QUESTION 2 (Beta of portfolio & Total, Systematic & unsystematic risk)

A study by a Mutual fund has revealed the following data in respect of three securities:

Security	σ (%)	Correlation with Index, Pm
A	20	0.60
B	18	0.95
C	12	0.75

The standard deviation of market portfolio (BSE Sensex) is observed to be 15%.

(i) What is the sensitivity of returns of each stock with respect to the market?

(ii) What are the covariances among the various stocks?

(iii) What would be the risk of portfolio consisting of all the three stocks equally?

(iv) What is the beta of the portfolio consisting of equal investment in each stock?

(v) What is the total, systematic and unsystematic risk of the portfolio in (iv)?

SOLUTION

(i) Sensitivity of each stock with market is given by its beta.

Standard deviation of market Index = 15%

Variance of market Index = 0.0225

Beta of stocks = $\sigma_i r / \sigma_m$

A = 20 × 0.60 / 15 = 0.80

B = 18 × 0.95 / 15 = 1.14

C = 12 × 0.75 / 15 = 0.60

(ii) Covariance between any 2 stocks = $\beta_1 \beta_2 \sigma^2_m$

Covariance matrix

Stock/Beta	0.80	1.14	0.60
A	400.000	205.200	108.000
B	205.200	324.000	153.900
C	108.000	153.900	144.000

(iii) Total risk of the equally weighted portfolio (Variance) = $400(1/3)^2 + 324(1/3)^2 + 144(1/3)^2 + 2(205.20)(1/3)^2 + 2(108.0)(1/3)^2 + 2(153.900)(1/3)^2 = 200.244$

(iv) β of equally weighated portfolio = $\beta_p = \dfrac{0.80 + 1.14 + 0.60}{3} = 0.8467$

(v) Systematic Risk $\beta_p^2 \sigma_m^2 = (0.8467)^2 (15)^2 = 161.302$

Unsystematic Risk = Total Risk – Systematic Risk = 200.244 – 161.302 = 38.942

QUESTION 3 (Portfolio return and Porfolio Beta)

Mr. Tempest has the following portfolio of four shares:

Name	Beta	Investment (Rs. Lac.)
Oxy Rin Ltd.	0.45	0.80
Boxed Ltd.	0.35	1.50
Square Ltd.	1.15	2.25
Ellipse Ltd.	1.85	4.50

The risk-free rate of return is 7% and the market rate of return is 14%. Required.
(I) Determine the portfolio return. (ii) Calculate the portfolio Beta.

SOLUTION

Market Risk Premium (A) = 14% -7% = 7%

Share	Beta	Risk Premium(Beta x A) %	Risk Free Return %	Return %	Return Rs.
Oxy Rin Ltd.	0.45	3.15	7	10.15	8,120
Boxed Ltd.	0.35	2.45	7	9.45	14,175
Square Ltd.	1.15	8.05	7	15.05	33,863
Ellipse Ltd.	1.85	12.95	7	19.95	89,775
Total Return					**1,45,933**

Total Investment Rs. 9,05,000

$$\text{(i) Portfolio Return} = \frac{\text{Rs. } 1,45,933}{\text{Rs. } 9,05,000} \times 100 = 16.13\%$$

(ii)Portfolio Beta : Portfolio Return = Risk Free Rate + Risk Premium X β = 16.13 %

7% + 7 β = 16.13 % β = 1.30

Alternative Approach

First we shall compute Portfolio Beta using the weighted average method as follows:

$$\text{Beta}_P = 0.45 \times \frac{0.80}{9.05} + 0.35 \times \frac{1.50}{9.05} + 1.15 \times \frac{2.25}{9.05} + 1.85 \times \frac{4.50}{9.05}$$

= 0.45 X 0.0884 + 0.35 X 0.1657 + 1.15 X 0.2486 + 1.85 X 0.4972
= 0.0398 + 0.058 + 0.2859 + 0.9198 = 1.3035

Accordingly

(i) Portfolio Return using CAPM formula will be as follows: $R_P = R_F + \text{Beta}_P (R_M - R_F)$
= 7% + 1.3035 (14%-7%) = 7% + 1.3035(7%)
7% + 9.1245% = 16.1245%

