Chapter 5 – Basic Concepts of Permutations and Combinations

Unit 1 – Permutations

Introduction

Permutations are used to determine the number of ways in which certain items can be arranged.

Factorial

The Factorial of a number is the product of all the numbers from 1 till that number. For example, the factorial of 5 = 1 × 2 × 3 × 4 × 5 = 120. The factorial of 5 is denoted either as 5!, or $\underline{|5}$.

Note: 0! = 1.

Rules of Counting

OR

This is the first rule of counting. "OR" always means "+".

AND

This is the second rule of counting. "AND" always means "×".

Permutations

The word Permutation means "each of several possible ways in which a number of things can be ordered or arranged".

When r items need to be arranged from a set of n items, it is written as ${}^nP_r$, and is calculated using the formula: $\frac{n!}{(n-r)!}$.

Therefore, ${}^nP_r = \frac{n!}{(n-r)!}$.

Note: ${}^nP_n = n!$

Questions to be solved from Scanner

1. Question 23
2. Question 54
3. Question 34

4. Question 35
5. Question 39 – Homework
6. Question 2
7. Question 42
8. Question 67
9. Question 68 – Homework
10. Question 50
11. Question 55
12. Question 59 – Homework
13. Question 73 – Homework

Permutations of items not all distinct

Questions to be solved from Scanner

1. Question 18
2. Question 29 – Homework
3. Question 71 – Homework
4. Question 16

Permutations with Restrictions

Note: The number of ways in which *n* items can be arranged so that two particular items are not together is $(n-2).(n-1)!$. – Question 7

Questions to be solved from Scanner

1. Question 28
2. Question 41
3. Question 38
4. Question 33 – Homework
5. Question 27 – Homework
6. Question 14 – Homework
7. Question 8 – Homework
8. Question 43 – Homework
9. Question 44
10. Question 47 – Homework
11. Question 56 – Homework
12. Question 57 – Homework
13. Question 64 – Homework
14. Question 77 – Homework
15. Question 84 – Homework
16. Question 85 – Homework

Circular Permutations

No. of ways in which *n* items can be arranged in a circular fashion is $(n-1)!$.

Note:

- The number of ways of arranging n persons along a round table so that no person has the same two neighbours is $\frac{1}{2}(n-1)!$.
- The number of necklaces formed with n beads of different colours is $\frac{1}{2}(n-1)!$.

Questions to be solved from Scanner

1. Question 10
2. Question 21

Permutations when Repetition is Allowed

No. of arrangements of n items when repetitions are allowed = n^n.

Question

How many 5 digit numbers can be formed from the numbers 1, 2, 3, 4, and 5, given that any digit can be used any number of times?

Unit 2 – Combinations

Combinations

The word "Combination" is used for selecting things, and not arranging things. The number of ways in which r objects can be selected from a set of n items is denoted by $^{n}C_{r}$, and is given by the formula: $\frac{n!}{r!(n-r)!}$.

Therefore, $^{n}C_{r} = \frac{n!}{r!(n-r)!}$.

Questions to be solved from Scanner

1. Question 86
2. Question 87
3. Question 17 – Homework
4. Question 20 – Homework
5. Question 25
6. Question 1
7. Question 74 – Homework
8. Question 3
9. Question 4
10. Question 5
11. Question 9
12. Question 24 – Homework
13. Question 31 – Homework
14. Question 32 – Homework
15. Question 37 – Homework
16. Question 48
17. Question 58
18. Question 62 – Homework
19. Question 63 – Homework
20. Question 70
21. Question 81 – Homework

Combinations of items not all distinct

Questions to be solved from Scanner

1. Question 13
2. Question 76

Some Standard Results

1. Number of ways of selecting some or all items from a set of *n* items –
   a. When there are 2 choices for each item: $(2^n - 1)$.
      i. Question 40
      ii. Question 60
   b. When there are 3 choices for each item: $(3^n - 1)$.
2. $^nC_r = {}^nC_{n-r}$
3. $^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$
   a. Question 36
   b. Question 15
   c. Question 75 – Homework – Same as Question 15
   d. Question 52 – Homework – Same as Question 15
   e. Question 45
   f. Question 69 – Homework – Similar to Question 45
4. $\frac{^nC_r}{^nC_{r+1}} = \frac{r+1}{n-r}$; $\frac{^nC_{r-1}}{^nC_r} = \frac{r}{n-r+1}$.
   a. Question 19 – Homework
5. If $^nC_x = {}^nC_y$, and $x \neq y$, then $x + y = n$
   a. Question 49
   b. Question 26
   c. Question 82 – Homework
6. If $^nP_x = {}^nP_y$, and $x \neq y$, then $x + y = 2n - 1$
   a. Question 22 – Homework
7. The number of diagonals in a polygon of *n* sides is $\frac{1}{2}n(n-3)$.
   a. Question 46 – Homework
8. Division of Items in Groups –
   a. Division of Distinct Items in Groups –
      i. Equal items in every group – The number of ways to divide *n* students into *k* groups of *h* students each is given by $\frac{n!}{k!(h!)^k}$

      Question
      In how many number of ways can 12 students be equally divided into three groups?
      Solution
      The number of ways to divide *n* students into *k* groups of *h* students each is given by $\frac{n!}{k!(h!)^k}$.
      We have, $n = 12$; $k = 3$; $h = 4$.

Therefore, $\frac{n!}{k!(h!)^k} = \frac{12!}{3!(4!)^3} = 5,775$.

ii. Unequal items in every group – The number of ways to divide *n* items into 3 groups → one containing *a* items, the second containing *b* items, and the third containing *c* items, such that $a+b+c=n$, is given by $\frac{n!}{a!b!c!}$.

Question

The number of ways in which 9 things can be divided into twice groups containing 2, 3, and 4 things respectively is _________.

Solution

The number of ways to divide *n* items into 3 groups → one containing *a* items, the second containing *b* items, and the third containing *c* items, such that $a+b+c=n$, is given by $\frac{n!}{a!b!c!}$.

Here, $n = 9$; $a = 2$; $b = 3$; $c = 4$

$\frac{n!}{a!b!c!} = \frac{9!}{2!\times 3!\times 4!} = 1,260$

b. Division of Identical Items in Groups – The number of ways to divide *n* identical objects into *k* groups of *h* items each is given by $\frac{n!}{(h!)^k}$

Question

In how many number of ways can 15 mangoes be equally divided among 3 students?

Solution

The number of ways to divide *n* identical objects into *k* groups of *h* items each is given by $\frac{n!}{(h!)^k}$.

We have, $n = 15$; $k = 3$; $h = 5$.

Therefore, $\frac{n!}{(h!)^k} = \frac{15!}{(5!)^3} = 7,56,756$.

9. The maximum number of points of intersection of *n* circles will be $n(n-1)$
    a. Question 65 – Homework
10. ${}^{n}P_{r} = {}^{n-1}P_{r} + r.{}^{n-1}P_{r-1}$
11. $\frac{{}^{n}C_{r}}{{}^{n}P_{r}} = \frac{1}{r!}$
    a. Question 79
    b. Question 61

Miscellaneous Questions

1. Question 11
2. Question 12
3. Question 51
4. Question 53
5. Question 66
6. Question 72
7. Question 78
8. Question 80
9. Question 88
10. Question 30
11. Question 6
12. Question 83