

Chapter 19 – Index Numbers and Time Series

Unit 1 – Index Numbers

Introduction

An index number is a ratio of two or more time periods, one of which is the base time period. The value at the base time period serves as the standard point of comparison.

The base time period is that time period from which the comparisons are to be made. For example, in 2009 the price of a McAloo Tikki burger was ₹20; in 2020, it's ₹40. Now, if I need to compare the price of 2020 with the price of 2009, 2009 will be the base time period, and 2020 will be current time period. The price in the base time period is denoted as P_0 . The price in the current time period is denoted as P_1 . The ratio of the price of the current period (2020, i.e., P_1) to the price of the base period (or reference period, i.e., 2009, i.e., P_0), is known as the Price Relative, and is denoted as

$$P_{01}. \text{ Therefore, } P_{01} = \frac{P_1}{P_0}.$$

Therefore, Price Relative = $\frac{P_n}{P_0}$. It is expressed as a percentage as follows: Price Relative

$$= \frac{P_n}{P_0} \times 100.$$

Questions to be solved from Scanner

1. Page 3.970 – Question 10
2. Page 3.986 – Question 40
3. Page 3.989 – Question 46
4. Page 3.984 – Question 37 – Homework
5. Page 3.993 – Question 56 – Homework

Simple Aggregative Method

$$\text{Simple Aggregative Price Index} = \frac{\sum P_n}{\sum P_0} \times 100$$

Questions to be solved from Scanner

1. Question 38

Simple Average of Price Relatives

$$\text{Index} = \frac{\sum \left(\frac{P_1}{P_0} \times 100 \right)}{N}$$

Questions to be solved from Scanner

1. Page 3.990 – Question 49
2. Page 3.976 – Question 19

Weighted Average Method

In this method, we assign a weight to the prices of the commodities. Thereafter, the average is calculated as follows:

$$\text{General Index} = \frac{\text{Sum of Products}}{\text{Sum of Weights}}$$

Questions to be solved from Scanner

1. Question 8
2. Question 14 – Homework

The weights are usually the quantities of the commodities. These indices can be classified into two broad groups:

1. Weighted Aggregative Index
2. Weighted Average of Relatives

Weighted Aggregative Index

In this method, weights are assigned to the prices of the commodities. The weights are usually either the quantities or the value of goods, sold either during the base year, or the given year, or an average of some years. Various alternative formulae used are as follows:

1. Laspeyres' Index: In this Index, base year quantities are used as weights:

$$\text{Laspeyres Index} = \frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$$

Questions to be solved from Scanner:

- a. Question 93
- b. Question 2 – Homework
- c. Question 62
- d. Question 80 – Homework
- e. Question 88 – Homework
2. Paasche's Index: In this Index current year quantities are used as weights:

$$\text{Paasche's Index} = \frac{\sum P_n Q_n}{\sum P_0 Q_n} \times 100$$

Questions to be solved from Scanner:

- a. Question 65

- b. Question 22 – Homework
 - c. Question 32 – Homework
 - d. Question 42 – Homework
3. Methods based on some typical Period:

$$\text{Index} = \frac{\sum P_n Q_t}{\sum P_0 Q_t} \times 100, \text{ where } t \text{ stands for some typical period of years, the quantities of}$$

which are used as weights.

The Marshall-Edgeworth index uses this method by taking the average of the base year and the current year.

$$\text{Marshall-Edgeworth Index} = \frac{\sum P_n (Q_0 + Q_n)}{\sum P_0 (Q_0 + Q_n)} \times 100$$

Questions to be solved from Scanner:

- a. Question 11
4. Bowley’s Price Index: This index is the arithmetic mean of Laspeyres’ and Paasche’s.

$$\text{Bowley's Index} = \frac{\text{Laspeyres' + Paasche's}}{2}$$

Questions to be solved from Scanner:

- a. Question 47
 - b. Question 30
5. Fisher’s ideal Price Index: This index is the geometric mean of Laspeyres’ and Paasche’s.

$$\text{Fisher's Index} = \sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{\sum P_n Q_n}{\sum P_0 Q_n}} \times 100$$

Questions to be solved from Scanner:

