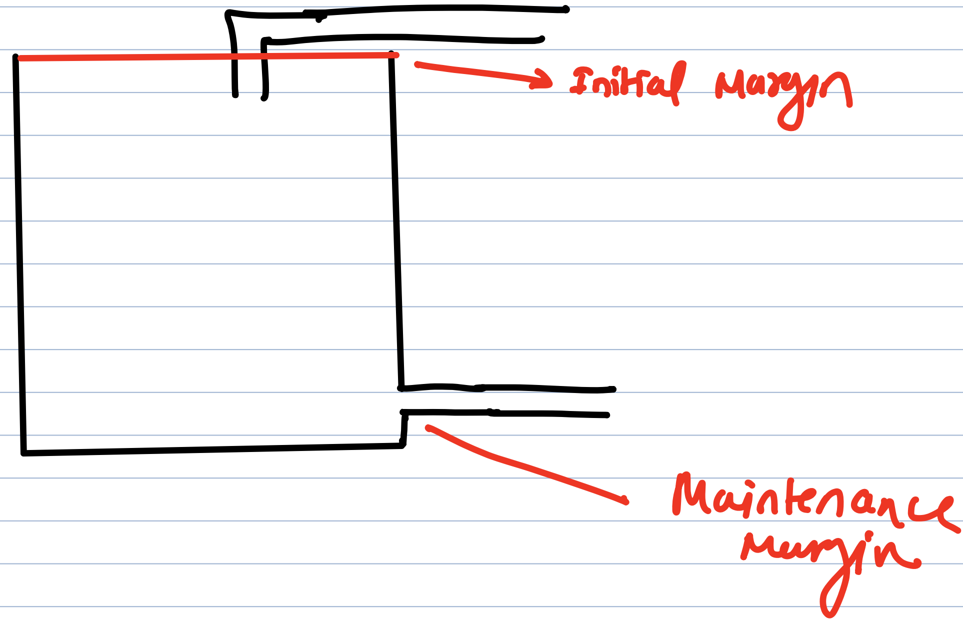
$= 2470 \, e^{(0.0416 - 0.0175)\frac{0}{365}}$

$= 2470 \times e^{0}$

$= 2470 \times 1$

$= 2470$

Q11



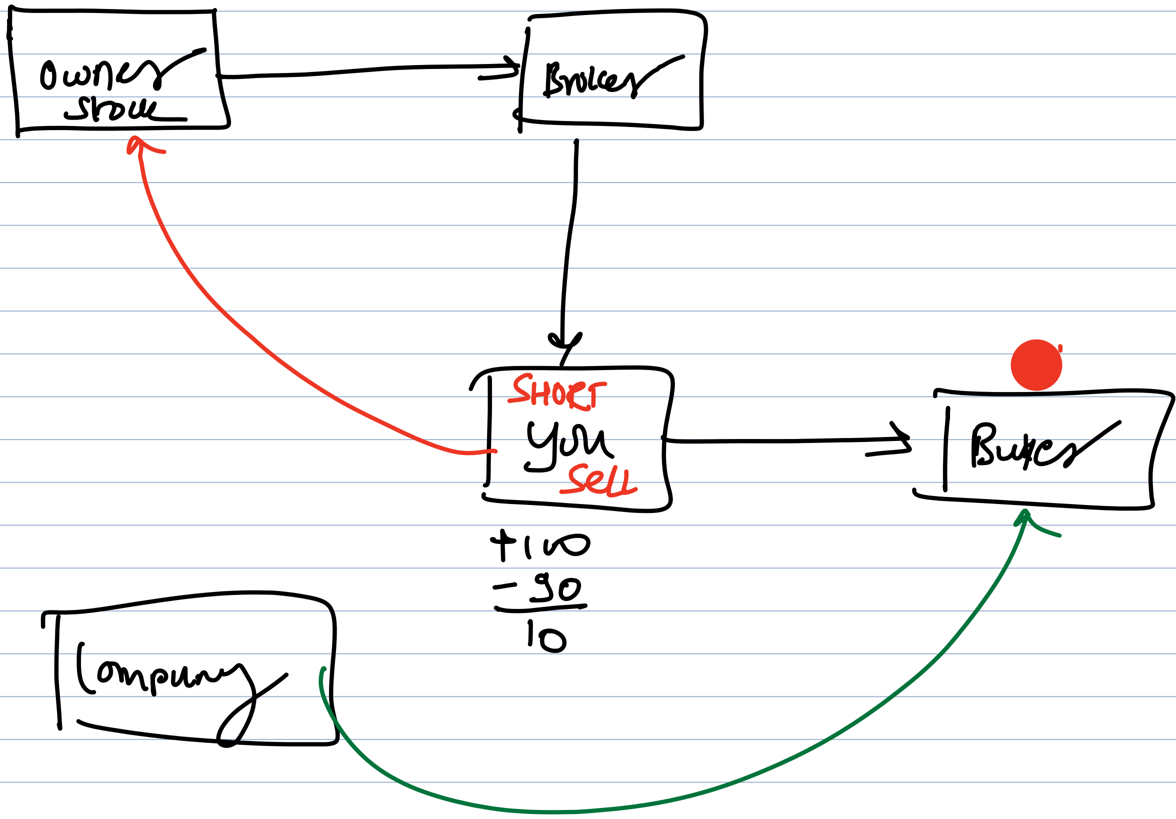
Date	Sensex	Change	Margin Bal	Margin Call
4.02	3296.50	—	16000	—
5.02	3294.40	[3294.40 − 3296.50]×50 = −105	15895	—
6.02	3230.40	[3230.40 − 3294.40]×50 = −3200	12695	—
07.02	3212.30	[3212.30 − 3230.40]×50 = −905	16000 [11790 + 4210]	4210 ✓