(ii) Portfolio Beta: As calculated above 1.3035

QUESTION 4 (Sharpe, Treynor & Alpha)

Mr. Abhishek is interested in investing Rs.2,00,000 for which he is considering following three alternatives:

(i) Invest Rs.2,00,000 in Mutual Fund X (MFX)

(ii) Invest Rs.2,00,000 in Mutual Fund Y (MFY)

(iii) Invest Rs.1,20,000 in Mutual Fund X (MFX) and Rs.80,000 in Mutual Fund Y (MFY)

Average annual return earned by MFX and MFY is 15% and 14% respectively. Risk free rate of return is 10% and market rate of return is 12%.

Covariance of returns of MFX, MFY and market portfolio Mix are as follow:

	MFX	MFY	Mix
MFX	4.800	4.300	3.370
MFY	4.300	4.250	2.800
Mix	3.370	2.800	3.100

You are required to calculate:

(i) variance of return from MFX, MFY and market return,

(ii) portfolio return, beta, portfolio variance and portfolio standard deviation,

(iii) expected return, systematic risk and unsystematic risk; and

(iv) Sharpe ratio, Treynor ratio and Alpha of MFX, MFY and Portfolio Mix.

SOLUTION

(i) Variance of Returns $\text{Cor}_{i,j} = \dfrac{\text{Cov}(i, j)}{\sigma_i \ \sigma_j}$

Accordingly, for MFX $1 = \dfrac{\text{Cov}(x, x)}{\sigma_x \ \sigma_x}$ $\sigma^2_X = 4.800$

Accordingly, for MFY $1 = \dfrac{\text{Cov}(Y, Y)}{\sigma_Y \ \sigma_Y}$ $\sigma^2_Y = 4.250$

Accordingly, for Market Return $1 = \dfrac{\text{Cov}(M, M)}{\sigma_M \ \sigma_M}$ $\sigma^2_M = 4.250$

Alternatively, by referring diagonally the given Table these values can identified as follows:

Variance $_X$ = 4.800 **Variance** $_Y$ = 4.250 **Variance** $_X$ = 3.100

(ii) Portfolio return, beta, variance and standard deviation

Weight of MFX in portfolio = $\frac{1,20,000}{2,00,000}$ = 0.60	Weight of MFY in portfolio = $\frac{80,000}{2,00,000}$ = 0.40

Accordingly Portfolio Return
0.60 X 15% + 0.40 X 14% = 14.60%

Beta of each fund

$\beta = \frac{\text{Cov (Fund, Market)}}{\text{Variance of Market}}$	$\beta_X = \frac{3.370}{3.100} = 1.087$	$\beta_Y = \frac{2.800}{3.100} = 0.903$

Portfolio Beta: 0.60 X 1.087 + 0.40 X 0.903 = 1.013

Portfolio Variance: $\sigma^2_{XY} = W^2_X\ \sigma^2_X + W^2_Y\ \sigma^2_Y + 2W_X\ W_Y\ Cov_{X,Y}$

$= (0.60)^2\ (4.800) + (0.40)^2\ (0.4250) + 2\ (0.60)\ (0.40)\ (4.300)$
$= 4.472$

Or Portfolio Standard Deviation : $\sigma_{XY} = \sqrt{4.472} = 2.115$

(iii) Expected Return, Systematic and Unsystematic Risk of Portfolio
Portfolio Return = 10% + 1.0134 (12% - 10%) = 12.03%
MF X Return = 10% + 1.087(12% - 10%) = 12.17%
MF Y Return = 10% + 0.903 (12% - 10%) = 11.81%

Systematic Risk = $\beta^2\ \sigma^2$
Accordingly,
Systematic Risk of MFX= $(1.087)^2$x 3.10 = 3.663
Systematic Risk of MFY= $(0.903)^2$x 3.10 = 2.528
Systematic Risk of Portfolio=$(1.013)^2$x 3.10 = 3.181

Unsystematic Risk = Total Risk- Systematic Risk
Accordingly,
Unsystematic Risk of MFX=4.80 - 3.663 = 1.137
Unsystematic Risk of MFY=4.250 - 2.528 = 1.722
Unsystematic Risk of Portfolio = 4.472 - 3.181 = 1.291