- a. Question 86
- b. Question 34 – Homework
- c. Question 43
- d. Question 55
- e. Question 41 – Homework
- f. Question 6
- g. Question 16

Weighted Average of Relatives

In this method, weighted arithmetic mean is used to calculate the index.

$$\text{Index} = \frac{\sum \left[\frac{P_n}{P_0} \times (P_0 Q_0) \right]}{\sum P_0 Q_0} \times 100$$

The Chain Index Numbers

Till now, we have been taking a fixed base; however, when conditions change rapidly, the fixed base does not suit the required needs. In such a case, changing base is more suitable. For example, the base for the year 1999 could be 1998; the base for the year 2000 could be 1999 (not 1998), the

base for the year 2001 could be 2000 (neither 1998, nor 1999), and so on. If it is desired to associate these relatives to a common base, the results are chained. Thus, under this method the relatives of each year are first related to the preceding year, called the link relatives, and then they are chained together by successive multiplication to form a chain index.

$$\text{Chain Index} = \frac{\text{Link Relative of the Current Year} \times \text{Chain Index of the Previous Year}}{100}$$

For example,

Year	Price	Link Relatives (Taking Previous Year as Base Year)	Chain Indices (Taking 1991 as Base Year)
1991	50	100.00	100.00
1992	60	$\frac{60}{50} \times 100 = 120.00$	$\frac{120}{100} \times 100 = 120.00$
1993	62	$\frac{62}{60} \times 100 = 103.33$	$\frac{103.33}{100} \times 120 = 124.00$
1994	65	$\frac{65}{62} \times 100 = 104.84$	$\frac{104.84}{100} \times 124 = 129.90$

Quantity Index Numbers

1. Simple Aggregate of Quantities

$$\text{Index} = \frac{\sum Q_n}{\sum Q_0} \times 100$$

2. Simple Average of Quantity Relatives:

$$\text{Index} = \frac{\sum Q_n}{n} \times 100$$

3. Weighted Aggregate Quantity Indices:

- a. With base year weight (Laspeyre's index)

$$\text{Index} = \frac{\sum Q_n P_0}{\sum Q_0 P_0} \times 100$$

- b. With current year weight (Paasche's index)

$$\text{Index} = \frac{\sum Q_n P_n}{\sum Q_0 P_n} \times 100$$

- c. Fisher's Ideal (Geometric mean of the above)

$$\text{Index} = \sqrt{\frac{\sum Q_n P_0}{\sum Q_0 P_0} \times \frac{\sum Q_n P_n}{\sum Q_0 P_n}} \times 100$$

4. Base-year weighted average of quantity relatives

$$\text{Index} = \frac{\sum \left[\frac{Q_n}{Q_0} \times (P_0 Q_0) \right]}{\sum P_0 Q_0} \times 100$$

Value Indices

Value = Price × Quantity

$$\text{Value Index} = \frac{\sum V_n}{\sum V_0} = \frac{\sum P_n Q_n}{\sum P_0 Q_0}$$

Limitations and Usefulness of Index Numbers

Limitations

1. As the indices are constructed mostly from deliberate samples, chances of errors creeping in cannot be always avoided.
2. Since index numbers are based on some selected items, they simply depict the broad trend and not the real picture.
3. Since many methods are employed for constructing index numbers, the result gives different values and this at times creates confusion.

Usefulness

1. Framing suitable policies in economics and business: They provide guidelines to make decisions in measuring intelligence quotients, research etc.
2. They reveal trends and tendencies in making important conclusions in cyclical forces, irregular forces, etc.
3. They are important in forecasting future economic activity. They are used in time series analysis to study long-term trend, seasonal variations and cyclical developments.
4. Index numbers are very useful in deflating i.e., they are used to adjust the original data for price changes and thus transform nominal wages into real wages.
5. Cost of living index numbers measure changes in the cost of living over a given period.

