CPM & PERT

DEFINITION OF A PROJECT:

A project can be defined as a set of activities or jobs that are performed in a certain sequence determined logically or technologically and it has to be completed within a specified time meeting the performance standards at a specified cost. A project is a new work for which organisation has generally no previous experience.

Examples of a project from fairly diverse fields could be cited. Some of them are given below:

1. Introducing a new product in the market.

2. Construction of a new bridge over a river or construction of a 25 storied building.

NETWORK DIAGRAM:

It is a diagram showing sequential flow of activities from one event to another along with the project duration.

PREPARATION OF NETWORK DIAGRAM:

The following important terms to be remembered before drawing a network diagram:

(a) Activity: Activity is a small work of a project which requires a certain time to complete and must consume some resource for its completion.

Every activity must be well defined and should not overlap with each other.
The following is to be remembered for an activity:

1. Activities are denoted by a left to right arrow sign and are symbolised as a, b, c, d, etc.
2. The length of arrow is of no significance and has no relation with the duration of the activity.

3. Each activity can have only one arrow and one number and therefore cannot be repeated.
4. Before an activity can be undertaken, all activities preceding it must be completed.
5. When more than one activity terminates at one event, it means that no activity emanating from that event can start unless all activities terminating there have been completed.

(b) Sequencing of Activities: The above activities will occur in an order following the requirement of the project. This is known as Sequencing of activity.

(c) Event: Event defines or shows the start and end of an activity. It does not consume any time or resource. It is also called as Node. For an activity there is a head event and a tail event. All activities commence from a tail event and terminate at the head event. When two or more activities start from the same event it is called a burst event and when two or more activities end in the same event it is called a merge event.

(d) Numbering of Events (FULKERSON's RULE): Events are numbered as 1,2,3,4..... , etc in such a way that the activity always move from lower number to higher number event. A higher numbered event should be in the vertical line or to the right of the lower numbered event.

(e) Duplicating: No two activities can have same head event and same tail event. This is called as Duplicating.

(f) Dangling: There can be only one start event and one end event of any project. Also all events except the first and the last must have at least one activity entering and at least one activity leaving. This is called as Dangling.

(g) Dummy Activities: Dummy activities are hypothetical activities which consume no resource or time. It is required to complete the logical sequence in a network diagram. A dummy activity is shown by a dashed arrow.

The errors of Duplicating and Dangling can be solved using dummy activities.

PROJECT DURATION CALCULATION AND CRITICAL PATH METHOD (CPM):

(a) Duration: The time required to complete an activity is known as its duration. The duration of an activity is written on its arrow sign.

(b) Path or Route: It is a combination of activities joining start and end point.

(c) Critical Path: The critical path on a network is that sequence of activities which has the longest duration. There may be more than one critical path in a project, each of which has the same longest duration.

(d) Project Duration or Duration of Critical Path: It is the minimum time required to complete the project. A project would complete only if all the activities involved in it are complete. The minimum time required to complete the project is the longest path from the start event to the end event.

(e) Forward Pass Calculation (Left to Right): It is represented by letter 'E'. Considering E = 0 for the initial event, we calculate E for every event considering the terminating activities. E of an activity is written on its Tail or Start Event

E of head event = E of tail event + Duration of the activity.

For convergent events, we take the maximum value of E.

E also stands for Earliest Start Time of an activity.

(f) Backward Pass Calculation (Right to Left): It is represented by letter 'L'. Considering L = E for the final event, we calculate L for every event considering the emanating activities. L of an activity is written on its Head or End Event

L of tail event = L of head event – Duration of the activity.

For divergent events, we take the minimum value of L.

L also stands for Latest Finish Time of an activity.

(g) For all critical path activities, the value of E and L would be equal.

(h) Earliest Finish time of an activity: The time by which an activity can be scheduled to finish, at the earliest.

Earliest Finish time of an activity **=** Earliest Start time + Duration of activity

(i) Latest Start time of an activity: The time by which an activity can be scheduled to finish, at the latest.

Latest Start time of an activity = Latest Finish time – Duration of activity

(j) Slack: It means idle time of an event i.e. the time by which the event can be delayed without causing a delay in the project.

Slack of an event = L – E of an event.

Slack of Head Event = L – E of Head Event

Slack of Tail Event = L – E of Tail Event

<u>**TOTAL FLOAT, FREE FLOAT, INDEPENDENT FLOAT:**</u>

(a) Total float: It implies the idle time in a path or in its non – critical activities. For critical activities there will not be any idle time. In other words, the non – critical activities in total can be delayed by some time without affecting or increasing the project duration.

It can be computed in two different ways:

1. Total float = Latest Finish Time – Earliest Finish Time

= Latest Start Time – Earliest Start Time

2. As a time difference between critical and non – critical path.

(b) Free float: The time by which the actual completion of an activity can be delayed without affecting the total float of succeeding activities.