(iv) Sharpe and Treynor Ratios and Alpha

Sharpe Ratio

$MFX = \frac{15\% - 10\%}{\sqrt{4.800}} = 2.282$	$MFY = \frac{14\% - 10\%}{\sqrt{4.250}} = 1.94$	$Portfolio = \frac{14.6\% - 10\%}{2.115} = 2.175$

Treynor Ratio

$MFX = \frac{15\% - 10\%}{1.087} = 4.60$	$MFY = \frac{14\% - 10\%}{0.903} = 4.43$	$Portfolio = \frac{14.6\% - 10\%}{1.0134} = 4.54$

Alpha

MFX = 15% - 12.17% = 2.83%	MFY = 14% - 11.81% = 2.19%	Portfolio = 14.6 % - 12.03% = 2.57%

QUESTION 5 (CAPM and Risk free rate of return)

Your client is holding the following securities:

Particulars of Securities	Cost (Rs.)	Dividends/Interest	Market price	Beta
Equity Shares:				
Gold Ltd.	10,000	1,725	9,800	0.6
Silver Ltd.	15,000	1,000	16,200	0.8
Bronze Ltd.	14,000	700	20,000	0.6
GOI Bonds	36,000	3,600	34,500	0.01

Average return of the portfolio is 15.7%, calculate:

(i) Expected rate of return in each, using the Capital Asset Pricing Model (CAPM).

(ii) Risk free rate of return.

SOLUTION

Particulars of Securities	Cost (Rs.)	Dividend	Capital gain
Gold Ltd.	10,000	1,725	-200
Silver Ltd.	15,000	1,000	1,200
Bronze Ltd.	14,000	700	6,000
GOI bonds	36,000	3,600	-1,500
Total	**75,000**	**7,025**	**5,500**

Expected rate of return on market portfolio

$$\frac{\text{Dividend Earned + Capital appreciation}}{\text{Intial investment}} \times 100 = \frac{\text{Rs. } 7{,}025 + \text{Rs. } 5{,}500}{\text{Rs. } 75{,}000} \times 100 = 16.7\%$$

Risk free return

$$\text{Average of Betas} = \frac{0.6 + 0.8 + 0.6 + 0.01}{4} = 0.50$$

Average return= Risk free return + Average Betas (Expected return - Risk free return)
15.7 = Risk free return + 0.50 (16.7- Risk free return)
Risk free return = 14.7%

*Alternatively, it can also be calculated through Weighted Average Beta.
Expected Rate of Return for each security is   Rate of Return $= R_f + B\ (R_m - R_f)$

Gold Ltd = 14.7 + 0.6 (16.7-14.7) = 15.90%	Silver Ltd. = 14.7 + 0.8 (16.7-14.7) = 16.30%
Bronze Ltd. = 14.7 + 0.6 (16.7-14.7) = 15.90%	GOI bonds = 14.7 + 0.01 (16.7-14.7) = 14.72%

* Alternatively, it can also be computed by using Weighted Average Method.

QUESTION 6 (Expected average return and standard deviation)

X Co., Ltd., invested on 1.4.2009 in certain equity shares as below:

Name of Co.	No. of shares	Cost (Rs.)
M Ltd.	1,000 (Rs.100 each)	2,00,000
N Ltd.	500 (Rs.10 each)	1,50,000

In September, 2009, 10% dividend was paid out by M Ltd. and in October, 2009, 30% dividend paid out by N Ltd. On 31.3.2010 market quotations showed a value of Rs. 220 and Rs. 290 per share for M Ltd. and N Ltd. respectively.

On 1.4.2010, investment advisors indicate (a) that the dividends from M Ltd. and N Ltd. for the year ending 31.3.2011 are likely to be 20% and 35%, respectively and (b) that the probabilities of market quotations on 31.3.2011 are as below:

Probability factor	Price/share of M Ltd.	Price/share of N Ltd.
0.2	220	290
0.5	250	310
0.3	280	330

You are required to:

(i) Calculate the average return from the portfolio for the year ended 31.3.2010;

(ii) Calculate the expected average return from the portfolio for the year 2010-11; and

(iii) Advise X Co. Ltd., of the comparative risk in the two investments by calculating the standard deviation in each case.