Deflating Time Series Using Index Numbers

$$\text{Deflated Value} = \frac{\text{Current Value}}{\text{Price Index of the Current Year}}$$

or

$$\text{Current Value} = \frac{\text{Base Price } (P_0)}{\text{Current Price } (P_n)}$$

Year	Wholesale Price Index	Gross National Product at Current Prices	Real Gross National Product
1970	113.1	7499	$\frac{7499}{113.1} \times 100 = 6630$
1971	116.3	7935	$\frac{7935}{116.3} \times 100 = 6823$
1972	121.2	8657	$\frac{8657}{121.2} \times 100 = 7143$

1973	127.7	9323	$\frac{9323}{127.7} \times 100 = 7301$
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Questions to be solved from Scanner

1. Page 3.995 – Question 60
2. Page 3.977 – Question 21
3. Page 3.994 – Question 58 – Homework
4. Page 3.1006 – Question 94
5. Page 3.965 – Question 3
6. Page 3.998 – Question 68 – Homework
7. Page 3.971 – Question 12 – Homework
8. Page 3.988 – Question 44 – Homework
9. Page 3.992 – Question 54 – Homework
10. Page 3.966 – Question 63 - Homework

Shifting and Splicing of Index Numbers**Shifting of Index Numbers**

$$\text{Shifted Price Index} = \frac{\text{Original Price Index}}{\text{Price Index of the Year on which it has to be shifted}} \times 100$$

Year	Original Price Index	Shifted Price Index to Base 1990
1988	125	$\frac{125}{140} \times 100 = 89.3$
1989	131	$\frac{131}{140} \times 100 = 93.6$
1990	140	$\frac{140}{140} \times 100 = 100.0$
1991	147	$\frac{147}{140} \times 100 = 105.0$

Splicing of Index Numbers

Splicing means combining two index covering different bases into a single series. Splicing two sets of price index numbers covering different periods of time is usually required when there is a major change in quantity weights. It may also be necessary on account of a new method of calculation or the inclusion of new commodity in the index.

Year	Old Price Index [1900 = 100]	Revised Price Index [1995 = 100]	Spliced Price Index [1995 = 100]
1990	100.0		$\frac{100}{114.2} \times 100 = 87.6$
1991	102.3		$\frac{102.3}{114.2} \times 100 = 89.6$
1992	105.3		$\frac{105.3}{114.2} \times 100 = 92.2$
1993	107.6		$\frac{107.6}{114.2} \times 100 = 94.2$

1994	111.9		$\frac{111.9}{114.2} \times 100 = 98.0$
1995	114.2	100.0	100.0
1996		102.5	102.5
1997		106.4	106.4
1998		108.3	108.3
1999		111.7	111.7
2000		117.8	117.8

Test of Adequacy

There are four tests:

1. Unit Test –

- This test requires that the formula should be independent of the unit in which (or, for which) prices and quantities are quoted.
- All the formulae satisfy this test, except for the simple (unweighted) aggregative index.

2. Time Reversal Test –

- It is a test to determine whether a given method will work both ways in time, forward and backward.
- The test provides that the formula for calculating the index number should be such that two ratios, the current on the base and the base on the current should multiply into unity.
- In other words, the two indices should be reciprocals of each other. Symbolically,

$$P_{01} \times P_{10} = 1, \text{ where, } P_{01} = \frac{P_1}{P_0}, \text{ and } P_{10} = \frac{P_0}{P_1}.$$

d. Check of Different Methods

i. Laspeyres' method

$$P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0}, P_{10} = \frac{\sum P_0 Q_0}{\sum P_1 Q_0}$$

$$P_{01} \times P_{10} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0} \neq 1$$

Therefore, Laspeyres' Method does not satisfy this test.

ii. Paasche's method

$$P_{01} = \frac{\sum P_1 Q_n}{\sum P_0 Q_n}, P_{10} = \frac{\sum P_0 Q_n}{\sum P_1 Q_n}$$

$$P_{01} \times P_{10} = \frac{\sum P_1 Q_n}{\sum P_0 Q_n} \times \frac{\sum P_0 Q_n}{\sum P_1 Q_n} \neq 1$$

Therefore, Paasche's Method does not satisfy this test.

iii. Fisher's Ideal

$$P_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}}, P_{10} = \sqrt{\frac{\sum P_0 Q_1}{\sum P_1 Q_1} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times \sqrt{\frac{\sum P_0 Q_1}{\sum P_1 Q_1} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0}} = 1$$

Therefore, Fisher's Idea does satisfy this test.

