Chapter 6 – Sequence and Series

FREE FAST TRACK Lectures: https://www.youtube.com/watch?v=ZZpvRpkgmaE&list=PLAKrxMrPL3fwOSJWxnr8j0C9a4si2Dfdz

Lecture 1 of Sequence and Series: https://youtu.be/d2ImTctYMR8

Lecture 2 of Sequence and Series: https://youtu.be/LzpooC2IMXM

Unit 1 – Arithmetic Progression

Arithmetic Progression

A sequence of numbers is known as an Arithmetic Progression if the difference between two consecutive terms is the same. For example,

1. 2, 4, 6, 8, 10… is an arithmetic progression as the difference between any two consecutive terms is the same, i.e. 2.
2. 1, 5, 9, 13… is an arithmetic progression as the difference between any two consecutive terms is the same, i.e. 4.

Since the difference between any two consecutive terms in an A.P. is the same, i.e. common, it is known as the ***common difference***.

The n^{th} term of an A.P. is given by:

$$t_n = a + (n-1)d$$

Here,
t_n = n^{th} term
a = first term
d = common difference

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Arithmetic Mean

If we select any three consecutive terms from an A.P., the middle term is known as the Arithmetic Mean. If there are three consecutive terms t_1, t_2, and t_3, the term t_2 is known as the arithmetic mean, and is given by $\frac{t_1+t_3}{2}$.

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Sum of the First *n* Terms

The sum of *n* terms of an A.P. is given by the formula:

$$S_n = \frac{n}{2}\left\{2a+(n-1)d\right\}.$$

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If the common difference is not given but the first and the last terms are given, the sum can be calculated using the formula:

$S_n = \frac{n}{2}(a+l)$, where l is the last term.

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1. Page 3.303 – Question 3

Unit 2 – Geometric Progression

A sequence of numbers is known as a Geometric Progression if the ratio between two consecutive terms is the same. For example,

1. 5, 15, 45, 135, … is a geometric progression as the ratio between any two consecutive terms is the same, i.e. 3. $\left(\frac{15}{5}=3, \frac{45}{15}=3, etc.\right)$
2. 1, 1/2, 1/4, 1/9, … is a geometric progression as the ratio between any two consecutive terms is the same, i.e. 1/2. $\left(\frac{1/2}{1}=1/2, \frac{1/4}{1/2}=1/2, etc.\right)$

Since the ratio between any two consecutive terms in a G.P. is the same, i.e. common, it is known as the ***common ratio***.

The n^{th} term of a G.P. is given by:

$t_n = ar^{n-1}$

Here,
t_n = n^{th} term
a = first term
r = common ratio

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3. Page 3.338 – Question 55
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Geometric Mean

If we select any three consecutive terms from a G.P., the middle term is known as the Geometric Mean. If there are three consecutive terms t_1, t_2, and t_3, the term t_2 is known as the geometric mean. There are two methods of calculating the geometric mean:

1. If there are three consecutive terms t_1, t_2, and t_3, the term t_2 is the geometric mean and is given by $\sqrt{t_1 \times t_3}$. For example, the Geometric Mean between the numbers 3 and 27 is given by $\sqrt{3\times 27}=9$.
2. Geometric mean is also calculable as the n^{th} root of the product of n numbers. For example, if I ask you to calculate the geometric mean between 3 and 27, it obviously implies that there are three numbers in total. Therefore, n = 3. Also, we know that the G.M. is 9 (as

calculated above). Now, let's check whether 9 is equal to the n^{th} root of the product of n numbers. $(3\times 9\times 27)^{\frac{1}{3}}=9$. Therefore, it is clear that the geometric mean can also be calculated as the n^{th} root of the product of n numbers.

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1. Page 3.368 – Question 98
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3. Page 3.339 – Question 57
4. Page 3.329 – Question 39 – Homework
5. Page 3.328 – Question 38
6. Page 3.326 – Question 34
7. Page 3.322 – Question 28 – Homework

Sum of the First *n* Terms

The sum of n terms of a G.P. is given by the formula:

1. When $r < 1$, $S_n = a\left(\frac{1-r^n}{1-r}\right)$
2. When $r > 1$, $S_n = a\left(\frac{r^n-1}{r-1}\right)$
3. When $r = 1$, it becomes an A.P. with $d = 0$, and the sum is calculated using the formula: $S_n=\frac{n}{2}\left\{2a+(n-1)d\right\}$. For example, to calculate the sum of series 2, 2, 2, … to 11 terms, we have $a = 2$; $d = 0$; $n = 11$; and the sum is given by $\frac{11}{2}\times\left\{(2\times 2)+(11-1)0\right\}=22$.