SOLUTION

Working:

Calculation of return on portfolio for 2009- 10	M	N	
Dividend received during the year	10	3	0.2
Capital gain/ loss by 31.03.10			0.5
Market value by 31.03.10	220	290	0.3
Cost of investment	200	300	
Gain/ loss	20	(-)10	
Yield	30	(-)7	
Cost	200	300	
% return	15%	(-)2.33%	
Weight in the portfolio	57	43	
Weighted average return			
Calculation of estimated return for 2010- 11 Expected dividend			
Capital gain by 31.03.11	20	3.5	
(220X0.2) + (250X0.5) + (280X0.3) - 220 = (253-220)			
(290X0.2)+ (310X0.5) + (330X0.3) - 290 = (312-290)	33	-	
Yield	-		
*Market value 01.04.10	53	25.5	
% return	220	290	
*Weight in portfolio(1,000X220): (500X290)	24.09%	8.79%	
Weighted average (Expected) return	60.3	39.7	
(*The market value on 31.03.10 is used as the base for calculating yield for 10-11			

(i) Average Return from Portfolio for the year ended 31.03.2010 is 7.55%.

(ii) Expected Average Return from portfolio for the year 2010- 11 is 18.02%

(iii) **Calculation of Standard Deviation**

M Ltd.

Exp.market value	Exp. gain	Exp div.	Exp. Yield (1)	Prob. Factor (2)	(1) X(2)	Dev. $P_M - \overline{P}_M$	Square Of dev. (3)	(2) X (3)
220	0	20	20	0.2	4	-33	1089	217.80
250	30	20	50	0.5	25	-3	9	4.50
280	60	20	80	0.3	24	27	729	218.70
					53			σ^2_m = 441.00

Standard Deviation (σ_M) 21

N Ltd.

Exp. market value	Exp. gain	Exp div.	Exp. Yield (1)	Prob. Factor (2)	(1) X(2)	Dev. $P_N - \overline{P_N}$	Square Of dev. (3)	(2) X (3)
290	0	3.5	3.5	0.2	0.7	-22	484	96.80
310	20	3.5	23.5	0.5	11.75	-2	4	2.00
330	40	3.5	43.5	0.3	13.05	18	324	97.20
					25.5			σ^2_N = 196.00

Standard Deviation (σ_N) 14

Alternatively based on return in percentage terms Standard Deviation can also be computed as follows:

M Ltd.

Exp. market value	Exp. gain	Exp div.	Exp. return (1)	Prob. Factor (2)	(1) X(2)	Dev. $P_M - \overline{P_M}$	Square Of dev. (3)	(2) X (3)
220	0	20	9.09	0.2	1.82	-15.01	225.30	45.06
250	30	20	22.73	0.5	11.37	-1.37	1.88	0.94
280	60	20	36.36	0.3	10.91	12.26	150.31	45.09
					24.10			σ^2_m = 91.09

Standard Deviation (σ_M) 9.54%

N Ltd.

Exp. market value	Exp. gain	Exp div.	Exp. return (1)	Prob. Factor (2)	(1) X(2)	Dev. $P_N - \overline{P_N}$	Square Of dev. (3)	(2) X (3)
290	0	3.5	1.21	0.2	0.24	-7.58	57.46	11.49
310	20	3.5	8.10	0.5	4.05	-0.69	0.48	0.24
330	40	3.5	15.00	0.3	4.50	6.21	38.56	11.57
					8.79			σ^2_N = 23.30

Standard Deviation (σ_N) 4.83%

Share of company M Ltd. is more risky as the S. D. is more than company N Ltd.

QUESTION 7 (Portfolio variance using Sharpe Index Model and Markowitz)

Following are the details of a portfolio consisting of three shares:

Share	Portfolio weight	Beta	Expected return in %	Total variance
A	0.20	0.40	14	0.015
B	0.50	0.50	15	0.025
C	0.30	1.10	21	0.100

Standard Deviation of Market Portfolio Returns = 10%.