3. Factor Reversal Test –

- a. This states that the product of price index and the quantity index should be equal to the corresponding value index, i.e., $\frac{\sum P_1 Q_1}{\sum P_0 Q_0}$.

b. Symbolically, $P_{01} \times Q_{01} = V_{01}$.

c. Check for Fisher's Method

$$P_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}}, Q_{01} = \sqrt{\frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum Q_1 P_1}{\sum Q_0 P_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times \sqrt{\frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum Q_1 P_1}{\sum Q_0 P_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum Q_1 P_1}{\sum Q_0 P_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{(\sum P_1 Q_1)^2}{(\sum P_0 Q_0)^2}}$$

$$= \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

Therefore, Fisher's Method satisfies this test as well.

- d. While selecting an appropriate index formula, the Time Reversal Test and the Factor Reversal test are considered necessary in testing the consistency.
- e. Since Fisher's Index number satisfies both the tests (Time Reversal, as well as Factor Reversal), it is called an Ideal Index Number.

4. Circular Test –

- a. As per this test, $P_{01} \times P_{12} \times P_{20} = 1$.

For example,

Question

If the 1970 index with base 1965 is 200, and 1965 index with base 1960 is 150, what will be the index of 1970 on base 1960?

(May, 2018)

Solution

Let the year 1960 be P_0 , the year 1965 be P_1 , and the year 1970 be P_2 .

We need to find out the index of 1970 (P_2), on base 1960 (P_0). Therefore, we need to find P_{02} .

As per the question,

- i. the 1970 index with base 1965 is 200. This means that $P_{12} = 200$.

ii. the 1965 index with base 1960 is 150. This means that $P_{01} = 150$.

As per the circular test, we know that $P_{01} \times P_{12} \times P_{20} = 1$.

$$\text{Therefore, } 150 \times 200 \times P_{20} = 1 \Rightarrow P_{20} = \frac{1}{150 \times 200} \times 100 = \frac{1}{300}.$$

$$\text{Therefore, } P_{02} = \frac{1}{P_{20}} = 300.$$

- b. Therefore, this property enables us to adjust the index values from period to period without referring to the original base every time.
- c. The test of this shiftability of base is called the circular test.
- d. This test is not met by Laspeyres, or Paasche's or the Fisher's ideal index.
- e. The simple geometric mean of price relatives and the weighted aggregative with fixed weights meet this test.



Unit 2 – Time Series

Introduction

A Time Series is a set of observations taken at specified times, usually at equal intervals.

Components of Time Series

There are various forces that affect the values of a phenomenon in a time series; these may be broadly divided into the following four categories, commonly known as the components of a time series.

1. Long term movement or Secular Trend
2. Seasonal variations
3. Cyclical variations
4. Random or irregular variations

Long term movement or Secular Trend or Simple Trend

1. Secular trend is the long-term tendency of the time series to move in an upward or downward direction.
2. It indicates how it has behaved over the entire period under reference.
3. A general tendency of a variable to increase, decrease or remain constant in long term is called trend of a variable. However, in a small interval of time, the variable may increase or decrease.
4. Some examples are:
 - a. Population of a country has an increasing trend over the years.
 - b. Due to modern technology, agricultural and industrial production is increasing.
 - c. Due to modern technology and health facilities, death rate is decreasing and life expectancy is increasing.
5. These are results of long-term forces that gradually operate on the time series variable.
6. A few examples of these long term forces (which make a time series to move in any direction over long period of the time) are:
 - a. long term changes per capita income,
 - b. technological improvements of growth of population,
 - c. changes in Social norms etc.
7. Most of the time series relating to Economic, Business and Commerce might show
 - a. an upward tendency in case of
 - i. population,
 - ii. production & sales of products,
 - iii. incomes,
 - iv. prices; or
 - b. a downward tendency in case of
 - i. share prices,

- ii. death,
 - iii. birth rate etc.
- due to global meltdown, or improvement in medical facilities etc.