Q16

	01	15	30
Spot	948	1012	974
Futures	980	1005	974
	×250	×250	×250
	245000	SHORT 251250 6250	243500
	× 3 (3)	× 1 (2)	× 2 (0)
	Long 735000	251250	SHORT 487000

Q19

100 → 90
10



Long +

SHORT −

Contract value £56000
5.6x10000

Margin 45% − £25200 Deposit
Commission − £ 1550 Expense

Investment − 26750

Dividend −2500
0.25x10000
Profit on short position +11000
[5.60 − 4.50] x10000
Deposit Refund + 25200
Commission − 1450
Net CF + £32250

Return = $\frac{32250 - 26750}{26750}$ = 20.56%

Q20

Spot	1800	1700 2000
Futures	1950 AMP	f: ? 1700

AMP[1950] > TMP[1908]

futures are overvalued → Sell futures

if AMP ≠ TMP

∴ futures are mispriced in market

∴ Arbitrage is possible

t=0

Invest / Sell / Buy

Borrow	+1800
Buy stock	−1800
Sell Futures [1950]	–
	0

t=6m	1700	2000
Repayment [1800×1.06]	−1908	−1908
Sell Stock	+1700	+2000
Futures settle [1950 − 1700] [1950 − 2000]	+250	−50
	42	42

Q22

Spot	13800	6
Index	14340 Amp	

Dividend: 13800 × 50% × 4.8% = 331.20 COF

Interest: 13800 × 12% × 6/12 = 828 COF

principal = 13800 COF

14296.80 TMP

10200 15600 → 10200 15600

AMP > TMP

Futures are overvalued

∴ Sell Futures

t=0

Borrow	+13800
Buy Index in Spot	−13800
Sell Futures [14340]	–

t=6m	10200	15600
Repayment 13800×1.06=	−14628	−14628
Dividend Received	+331.20	+331.20
Sell Index	+10200	+15600
Settle Futures	+4140 [14340 − 10200]	−1260 [14340 − 15600]
Arbitrage gain	43.20	43.20

- AMP = TMP → Arbitrage is not possible
- AMP ≠ TMP → Arbitrage is possible
  - AMP > TMP → Overvalued
    - t = 0: Sell Futures; Buy spot; Borrow @ spot
    - t = Expiry: Close all the open positions
      - Repayment of borrowings — COF
      - Receive dividend (if any) — CIF
      - Sell stock/Index @ spot — CIF
      - Settle Futures [Buy] — CIF/COF
      - gain
  - AMP < TMP → Undervalued
    - t = 0: Buy Futures; Sell spot; Invest @ spot
    - t = expiry: Close all the open positions
      - Investment proceeds — CIF
      - Buy stock @ spot — COF
      - Pay Dividend (if any) — COF
      - Settle Futures (SHORT) — CIF/COF
      - gain

b) Implied Rf $F = S + S \times r \times t - S \times \text{yield}$

$14340 = 13800 + 13800 \times r \times 6/12 - 13800 \times 50\% \times 4.8\%$

$14340 = 13800 + 6900r - 331.20$

$6900r = 871.20$

$r = 12.63\%$

Q24

15/09 — 30 — 15/10 — 31 — 15/11 — 30 — 15/12 = 91

Spot 1195 → Div — $1195 \times 3\% \times \frac{91}{365} = 8.94$

Int — $1195 \times 9.5\% \times \frac{91}{365} = 28.30$

Futu 1225 Amp

Pric — 1195 TMP

$$f = s\left[1 + (r-y)t\right] \times 100$$

$$= 1195\left[1 + (0.095 - 0.03) \times \frac{91}{365}\right] \times 100$$

$$= 121437$$

Q26

Spot = 60 — 57 — +57

futures = 59 SHORT — 58 LONG = +1

58

Q27

Spot E 90

$\beta_E = 1.1$ 99

Futures 4300

lotsize 50

Contract value 215000

$$N = \frac{\text{value to be Hedged} \times \text{Risk to be Reduced}}{1 \text{ contract value}}$$

$$= \frac{90000000 \times [1.1 - 0]}{215000}$$

$$= \frac{99000000}{215000}$$

No. of contracts $= 4604.65$ i.e. 4605 SHORT

			gain (loss)
Spot E 90000000 LONG		11% ↓	(99000000)
Futures Index	99,000000 SHORT	10% ↓	99000000
			0

Q28

Spot	994450	LONG	$2 \times 1.102 = 2.204\%$ ↓	(21918)
Futures F	8767.07 × lot size 25 = 219177 × No. of contract 5 = 1095885	SHORT	2% ↓	P 21918

$$N = \frac{\text{Value to be Hedged} \times \text{Risk to be reduced}}{\text{Contract Value}}$$

$$5 = \frac{994450 \times [\beta_0 - \beta_N]}{8767.07 \times 25}$$

$$5 = \frac{994450 \times [\beta_0 - 0]}{8767.07 \times 25}$$

$$1095885 = 994450\ \beta_P$$

$$\beta_P = 1.102 = \Sigma w_i \beta_i$$

$$= w_{1-9}\,\beta_{1-9} + w_{10}\,\beta_{10}$$

$$1.102 = 0.90 \times 1.10 + 0.10 \times \beta_{10}$$

$$\beta_{10} = 1.12$$

1-9 ✓

10th ✓

1-10 ✓

Q29

Spot $\beta = 1.2$	27387000	LONG	1.2% ↓	(328644)
Futures	3286400	SHORT	1% ↓	328644
				0