You are given the following additional data:

Covariance (A, B) = 0.030 Covariance (A, C) = 0.020 Covariance (B, C) = 0.040 Calculate the following:

(i) The Portfolio Beta

(ii) Residual variance of each of the three shares

(iii) Portfolio variance using Sharpe Index Model

(iv) Portfolio variance (on the basis of modern portfolio theory given by Markowitz)

SOLUTION

(i) Portfolio Beta

0.20 x 0.40 + 0.50 x 0.50 + 0.30 x 1.10 = 0.66

(ii) Residual Variance

To determine Residual Variance first of all we shall compute the Systematic Risk as follows:

$\beta_A^2 = \sigma_M^2 = (0.40)^2\ (0.01) = 0.0016$	$\beta_B^2 = \sigma_M^2 = (0.50)^2\ (0.01) = 0.0025$	$\beta_C^2 = \sigma_M^2 = (1.10)^2\ (0.01) = 0.0121$

Residual Variance

A 0.015 – 0.0016 = 0.0134	B 0.025 – 0.0025 = 0.0225	C 0.100 – 0.0121 = 0.0879

(iii) Portfolio variance using Sharpe Index Model

Systematic Variance of Portfolio = $(0.10)^2 \times (0.66)^2$ = 0.004356

Unsystematic Variance of Portfolio = $0.0134 \times (0.20)^2 + 0.0225 \times (0.50)^2 + 0.0879 \times (0.30)^2$
= 0.014072

Total Variance = 0.004356 + 0.014072 = 0.018428

(iv) Portfolio variance on the basis of Markowitz Theory

$$= (W_A \times W_A \times \sigma_A^2) + (W_A \times W_B \times Cov_{AB}) + (W_A \times W_C \times Cov_{AC}) + (W_B \times W_A \times Cov_{AB})$$

$$+ (W_B \times W_B \times W_B \times s_B^2) + (W_B \times W_C \times Cov_{BC}) + (W_C \times W_A \times Cov_{CA})$$

$$+ (W_C \times W_B \times Cov_{CB}) + (W_C \times W_C \times s_C^2)$$

= (0.20 x 0.20 x 0.015) + (0.20 x 0.50 x 0.030) + (0.20 x 0.30 x 0.020) + (0.20 x 0.50 x 0.030) + (0.50 x 0.50 x 0.025) + (0.50 x 0.30 x 0.040) + (0.30 x 0.20 x 0.020) + (0.30 x 0.50 x 0.040) + (0.30 x 0.30 x 0.10)

= 0.0006 + 0.0030 + 0.0012 + 0.0030 + 0.00625 + 0.0060 + 0.0012 + 0.0060 + 0.0090
= 0.0363

QUESTION 8 (APT, CAPM & determining compostion of portfolio)

Mr. Nirmal Kumar has categorized all the available stock in the market into the following types:

(i) Small cap growth stocks
(ii) Small cap value stocks
(iii) Large cap growth stocks
(iv) Large cap value stocks

Mr. Nirmal Kumar also estimated the weights of the above categories of stocks in the market index. Further, the sensitivity of returns on these categories of stocks to the three important factor are estimated to be:

Category of Stocks	Weight in the Market Index	Factor I (Beta)	Factor II (Book Price)	Factor III (Inflation)
Small cap growth	25%	0.80	1.39	1.35
Small cap value	10%	0.90	0.75	1.25
Large cap growth	50%	1.165	2.75	8.65
Large cap value	15%	0.85	2.05	6.75
Risk Premium		6.85%	-3.5%	0.65%

The rate of return on treasury bonds is 4.5% . Required:

(a) Using Arbitrage Pricing Theory, determine the expected return on the market index.

(b) Using Capital Asset Pricing Model (CAPM), determine the expected return on the market index.

(c) Mr. Nirmal Kumar wants to construct a portfolio constituting only the 'small cap value' and 'large cap growth' stocks. If the target beta for the desired portfolio is 1, determine the composition of his portfolio.