Seasonal Variations

1. Over a span of one year, seasonal variation takes place due to the rhythmic forces which operate in a regular and periodic manner.
2. These forces have the same or almost similar pattern year after year.
3. It is common knowledge that the value of many variables depends in part on the time of year.
4. For Example, Seasonal variations could be seen and calculated if the data are recorded quarterly, monthly, weekly, daily or hourly basis.
5. So, if in a time series data, only annual figures are given, there will be no seasonal variations.
6. The seasonal variations may be due to various seasons or weather conditions; for example, sale of cold drink would go up in summers & go down in winters.
7. These variations may also be due to
 - a. man-made conventions,
 - b. habits,
 - c. customs, or
 - d. traditions.

For example, sales might go up during Diwali & Christmas or sales of restaurants & eateries might go down during Navratri's.
8. The methods of seasonal variations are:
 - a. Simple Average Method
 - b. Ratio to Trend Method
 - c. Ratio to Moving Average Method
 - d. Link Relatives Method

Cyclical variations

1. Cyclical variations are the periodic movements. These are also generally termed as business cycles.
2. These variations in a time series are due to ups & downs recurring after a period from Season to Season.
3. Though they are more or less regular, they may not be uniformly periodic.
4. These are oscillatory movements which are present in any business activity, and is termed as business cycle.
5. It has got four phases consisting of
 - a. prosperity (boom),
 - b. recession,
 - c. depression, and
 - d. recovery.
6. All these phases together may last from 7 to 9 years may be less or more.

Random or Irregular Variations

1. These are irregular variations which occur on account of random external events.

2. These variations either go very deep downward or too high upward to attain peaks abruptly.
3. These fluctuations are a result of unforeseen and unpredictable forces which operate in absolutely random or erratic manner.
4. They do not have any definite pattern and it cannot be predicted in advance.
5. These variations are due to floods, wars, famines, earthquakes, strikes, lockouts, epidemics etc.

Models of Time Series

There are two models which are generally used for decomposition of time series into its four components. The objective is to estimate and separate the four types of variations and to bring out the relative impact of each on the overall behaviour of the time series.

1. Additive model
2. Multiplicative model

Additive Model

In additive model, it is assumed that the four components are independent of one another. Under this assumption, the four components are arithmetically additive, i.e., magnitude of time series is the sum of the separate influences of its four components, i.e., $Y_t = T + C + S + I$, where,

Y_t is time series;

T is trend variation;

C is cyclical variation;

S is seasonal variation;

I is random or irregular variation.

Multiplicative Model

In this model, it is assumed that the forces that give rise to four types of variations are interdependent, so that the overall pattern of variations in the time series is a combined result of the interaction of all the forces operating on the time series. Therefore, time series is the product of its four components, i.e., $Y_t = T \times C \times S \times I$.

Multiplication model which is used more frequently.

Measurement of Secular Trend

The following are the methods most commonly used for studying & measuring the trend component in a time series:

1. Graphic or a Freehand Curve Method
2. Method of Semi Averages
3. Method of Moving Averages
4. Method of Least Squares

Graphic or a Freehand Curve Method

- The data of a given time series is plotted on a graph and all the points are joined together with a straight line.