Futures: $6086 \times 50 = 304300$

$$N = \frac{27387000 \times 1.2}{304300} = 108 \text{ SHORT}$$

Q30

Spot portfolio (1.5) 5050000 LONG

Sensex 25000 → 22500

Futures sensex 25250 → 22556.25

$$\begin{array}{r} 25250 \\ \times\ 50 \\ \hline 1262500 \\ \times\ 6 \\ \hline 7575000 \end{array} \text{ SHORT} \qquad \begin{array}{r} 22556.25 \\ \times\ 50 \\ \hline 1127812.5 \\ \times\ 6 \\ \hline 6766875 \end{array} \text{ LONG} = 808125$$

① $F = S[1 + (r - y)t]$

$= 25000[1 + (0.09 - 0.06)4/12]$

TMP $= 25250$

TMP
Futures are fairly traded
$\therefore$ AMP = FMP
AMP = 25250

② $$N = \frac{5050000 \times 1.5}{25250 \times 50} = \frac{7575000}{1262500} = 6 \text{ SHORT}$$

③ F of$_{3m}$ $= S[1+(r-y)t]$

$= 22500[1+(0.09-0.06)\,1/12]$

$= 2255625$

Gain $= [25250-22556.25]\times 50\times 6 = 808125$

Q31

0.849

Spot portfolio 18854860 LONG
Nifty 5900

futures Nifty ?

CCREC

$e^{0.0998} = 1.105$
i.e. 10.5%

$\ln[1.105] = 9.98\%$

2] $F = Se^{rt}$

$= 5900\times e^{0.0998\times\frac{58}{365}}$

$= 5900\times e^{0.01586}$

$= 5900\times 1.01598$

Tmp $= 5994.28$

$\beta_0 - \beta_n$ $\quad 0.849 - 0 = 0.849$

β

3] $N = \dfrac{18854860\times 0.849}{F\,Amp \times \text{lot size}}$

$= \dfrac{16007776.}{5994.28\times 200}$

Short $=$ 13.35 i.e. 14 contracts

$\beta_0 - \beta_n$

$0.849 - 0.60 = 0.249$

4] $N = \dfrac{\text{Value to be Hedged} \times \text{Risk to be Reduced}}{\text{Contract Value}}$

Q32

APR 30APR 31 MAY 30 JUNE

Nifty 8500

Fmay

Fjune

$F_{may} : Se^{rt}$

$= 8500\, e^{0.20 \times \frac{2}{12}}$

$= 8500\, e^{0.0333}$

$= 8500 \times 1.03387$

TMP F_{may} $= 8787.90$

$F_{june} = Se^{rt}$

$= 8500 \times e^{0.20 \times \frac{3}{12}}$

$= 8500 \times e^{0.05}$

$= 8500 \times 1.05127$

$= 8935.80$

$e^{0.0333}$	$e^{0.0300}$	1.03045
	$-\,e^{0.0400}$	1.04081
0.0033	0.0100	0.01036

$= 1.03045 + \frac{0.01036}{0.0100} \times 0.0033$

$= 1.03387$

34

01/09 01/10 01/11 01/12 01/01

Spot Portfolio 11600000

BSE sensex ? 1 58000 2 56500 ✓

$\frac{56641.25}{1.0025}$

Futures Sensex 58580

lot size 50

Contract value 2929000

1]

$F = S\left[1 + (r-y)t\right]$

$58580 = S\left[1 + (0.09 - 0.06) \times \frac{4}{12}\right]$

$58580 = S \times 1.01$

$S = 58000$

$H = \frac{PV \times \text{Risk to be Reduced}}{\text{contract value}}$

Spot	7000000 L	12.50% ↑	875000
Future Hedging	875000 S	10% ↑	(875000)
			0

$\Delta P = \Delta M \times \beta$

$= 10\% \times 1.25 = 12.5\%$

$\Delta M = \frac{\Delta P}{\beta} = \frac{5\%}{1.25} = 4\%$

Q36

Portfolio xy 50000000 1.15

i] Reduce Beta to 0.85

$\beta_P = \Sigma w_i \beta_i$

$0.85 = w_0 \beta_x + w_{rf} \beta_{rf}$

$0.85 = w_0 \times 1.15 + 0$

$w_0 = 0.7391$ i.e. 73.91%

PM → Add funds

Old	New
E = 5	E = 5
	Rf = 1
	6

Derivatives → CHANGE in Composition.

Old	New	
E = 5	E = 4	0.7391
	Rf = 1	
	5	

Value of new portfolio = 5 crore
weight of old portfolio = 73.91%
Value of old portfolio 3.6955
∴ Add Rf securities 1.3045

i.e. sell Equity & Buy rf security worth ₹ 1.3045 crores

b) Futures $N = \frac{50000000 \times (1.15 - 0.85)}{21000 \times 150}$

$= \frac{15000000}{3150000}$

$= 4.76$ i.e. 5 contracts SHORT

0.30
0.30

2] Increase 1.45

$\beta_M = \beta_o \times W_o + 0$
$1.45 = 1.15 \times W_o$
$W_o = 1.2609$

Value of old portfolio $5 \times 1.2609 = 6.3045$
SHORT rf securities $6.3045 - 5 = \frac{-1.3045}{5}$