SOLUTION

(a) Method I

Stock's return

Small cap growth = 4.5 + 0.80 x 6.85 + 1.39 x (-3.5) + 1.35 x 0.65 = 5.9925%

Small cap value = 4.5 + 0.90 x 6.85 + 0.75 x (-3.5) + 1.25 x 0.65 = 8.8525%

Large cap growth = 4.5 + 1.165 x 6.85 + 2.75 x (-3.5) + 8.65 x 0.65 = 8.478%

Large cap value = 4.5 + 0.85 x 6.85 + 2.05 x (-3.5) + 6.75 x 0.65 = 7.535%

Expected return on market index

0.25 x 5.9925 + 0.10 x 8.8525 + 0.50 x 8.478 + 0.15 x 7.535 = 7.7526%

Method II

Expected return on the market index

= 4.5% + [0.1x0.9 + 0.25x0.8 + 0.15x0.85 + 0.50x1.165] x 6.85 + [(0.75 x 0.10 +1.39 x 0.25 + 2.05 x 0.15 + 2.75 x 0.5)] x (-3.5) + [{1.25 x 0.10 + 1.35 x 0.25 + 6.75 x 0.15 + 8.65 x 0.50)] x 0.65

= 4.5 + 6.85 + (-7.3675) + 3.77 = 7.7525%.

(b)

Small cap growth	4.5 + 6.85 x 0.80 = 9.98%	Large cap growth	4.5 + 6.85 x 1.165 = 12.48%
Small cap value	4.5 + 6.85 x 0.90 = 10.665%	Large cap value	4.5 + 6.85 x 0.8 = 10.3225%

Expected return on market index

= 0.25 x 9.98 + 0.10 x 10.665 + 0.50 x 12.45 + 0.15 x 10.3225 = 11.33%

(c) Let us assume that Mr. Nirmal will invest X_1% in small cap value stock and X_2% in large cap growth stock

$X_1 + X_2 = 1$

$0.90\ X_1 + 1.165\ X_2 = 1$

$0.90\ X_1 + 1.165(1 - X_1) = 1$

$0.90\ X_1 + 1.165 - 1.165\ X_1 = 1$

$0.165 = 0.265\ X_1$

$$\frac{0.165}{0.265} = x_1$$

$0.623 = X_1$, $X_2 = 0.377$

62.3% in small cap value

37.7% in large cap growth.

QUESTION 9 (Characteristic line & systematic unsystematic risk of security)

The returns on stock A and market portfolio for a period of 6 years are as follows:

Year	Return on A (%)	Return on market portfolio (%)
1	12	8
2	15	12
3	11	11
4	2	-4
5	10	9.5
6	-12	-2

You are required to determine:

(I) Characteristic line for stock A

(ii) The systematic and unsystematic risk of stock A.

SOLUTION

Characteristic line is given by

$\alpha + \beta\ Rm$

$$\beta i = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(\bar{x})^2}$$

$$\alpha i = \bar{y} - \beta\bar{x}$$

Return on A (Y)	Return on market (X)	xy	x2	(x- x)	(x- x)2	(y- y)	(y- y)2
12	8	96	64	2.25	5.06	5.67	32.15
15	12	180	144	6.25	39.06	8.67	75.17
11	11	121	121	5.25	27.56	4.67	21.81
2	-4	-8	16	-9.75	95.06	-4.33	18.75
10	9.5	95	90.25	3.75	14.06	3.67	13.47
-12	-2	24	4	-7.75	60.06	-18.33	335.99
38	34.5	508	439.25		240.86		497.34

$\bar{y}$ = 38/ 6 = 6.33

$\bar{x}$ = 34.5/6 = 5.75

$$\beta = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(\bar{x})^2} = \frac{508\text{-}6\ (5.75)\ (6.33)}{439.25 - 6\ (5.75)^2} = \frac{508 - 218.385}{439.25 - 198.375} = \frac{289.615}{240.875} = 1.202$$

$\alpha = \bar{y} - \beta\bar{x} = 6.33 - 1.202\ (5.75) = -0.5$

Hence the characteristic line is -0.58 + 1.202 (R_m)

Total RIsk of Market =

$$\sigma_{m^2} = \frac{\sum (x - \bar{x})^2}{n} = \frac{240.86}{6} = 40.14(\%)$$

Systematic Risk = $\beta i^2\ \sigma_2 = (1.202)^2 \times 40.14 = 57.99(\%)$

$$\text{Total Risk of Stock} = \frac{497.34}{6} = 82.89(\%)$$

Unsystematic Risk is

= Total Risk – Systematic Risk

= 82.89 – 57.99 = 24.90(%)