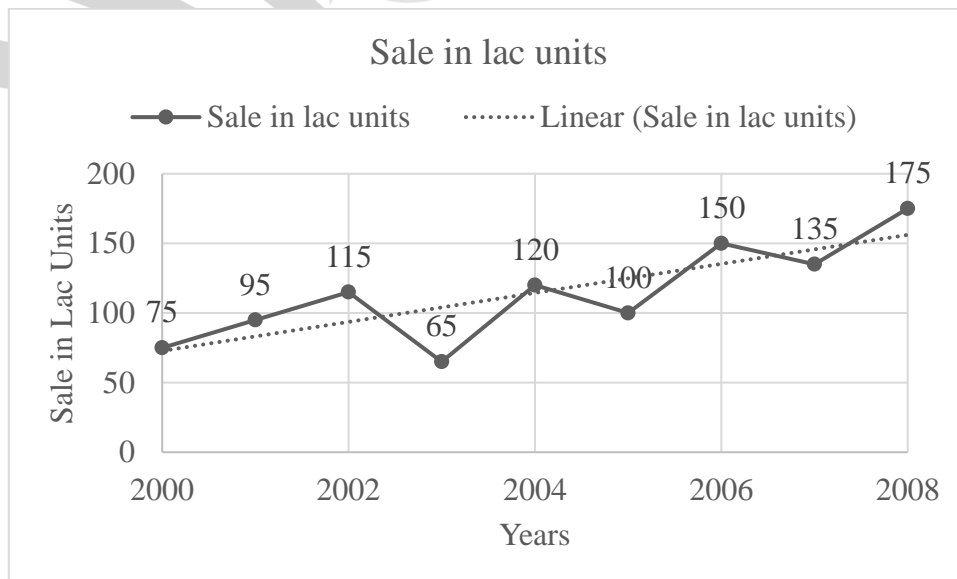
- This curve would be irregular as it includes short run oscillations.
- These irregularities are smoothed out by drawing a freehand curve or line along with the curve previously drawn.
- This curve would eliminate the short run oscillations and would show the long period general tendency of the data.
- While drawing this curve, it should be kept in mind that the curve should be smooth and the number of points above the trend curve should be more or less equal to the number of points below it.
- Merits:
 - It is very simple and easy to understand.
 - It does not require any mathematical calculations.
- Disadvantages:
 - This is a subjective concept. Hence different persons may draw freehand lines at different positions and with different slopes.
 - If the length of period for which the curve is drawn is very small, it might give totally erroneous results.

Question 1

The following are figures of a Sale for the last nine years. Determine the trend by line by the freehand method.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
Sale in lac units	75	95	115	65	120	100	150	135	175

Solution



Method of Semi Averages

- Under this method, the whole time series data is classified into two equal parts and the averages for each half are calculated.

- If the data is for even number of years, it is easily divided into two. If the data is for odd number of years, then the middle year of the time series is left and the two halves are constituted with the period on each side of the middle year.
- The arithmetic mean for a half is taken to be representative of the value corresponding to the midpoint of the time interval of that half.
- Thus, we get two points. These two points are plotted on a graph and then are joined by straight line which is our required trend line.

Question 2

Fit a trend line to the following data by the method of Semi-averages.

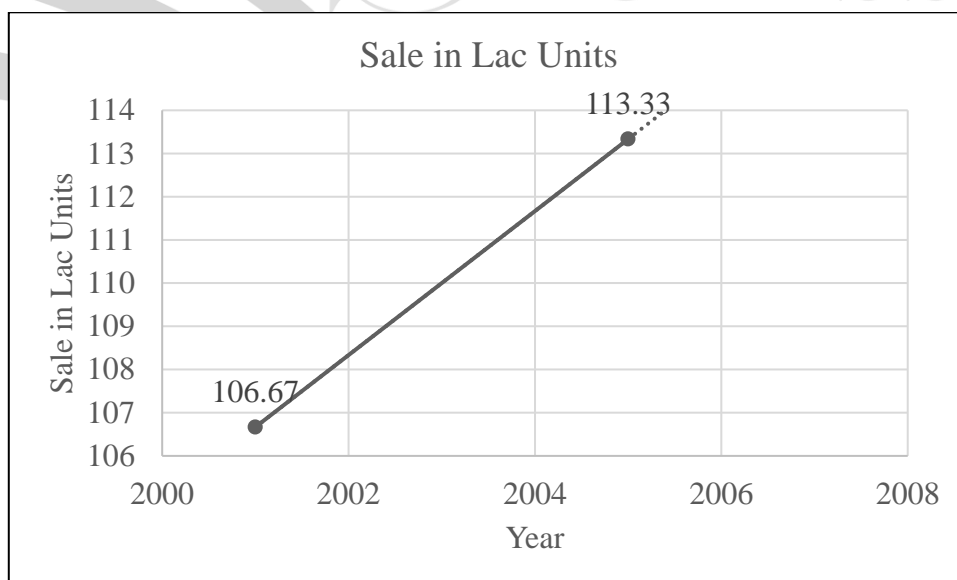
Year	2000	2001	2002	2003	2004	2005	2006
Sale in lac units	100	105	115	110	120	105	115

Solution

Here, since there are 7 years (odd), we'll leave out the middle one, i.e., the year 2003, and take the average of the first three years and the last three years.