E5 E 5-1=4
Rf =+1

E5 E 5+1=6
Rf $\frac{-1}{5}$

$N = \frac{50000000\,(1.15 - 1.45)}{21000 \times 150}$

$= \frac{-15000000}{3150000}$

$= -4.76$ Long

same

A	5	0.25	1 = 4 0.25 B
B	5	0.25	1 = 4 0.25 B
C	5	0.25	1 = 4 0.25 B
D	5	0.25	1 = 4 0.25 B
	20		16

4

1.30

70%

0.91

$\beta = 1.30$

Portfolio	S	5000 LONG	2.6% ↑	5130	130
Nifty	F	1950 SHORT	2% ↑	1989	(39)
					91

5000 x 0.39 =

8125 x 200 x 120

F x L x N

5000

$\Delta P = \frac{91}{5000} = 1.82\%\uparrow$

5130 − 39

5091

$\Delta M = 10\%\uparrow$ 2%

$\beta = 1.5$

$\Delta P = 15\%\uparrow$ 1.82%

$\frac{\Delta P}{\Delta M} = \beta$

$\frac{1.82\%}{2.00\%} =$

$= 0.91$

Q 39 0.50

S	4740000 L	5% ↓	(237000)
F	2370000 S	10% ↓	237000
			0

Q43

X inc	100000 x 22 = 2200000 Long	?	2% ↓	(44000)
APlc	50000 x 40 = 2000000 SHORT	2	3% ↑	(60000)
	200000 LONG			
Nifty Futures	1000 x x = 1000x SHORT LONG		1.5% ↓ ↑	15x (10500)
				(114500)

$15x - 104000 = -114500$

$15x = -10500$

$x = -700$

i.e. LONG 700 Nifty Futures Contract

−700 SHORT = 700 LONG

$N = \frac{\text{Value to Hedge} \times \text{Risk to Reduce}}{\text{Contract value}}$

$-700 = \frac{2200000 \times \beta_x - 2000000 \times 2}{1000}$

$-700000 = 2200000\beta_x - 4000000$

$3300000 = 2200000\beta_x$

$\beta_x = 1.5$

Spot	Equity	170 L	β=1.6	3.2% ↓	164.56 =	(5.44)
	Cash	30 L	β=0		26.00 =	(4)
		200	β=1.36	4.72% ↓	190.56	(9.44)
Futures	Nifty	200 L		2% ↓		(4)

$\Delta M = 10$ 2% ↓

$\beta = 1.5$ 1.60

$\Delta E = 15\%$ 3.2%

$$\Delta P = \frac{9.44 \downarrow}{200} = 4.72\%$$

$$\beta p = \frac{\Delta P}{\Delta M} = \frac{4.72}{2} = 2.36$$

OPTIONS

Spot	S= 100		S= 70	140
Forward	F= 110 LONG COF= NIL	SHORT	F= 70 (40)	140 30
Futures	F= 110 LONG COF= Initial Margin (Deposit)	SHORT	F= 70 (40)	140 30
Options CALL	K= 110 LONG COF= premium (Expense) -10		No Action NE Lapse 0	Action Exercise. Payoff 30
Insurance	K= 500000 LONG COF= premium (expense)		NO Medical Expenses 0	Medical Expenses 200000

Payoff

= Max[S-K,0]

Options

Buyer
Holder

Seller
Writer

Call
Right to Buy Asset

Put
Right to Sell Asset

Stocks
Index
Commodity
Currency

Premium

Basic

CALL = $MAX[S-K, 0]$
PUT = $MAX[K-S, 0]$

Methods

Payoff

CALL
Buy Asset
K → Lower than Spot
Exercise: $K < S$

PUT
Sell Asset
K → Higher than Spot
$K > S$

	Call	Put
Payoff	Max[S-K,0]	Max[K-S,0]
premium	Deduct	Deduct
Net payoff	net profit	net profit

Q45

S: 500 S=360

Buy +C	K=560	-40	LAPSE,	0
sell +P	K=460	-10	Exercise,	100
		-50		100
		x 100		x 100
		-5000		10000

Q47

K=220	200	210	220	230	240
Call K<S					
Action	NE	NE	NE	E	E
Gross payoff	0	0	0	10	20
premium	-6	-6	-6	-6	-6
Net payoff	-6	-6	-6	4	14
put K>S					
Action	E	E	E	E	E
Gross payoff	20	10	0	0	0
premium	-5	-5	-5	-5	-5
Net payoff	15	5	-5	-5	-5
Total Net payoff	9	-1	-11	-1	9

Call K=220 COF
↓ +6 COF
K<S 226

put K=220 COF
−5 COF
215
Below

	Above			Above			
S	210	$\boxed{220}$	222	225	$\boxed{226}$	228	230
K	220	220	220	220	220	220	220
Action	NE	NE	E	E	E	E	E
Payoff	0	0	2	5	6	8	10
premium	-6	-6	-6	-6	-6	-6	-6
net payoff	-6	-6	-4	-1	0	2	4
	Not Exer.		Exercise Not Gainfully		Exercise Gainfully		

Above 220 = Exercise
Above 260 = Gainfully Exercise

Q51

a. Expiration Date CF (Gross Payoff)

K = 60		50	55	60	65	70
K < S	+ call	0	0	0	5	10
	− call	0	0	0	−5	−10
K > S	+ Put	10	5	0	0	0
	− Put	−10	−5	0	0	0

b) Investment value

K = 60	50	55	60	65	70
+ call	−9	−9	−9	−9	−9
− call	+9	+9	+9	+9	+9
+ Put	−1	−1	−1	−1	−1
− Put	+1	+1	+1	+1	+1