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4. Page 3.321 – Question 26

Sum of Infinite Terms of a G.P.

The sum of infinite terms of a G.P. is given by the formula:

1. When $r < 1$, $S_\infty = \frac{a}{1-r}$
2. When $r \geq 1$, $S_\infty = \infty$

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8. Page 3.323 – Question 29
9. Page 3.304 – Question 4
10. Page 3.362 – Question 90
11. Page 3.331 – Question 43

Unit 3 – Special Series

Following are some of the Standard Results:

1. Sum of first *n* natural or counting numbers $\left(1+2+3+4+...+n\right)=\dfrac{n\left(n+1\right)}{2}$
2. Sum of first *n* odd numbers $\left\{1+3+5+...+\left(2n-1\right)\right\}=n^2$
3. Sum of the Squares of first *n* natural numbers $\left(1^2+2^2+3^2+4^2+...+n^2\right)=\dfrac{n\left(n+1\right)\left(2n+1\right)}{6}$
4. Sum of the Cubes of first *n* natural numbers $\left(1^3+2^3+3^3+4^3+...+n^3\right)=\left\{\dfrac{n\left(n+1\right)}{2}\right\}^2$
5. Sum of the series such as: 1 + 11 + 111 + … to *n* terms, or 2 + 22 + 222 + … to *n* terms, or 3 + 33 + 333 + … to *n* terms, and so on: $\dfrac{Number}{9}\times\left\{\dfrac{10\left(10^n-1\right)}{9}-n\right\}$. For example:

    a. 1 + 11 + 111 + … to *n* terms = $\dfrac{1}{9}\times\left\{\dfrac{10\left(10^n-1\right)}{9}-n\right\}$

    b. 2 + 22 + 222 + … to *n* terms = $\dfrac{2}{9}\times\left\{\dfrac{10\left(10^n-1\right)}{9}-n\right\}$

    c. 3 + 33 + 333 + … to *n* terms = $\dfrac{3}{9}\times\left\{\dfrac{10\left(10^n-1\right)}{9}-n\right\}$

    Questions to be Solved from Scanner

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    d. Page 3.307 – Question 8

6. Sum of the series 0.1 + 0.11 + 0.111 + … to *n* terms = $\dfrac{1}{9}\times\left[n-\left\{\dfrac{1-\left(0.1\right)^n}{9}\right\}\right]$.

    Example: Calculate the sum of 0.7 + 0.77 + 0.777 + … to *n* terms.
    Solution:
    0.7 + 0.77 + 0.777 + … to *n* terms = 7 × (0.1 + 0.11 + 0.111 + … to *n* terms)

    Therefore, 0.7 + 0.77 + 0.777 + … to *n* terms = $\dfrac{7}{9}\times\left[n-\left\{\dfrac{1-\left(0.1\right)^n}{9}\right\}\right]$

Similarly, sum of series 0.2 + 0.22 + 0.222 + … to n terms = $\frac{2}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$

Sum of series 0.4 + 0.44 + 0.444 + … to n terms = $\frac{4}{9}\times\left[n-\left\{\frac{1-(0.1)^n}{9}\right\}\right]$.

Questions to be Solved from Scanner

a. Page 3.314 – Question 16

7. Sum of an Arithmetico-Geometric Series to infinity = $\frac{ab}{1-r}+\frac{dbr}{(1-r)^2}$.

Where, $a = t_1$ of the A.P.; $b = t_1$ of the G.P.

For example, the series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+...\infty$ is an Arithmetico-Geometric series. The numerators 1, 4, 7, 10, … are in A.P., with a = 1, and d = 3. The denominators 1, 5, 5^2, 5^3,… are in G.P., with b = 1, and r = 5. However, since this G.P. is in the denominator, the common ratio of the entire series will be 1/5. Therefore, we have a = 1, b = 1, d = 3, and r = 1/5. Putting these values in the formula $S_\infty=\frac{ab}{1-r}+\frac{dbr}{(1-r)^2}$, we have:

$$S_\infty=\frac{1\times1}{1-(1/5)}+\frac{3\times1\times(1/5)}{\{1-(1/5)\}^2}=\frac{35}{16}.$$

8. The n^{th} element of the sequence –1, 2, –4, 8, … is $(-1)^n.2^{n-1}$