Average for the first three years = $\frac{100+105+115}{3} = 106.67$. This is taken to be representative of the value corresponding to the midpoint of the time interval of the first half, i.e., the year 2001.

Average for the last three years = $\frac{120+105+115}{3} = 113.33$. This is taken to be representative of the value corresponding to the midpoint of the time interval of the second half, i.e., the year 2005.



Method of Moving Averages

- A moving average is an average (Arithmetic mean) of fixed number of items (known as periods) which moves through a series by dropping the first item of the previously averaged group and adding the next item in each successive average.

- The value so computed is considered the trend value for the unit of time falling at the centre of the period used in the calculation of the average.
- **3 Year Moving Average:**
For computing 3 yearly moving average, the value of 1st, 2nd and 3rd years are added up, and arithmetic mean is found out and the answer is placed against the 2nd year; then value of 2nd, 3rd and 4th years are added up, and arithmetic mean is derived and this average is placed against 3rd year (i.e. the middle of 2nd, 3rd and 4th) and so on.
- **4 Year Moving Average:**
For computing 4 yearly moving average, the values of 1st, 2nd, 3rd and 4th years are added up, and the total is written between the second and the third year in the third column. Thereafter, the values of the 2nd, 3rd, 4th, and 5th years are added up, and the total is written between the 3rd and the 4th year in the third column. Thereafter, a fourth column is prepared, which contains the totals of groups of two values in the third column. A fifth column is then prepared to calculate the average, which is given by dividing the figure in the fourth column by the total number of years.
- **5 Year Moving Average:**
For computing 5 yearly moving average, the values of the 1st, 2nd, 3rd, 4th, and 5th years are added up, and arithmetic mean is found out and answer is placed against the 3rd year; then value of 2nd, 3rd, 4th, 5th, and 6th years are added up, and the arithmetic mean is derived and this average is placed against 4th year, and so on.
- This technique is called centring & the corresponding moving averages are called moving average centred.

Question 3

The wages of certain factory workers are given as below. Using 3 yearly moving average, indicate the trend in wages.

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012
Wages	1200	1500	1400	1750	1800	1700	1600	1500	1750

Solution

Year	Wages	3 Yearly Moving Totals	3 Yearly Moving Average, i.e., Trend
2004	1200		
2005	1500	$(1200 + 1500 + 1400) = 4100$	$4100 \div 3 = 1366.67$
2006	1400	$(1500 + 1400 + 1750) = 4650$	$4650 \div 3 = 1550.00$
2007	1750	$(1400 + 1750 + 1800) = 4950$	$4950 \div 3 = 1650.00$
2008	1800	$(1750 + 1800 + 1700) = 5250$	$5250 \div 3 = 1750.00$
2009	1700	$(1800 + 1700 + 1600) = 5100$	$5100 \div 3 = 1700.00$
2010	1600	$(1700 + 1600 + 1500) = 4800$	$4800 \div 3 = 1600.00$
2011	1500	$(1600 + 1500 + 1750) = 4850$	$4850 \div 3 = 1616.67$
2012	1750		

Question 4

Calculate 4 yearly moving average of the following data.

Year	2005	2006	2007	2008	2009	2010	2011	2012
Wages	1150	1250	1320	1400	1300	1320	1500	1700

Solution**Calculation of 4 Year Centred Moving Average**

Year (1)	Wages (2)	4 Year Moving Total (3)	2 Year Moving Total of Column 3 (Centred) (4)	4 Year Moving Average (Centred) (5) = (4) ÷ 8
2005	1,150	–	–	–
2006	1,250	–	–	–
		1,150 + 1,250 + 1,320 + 1,400 = 5,120		
2007	1,320		5,120 + 5,270 = 10,390	10,390 ÷ 8 = 1,298.75
		1,250 + 1,320 + 1,400 + 1,300 = 5,270		
2008	1,400		5,270 + 5,340 = 10,610	10,610 ÷ 8 = 1,326.25
		5,340		
2009	1,300		10,860	1,357.50
		5,520		
2010	1,320		11,340	1,417.50
		5,820		
2011	1,500			
2012	1,700			

Question 5

Calculate five yearly moving averages for the following data.