Spot LONG → (loss) 10

D SHORT → (gain) 10

0

Spot LONG → (loss) 10

D LONG → (gain) 10

Call Put

		800	1050
Spot	1000 Long	+800	+1050
Put	950 Long	+150	0
	−8	950	1050

$S = 100$ $\quad$ 80 $\quad$ 100 $\quad$ 120 $\quad$ 130

Call $K = 100$ $\quad$ 0 $\quad$ 0 $\quad$ 20 $\quad$ 30

$C_0 = 20$ AND

Investment $\quad$ Return

CF

$S = 100$ $\quad$ 90 $\quad$ 110 $\quad$ 130 $\quad$ 170

Call $K = 110$ $\quad$ 0 $\quad$ 0 $\quad$ 20 $\quad$ 60

$C_0 =$ $\quad$ $\times$ $\quad$ $\times$ $\quad$ $\times$ $\quad$ $\times$

0.30 $\quad$ 0.20 $\quad$ 0.25 $\quad$ 0.25

$\boxed{24}$

Q53

① $S = 132$ $\quad$ $\underline{S = 160.50}$ $\quad$ $S = K = 150$

E

$K = 150$

② $MAX[S - K, 0]$ $\quad$ $C_0 = 150 - 150, 0$

$Max[160.50 - 150, 0]$

$Max[10.50, 0]$

10.50

(3)

K	S	Action	Payoff	prob.	PxP
150	120	NE E	0 -30	0.05	0 -1.5
150	140	NE E	0 -10	0.20	0 -2
150	160	E	10	0.50	5 5
150	180	E	30	0.10	3 3
150	190	E	40	0.15	6 6
				C_0	14 10.50

Q55

Investment

$C_0 = COF$

Payoff

CIF = Break Even 0

10

$C_0 = 10$

COF

Payoff = 10

CIF

PV

BINOMIAL MODEL

$C_0 = \underline{S-K}$

$K = 110$

$C_0 = ?$

$S_0 = 100$

120 E $10 \times 0.40\% = 4$

S_u C_u p

$1-p$

90 NE $0 \times 0.60\% = 0$

S_d C_d

4

$3.64 \xleftarrow[1\,yr]{10\%} 4$

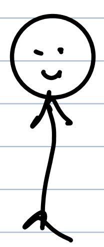
3.90 $\quad \dfrac{4}{1.025}$ $\quad 10\%$ $\quad 3m$

$\dfrac{4}{e^{rt}}$ $\quad 10\%$ $\quad 3m$ $\quad$ CCRRI

$$C_0 = \frac{4 \times 0.40 + 0 \times 0.60}{1.10}$$

$$C_0 = \frac{C_u \times P + C_d(1-P)}{1+r \to e^{rt}}$$

Risk Neutral Method



Equity 100 ——— $120 \times 0.7333 = 88$

$90 \times 0.2667 = 24$

112

FD 100 ——— 112

12%

$$FD\ CF = Equity\ CF$$

$$100(1.12) = S_u \times p + S_d(1-p)$$

$$S_0(1+r) = S_u \times p + S_d(1-p)$$

$$(1) \quad S_0(1+r) = S_u p + S_d - S_d p$$

$$S_0(1+r) - S_d = p(S_u - S_d)$$

$$\frac{S_0(1+r) - S_d}{S_u - S_d} = p$$

$$\frac{\frac{\cancel{S_0}(1+r)}{\cancel{S_0}} - \frac{S_d}{S_0}}{\frac{S_u}{S_0} - \frac{S_d}{S_0}} = p$$

② $$\boxed{\frac{(1+r) - d}{u - d} = p}$$

$$\frac{e^{rt} - d}{u - d}$$

$$\frac{1.12 - 90/100}{120/100 - \frac{90}{100}} = p$$

$$\frac{1.12 - 0.90}{1.20 - 0.90} = p$$

$$p = 0.7333$$

$$\frac{e^{0.08 \times 0.25} - \frac{400}{420}}{\frac{500}{420} - \frac{400}{420}} = p$$

$$\frac{1.0202 - 0.9524}{1.1905 - 0.9524} = p$$

$$\frac{0.0678}{0.2381} = p$$

$$p = 28.48\%$$

call | stock

$$C_0 = \frac{C_u \times p + C_d(1-p)}{1+r}$$

$$S_0 = \frac{S_u \times p + S_d(1-p)}{1+r}$$

$$\downarrow$$

$$p = \frac{(1+r) - d}{u - d}$$

Call

K	S	Action	Payoff	
100	90	NE	0	Out of The Money
$K > S$				
100	100	NE	0	AT The Money
$K = S$				
100	140	E	40	In The Money
$K < S$				

Q58

$$S_0 = \frac{S_u p + S_d(1-p)}{1+r}$$

$$26000 = \frac{27300p + 24700 - 24700p}{1.03}$$

$$26780 = 2600p + 24700$$

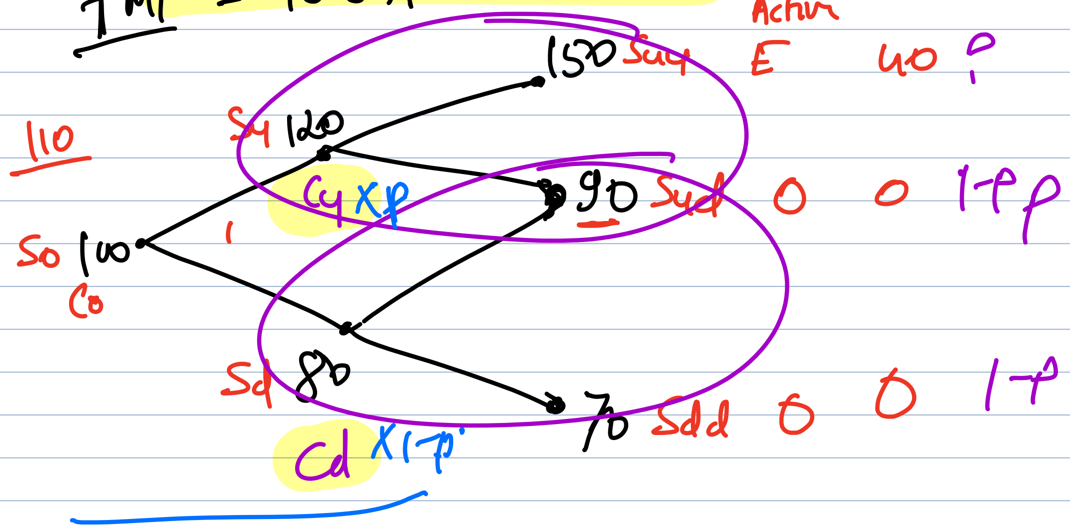
$$\frac{2080}{2600} = p$$

$$p = 0.80$$

$$C_0 = \frac{C_u \times p + C_d(1-p)}{1+r}$$

$$= \frac{1300 \times 0.80 + 0}{1.03}$$

IMP = 1009.71 i.e. 1010



Q59

$$C_0 = \frac{C_u \times p + C_d(1-p)}{1+r}$$

$$= \frac{6.33 \times 0.75 + 1.21 \times 0.25}{1.05}$$

$$C_0 = 4.81$$

$$C_u = \frac{C_{uu} \times p + C_{ud}(1-p)}{1+r}$$

$$= \frac{8.30 \times 0.75 + 1.70 \times 0.25}{1.05}$$

$$= 6.33$$

$$C_d = \frac{C_{ud} \times p + C_{dd}(1-p)}{1+r}$$

$$= \frac{1.70 \times 0.75 + 0}{1.05}$$

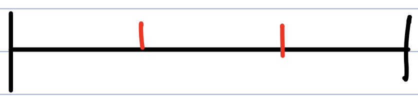
$$= 1.21$$

$$S_u = \frac{S_{uu}\, p + S_{ud}(1-p)}{1+r}$$

$$33 = \frac{36.30p + 29.70 - 29.70p}{1.05}$$

$$34.65 = 6.60p + 29.70$$

$$p = 75\%$$



European B S Exercise only of Expiry

American B S S S S S S S S S S Exercise on or before expiry

$S = 140$ $\qquad$ 110 $\qquad$ 150 $\qquad$ 70

$K = 110 \leftarrow e^{-rt}$ $\qquad$ E $\qquad$ NE

40 $\qquad$ 0

① $S - K \times e^{-rt}$

$$\boxed{C_0 = S \times N(d_1) - K e^{-rt} \times N(d_2)}$$

$$d_1 = \frac{\ln\left[\frac{S}{K}\right] + \left[r + \frac{\sigma^2}{2}\right]}{\sigma\sqrt{t}}$$

$\ln(1.105) = 0.0998$

$e^{0.0998} = 10.5\%$

Q 63

$S = 415$

Call $\quad K = 400$

$C_0 = 25$ Amp

Buy

Amp < TMP

BM ×

BSM

$$C_0 = S \times N(d_1) - K \times \frac{1}{e^{rt}} \times N(d_2)$$

$$= 415 \times 0.6937 - 400 \times \frac{1}{e^{0.05 \times 0.25}} \times$$

$=$

$=$

$=$

$$d_1 = \frac{\ln\left[\frac{S}{K}\right] + \left[r + \frac{\sigma^2}{2}\right]t}{\sigma\sqrt{t}}$$

$$= \frac{\ln\left[\frac{415}{400}\right] + \left[0.05 + \frac{0.22^2}{2}\right]0.25}{0.22\sqrt{0.25}}$$

$$= \frac{\ln\left[1.0375\right] + 0.01855}{0.11}$$

$$\begin{array}{ccc} & 1.0300 & 0.029559 \\ \underline{1.0375} & \underline{1.0400} & \underline{0.039221} \\ 0.0075 & 0.0100 & 0.009662 \end{array}$$

$$= 0.029559 + \frac{0.009662}{0.0100} \times 0.0075$$

$$= 0.03681$$

$$= \frac{0.03681 + 0.01855}{0.11}$$

$$d_1 = 0.5033$$

$$d_2 = d_1 - \sigma\sqrt{t}$$
$$= 0.5033 - 0.11$$
$$d_2 = 0.3933$$

$$N(d_1) = N(0.5033)$$
$$= 0.6927$$

$$\begin{array}{ccc} & 0.5000 & 0.6915 \\ \underline{0.5033} & \underline{0.5100} & \underline{0.6950} \\ 0.0033 & 0.0100 & 0.0035 \end{array}$$

$$0.6915 + \frac{0.0035}{0.0100} \times 0.0033$$

$$= 0.6927$$

2)

$$C_0 = S \times N(d_1) - Ke^{-rt} N(d_2)$$

$$= 380 \times \_\_\_\_ - \frac{400}{e^{0.0125}} \times \_\_\_\_$$

=

=

=

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$= \frac{\ln\left(\frac{380}{400}\right) + 0.01855}{0.11}$$

$$= \frac{\ln(0.95) + 0.01855}{0.11}$$

$$= \frac{-0.05129 + 0.01855}{0.11}$$

$$d_1 = -0.2976$$

$$d_2 = d_1 - \sigma\sqrt{t}$$
$$= -0.2976 - 0.11$$
$$d_2 = -0.4076$$

$$N(-0.2976) = 1 - N(0.2976)$$
$$= 1 - 0.6170$$
$$= 0.3830$$

	0.2900	0.6141
0.2976	0.3000	0.6179
0.0076	0.0100	0.0038

$$0.6141 + \frac{0.0038}{0.0100} \times 0.0076$$

$$= \underline{0.6170}$$

(3)

(3)