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Value	123	140	110	98	104	133	95	105	150	135

Solution

Year	Value ('000 ₹)	5 Year Moving Totals ('000 ₹)	5 Year Moving Average ('000 ₹)
2003	123	–	–
2004	140	–	–
2005	110	123 + 140 + 110 + 98 + 104 = 575	575 ÷ 5 = 115.0
2006	98	140 + 110 + 98 + 104 + 133 = 585	585 ÷ 5 = 117.0
2007	104	110 + 98 + 104 + 133 + 95 = 540	540 ÷ 5 = 108.0
2008	133	98 + 104 + 133 + 95 + 105 = 535	535 ÷ 5 = 107.0
2009	95	104 + 133 + 95 + 105 + 150 = 587	587 ÷ 5 = 117.4
2010	105	133 + 95 + 105 + 150 + 135 = 618	618 ÷ 5 = 123.6
2011	150	–	–
2012	135	–	–

Method of Least Squares

This method is used for finding a straight-line equation which represents the given data. The straight-line equation is given by $Y_c = a + bX$.

The values of a and b can be found as follows:

$$a = \frac{\sum Y}{N}, \quad b = \frac{\sum XY}{\sum X^2}$$

Here,

$\sum Y$ = Sum of actual values of Y variable, i.e. Sales, Profit, etc.

N = No. of years or months or any other period

$\sum X$ = Sum of values of X .

Note: When the number of years is odd, then X is the deviation of every year from the central year; however, if the number of years is even, then, first the deviations are taken from the average of the two middle most years, and then these deviations are multiplied by 2. This gives us the value of X .

$\sum XY$ = Sum of the products X and Y

$\sum X^2$ = Sum of squares of deviations from X

Question 6

Fit a straight-line trend to the following data by Least Square Method and estimate the sale for the year 2019.

Year	2013	2014	2015	2016	2017
Sales (₹ in lakhs)	10	20	30	50	40

Solution

Year	Sales (Y)	Deviations of the Years from 2015 (X)	XY	X ²
2013	10	2013 – 2015 = –2	–20	4
2014	20	2014 – 2015 = –1	–20	1
2015	30	2015 – 2015 = 0	0	0
2016	50	2016 – 2015 = 1	50	1
2017	40	2017 – 2015 = 2	80	4
Total	150		90	10

Now, we'll find out the values of a and b as follows:

$$a = \frac{\sum Y}{N} \Rightarrow a = \frac{150}{5} = 30$$

$$b = \frac{\sum XY}{\sum X^2} \Rightarrow b = \frac{90}{10} = 9$$

Therefore, the best fit line is $Y_c = 30 + 9X$.

Now, 2019 means that X is $2019 - 2015 = 4$. Therefore, estimated sales of 2019 is given by $Y_{2019} = 30 + (9 \times 4) = 30 + 36 = 66$.

Question 7

Fit a straight-line trend to the following data by Least Square Method and estimate the sale for the year 2012.

Year	2005	2006	2007	2008	2009	2010
Sale (in '000s)	70	80	96	100	95	114

Solution

Year	Sale (in '000s) (Y)	Difference from 2007.5	X = Deviations × 2	XY	X ²
2005	70.00	2005 - 2007.5 = -2.50	-5.00	-350.00	25.00
2006	80.00	2006 - 2007.5 = -1.50	-3.00	-240.00	9.00
2007	96.00	-0.50	-1.00	-96.00	1.00
2008	100.00	0.50	1.00	100.00	1.00
2009	95.00	1.50	3.00	285.00	9.00
2010	114.00	2.50	5.00	570.00	25.00
Total	555.00	0.00	0.00	269.00	70.00

Now, we'll find out the values of a and b as follows:

$$a = \frac{\sum Y}{N} \Rightarrow a = \frac{555}{6} = 92.5$$

$$b = \frac{\sum XY}{\sum X^2} \Rightarrow b = \frac{269}{70} = 3.843$$

Therefore, the best fit line is $Y_c = 92.5 + 3.843X$.

Now, 2012 means that the deviation from 2007.5 is $2012 - 2007.5 = 4.5$, and therefore, X is $4.5 \times 2 = 9$. Therefore, estimated sales of 2012 is given by $Y_{2012} = 92.5 + (3.843 \times 9) = 127.09$.