Put

$Ke^{-rt} \times (1 - N d_2) - S(1 - N(d_1))$ PCPT

X

Put Call Parity Theory

k = same

t =

PUT CALL PARITY THEORY

5%

Portfolio 1

t=0		
Buy stock	−100	105
Buy put [110]	−10	12
	−110	−117

t=1yr	70	140
Sell stock	+70	+140
put	E	NE
	+40	0
	+110	+140

Portfolio 2

Invest pv of K	−104.76
Buy call [K=110]	−10.00
	−114.76

t=1	70	140
Investment proceeds	+110	+110
call	NE	E
	0	+30
	+110	+140

Buy stock + Buy put = Buy call + Invest pv of K

$$S_0 + P_0 = C_0 + Ke^{-rt}$$

$$380 + P_0 = 10.52 + 400 \times \frac{1}{1.012578}$$

408 10

−9.90 pv

✓

Q64

$C = SN(d_1) - Ke^{-rt} N(d_2)$

$=$

$=$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$= \frac{\ln\left(\frac{80}{75}\right) + \left(0.12 + \frac{0.40^2}{2}\right)0.50}{0.40\sqrt{0.50}}$$

$$= \frac{0.0646 + 0.10}{0.2828}$$

$d_1 = 0.5820$

$d_2 = d_1 - \sigma\sqrt{t}$

$= 0.5820 - 0.2828$

$d_2 = 0.2992$

$N(d_1) = N(0.5820)$

0.55	0.2912
0.60	0.2743

$\frac{0.25}{1}$ 1 call 4

PORFOLIO REPLICATING MODEL

$S_0 = 50$
$S_u = 60$
$S_d = 40$
$K = 55$
$C_0 = ?$
$r = 10\%$
$t = 1$ years

Portfolio

Buy stock 0.25 x 50	-12.50
Borrowings	+9.09
Investment	-3.41

t=1	40	60
Sell stock 0.25	+10	+15
Repayment [9.09 x 1.10]	-10	-10
	0	5

Call

Buy 1 Call [55]	?	10
	?	+10

t=1	40	60
Settle call	Lapse	Exercise
Payoff	0	5
	0	5
	0	-5

$C_0 = 50 \times 0.25 - 9.09$

$C_0 = S_0 \times \Delta - \text{Borrowings}$

Hedge Ratio $\Delta = \frac{C_u - C_d}{S_u - S_d} = \frac{5-0}{60-40} = \frac{5}{20} = 0.25$

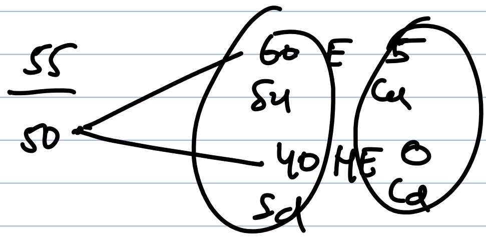
Buy 0.25 share of
Sell 1 call for perfect Hedging

$\frac{S_d \times \Delta - C_d}{1+r}$

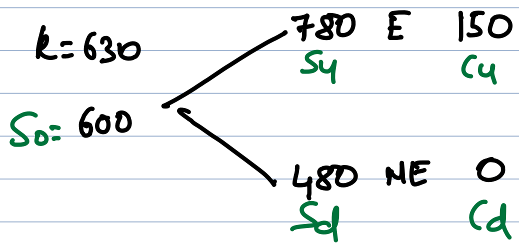
$\frac{40 \times 0.25 - 0}{1.10}$

$\frac{10}{1.10}$

$= 9.09$



Q65



1] $\Delta = \frac{C_u - C_d}{S_u - S_d} = \frac{150-0}{780-480} = \frac{150}{300} = 0.50$

Perfect Hedge situation = Buy 0.5 Share
Sell 1 call option

2] $C_0 = S \times \Delta - \text{Borrowings}$

$= 600 \times 0.5 - \frac{S_d \times \Delta - C_d}{1+r}$

$= 300 - \frac{480 \times 0.50 - 0}{1.025}$

$= 300 - 234.15$

$C_0 = 65.85$

AMP(70) > TMP(65.85)
call overvalued
sell call

Buy stock −300
Borrow +234.15
t=0 −65.85   Sell call +70 = 4.15

780
Sell stock +390
Repayment −240
+150

780 −630
Settle
−150

3)

Buy C_0 = −65.85
Invest
COF

780 E 150 × 0.60 = 90
480 ME 0 × 0.40 = 0
90
CIF

Return = $\frac{90-65.85}{65.85}$
=

66 t=0
Buy 0.5 share
Sell 1 call [550]

t=3m

	650	450
Sell 0.5 share	325	225
Settle	−100	0
	225	225

INTEREST RATE RISK MANAGEMENT

Loan → Buy Loan → LONG → Borrower → $\boxed{\text{Borrowing}}$ → Interest Rate → To Fall

wants

Risk

expected to Rise

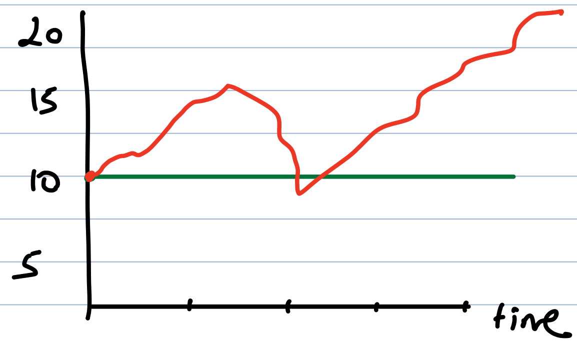
Invest → Sell Loan → SHORT → Investor / Depositor → $\boxed{\text{Investing}}$ → Interest Rate → To Rise

Risk

expected to Fall

$\boxed{\text{loss}}$

Fixed rate
Floating rate



Loan Amount: 60 crores

0 — 6 — 3

Spot 9% RR 9.60% 8.80%

Spot
Hedging
Derivatives
- OTC Forward → 6x9 FRA 9.30% FR
- ET Futures
- ET options
- OTC swaps

You = Buyer = Long FRA
Bank = Seller = Short FRA

Forward Rate = 9.30% → Obligation

	RR-FR	RR-FR	
gain/(loss)	0.30%	(0.50%)	p.a.
	0.0075%	(0.0125%)	3m
Amount	450000	(750000)	CF = 9m
Discount	$\left[1+0.096\times\frac{3}{12}\right]$	$\left[1+0.088\times\frac{3}{12}\right]$	
	1.024	1.022	
Settlement	439453	(733855)	CF = 6m

6 x 9
- 9 → Loan period will end 9 months from now
- 6 → Loan period will start 6 months from now

0 — 6 — 9

- 0: FRA Start
- 6: FRA Expire, Loan Start
- 9: Loan End

Settlement = 439453

$$= \frac{450000}{1.024}$$

$$= \frac{600000000 \times 0.0075\%}{\left[1 + 0.096 \times \frac{3}{12}\right]}$$

Notional principal Amount

RR FR

months till maturity / months in a year

$$= \frac{600000000 \times \left[0.096 - 0.093\right] \times \frac{3}{12} \text{ or } \frac{90}{360}}{\left[1 + 0.096 \times \frac{3}{12}\right]}$$

days till maturity / days in a year

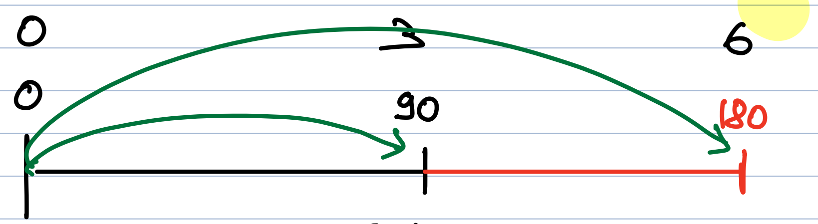
LONG FRA +ve -ve = $\dfrac{N\left[RR - FR\right]\frac{dtm}{dy}}{\left[1 + RR \times \frac{dtm}{dy}\right]}$

+ve: gain, -ve: Loss

SHORT FRA +ve -ve = $\dfrac{N\left[FR - RR\right]\frac{dtm}{dy}}{\left[1 + RR \times \frac{dtm}{dy}\right]}$

+ve: gain, -ve: Loss

Q3



90day LIBOR

SHORT

Hedging SHORT FRA 1.5%

1.25%

$$\text{Settlement}_{\underline{\text{SHORT}}} = \frac{N\left[FR - RR\right] \times \frac{d/m}{d/y}}{\left[1 + RR \times \frac{d/m}{d/y}\right]}$$

$$= \frac{15000000 \times \left[1.50\% - 1.25\%\right] \times \frac{90}{360}}{\left[1 + 0.0125 \times \frac{90}{360}\right]}$$

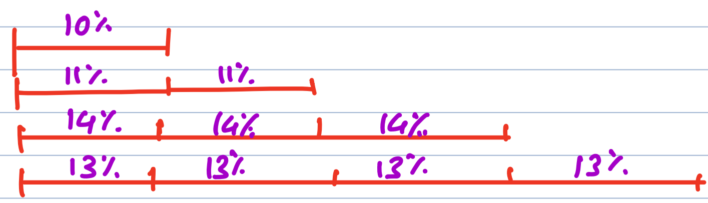
$$= \frac{\$9375}{1.003125}$$

$$= \$9345.79$$

THEORETICAL FORWARD RATE

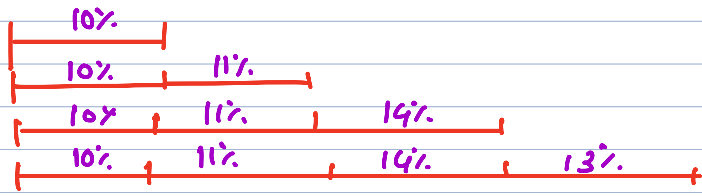
Spot yield

year	yield
1	10%
2	11%
3	14%
4 Four	13%



Forward yield

year	yield
1	10%
2	11%
3	14%
4 Fourth	13%



$$(1+F_1)(1+F_2) = (1+S_2)(1+S_2)$$
$$(1.10)(1.11) = (1+S_2)^2$$
$$1.221 = (1+S_2)^2$$
$$\boxed{S_2 = 10.50\%}$$

Theoretical Rate

I 100 $S_2 = 10\%$ $100 \times 1.10 \times 1.10 = 121$

B 100 $S_1 = 9\%$ $100 \times 1.09 \times 1.14 = 124.26$

FRA = 14%

+109
−109

3.26

FRA 8% $100 \times 1.09 \times 1.08 = 117.72$

FRA 11.01% $100 \times 1.09 \times 1.1101 = 121$

$$121 = 121$$
$$\cancel{100} \times 1.1 \times 1.1 = \cancel{100} \times 1.09 \times [1+F_2]$$
$$[1+S_2][1+S_2] = [1+S_1][1+F_2]$$
$$\frac{[1+S_2]^2}{[1+S_1]} = 1+F_2$$
$$F_2 = \frac{[1+S_2]^2}{[1+S_1]} - 1$$
$$= \frac{(1.10)^2}{(1.09)} - 1 = 11.01\%$$

$$F_2 = \frac{[1+S_2]^2}{[1+F_1]} - 1$$