Free Float = Total Float – Slack of Head Event

(c) Independent Float: The time by which the actual completion of an activity can be delayed without affecting the total float of preceding activities.

Independent Float = Free float – Slack of Tail Event or 0 whichever is larger.

UPDATING A NETWORK:

After a project has commenced revisions might have to be made to the project network. Activity duration might have to be reviewed and revised as sometimes activities might be completed before schedule or be delayed for various reasons. Periodic update of the network may be required during the progress of the project.

PROJECT CRASHING:

If costs are associated with activities it becomes pertinent to investigate the effect of the increase or decrease in the total duration of a project on the total cost of the project. The different types of times and costs involved are:

(a) Normal Time: The minimum time required to complete an activity at normal cost.

(b) Crash Time: The minimum time required to complete an activity.

(c) Normal Cost: The direct cost of completing the activity in normal time.

(d) Crash Cost: The direct cost of completing the activity within the crash time.

(e) Cost Slope: The increase in cost for every unit of time saved by crashing the activity.

Cost Slope = Crash Cost - Normal Cost / Normal Time Crash Time

(f) Optimum Duration: The duration of the project corresponding to the optimum cost.

(g) Optimum Cost: The minimum possible cost to complete the project.

(h) Total Direct Cost: Total Normal cost + Total Crash cost

(i) Total Project Cost: Total Normal cost + Total Crash cost + Total Indirect cost

Process of Crashing

The following steps are followed:

1. Prepare the project network.

2. Find the critical path and the normal duration of the project.

3. Calculate the cost slope for all the activities given in the network.

4. First identify those activities on the critical path which have a cost slope less than the indirect cost. The overall cost can be reduced only if the cost slope of the crashed activity is less than the indirect cost.

5. Start by crashing that activity which has the least crashing cost slope and progress with ones in order of increasing cost slopes (Keeping in mind that an activity can be crashed only till its crash time).

6. If at any point there happens to be more than one critical path then select different sets of activities such that crashing all the activities in each set reduces all the critical paths at the same time.

To make the sets select an activity from each path. If the same activity exists in any other path, put a dash there. If you cannot put a dash, discard that particular combination. Crash that set which has the least total crashing cost.

7. Stop crashing if any one of the longest path is exhausted fully i.e. crashed till its crash time.

8. Prepare a cost table which shows the direct crashing cost, the direct normal cost and the total indirect cost for all the reduced project durations. The duration which gives the least cost is the optimum project duration and the corresponding total cost is the optimum project cost.

9. To find the Optimum cost of the project, stop crashing at the point where total crashing cost is more than the indirect cost.

10. To find the Minimum Duration of the project, regardless of cost, continue crashing even if the crashing cost is more than the indirect cost. However, crashing will stop the moment any one of the longest path is fully exhausted.

PROJECT EVALUATION AND REVIEW TECHNIQUE (PERT):

PERT is more relevant for projects where there is a good measure of uncertainty in the estimation of activity duration. Here, we obtain three estimates of activity duration:

1. The Optimistic Time Estimate: This is the shortest possible time in which an activity can be completed. It is estimated under the best case situation and is denoted by t_o

2. The Pessimistic Time Estimate: This is the longest possible time in which an activity can be completed. It is estimated under the worst case situation and is denoted by t_p

3. The Most Likely Time Estimate: This is the normal time in which an activity can be completed. It is estimated under the most probable situation and is denoted by t_m

From the above three estimates an average or expected time of activity is calculated for any subsequent analysis.

A. Expected Time of activity: This is the duration of the activity found by calculating the weighted average of the three time estimates and is given by:

Duration of activity = $T_E = t_o + 4t_m + t_p / 6$

B. Expected Project Time: This is the longest duration or the critical path duration and is found by adding the expecte d times of the critical activities.

C. Variances and Standard Deviations:

1. Standard Deviation of an activity:

S.D. = $(t_p - t_o) / 6$

2. Variance of an activity = $(S.D.)^2$

3. Variance of the critical path: This is obtained by adding variances of all the activities on the critical path.

4. Standard Deviation of the Critical Path: This is obtained by the formula given below and not by adding the SD's of the critical path activities.

$S.D. = \sqrt{\text{Variance of critical path}}$

5. Variances of events for T_E **and** T_L

Sum of the variances along the path of T_E and T_L. If there are more than one longest path then select one having a larger sum of variances.

The Earliest time T_E = the duration of the longest path leading to the event from the start.

The Latest time T_L = the duration of the longest path emanating from the event to the end.

D. To find the Probabilities of completion of Project in the given time:

1. The time of completion of project for which the probability is required is expressed as P(x)

2. To find the probability for a given value of x, first find z using the following formula:

$Z = X - T_{cp} / S_{cp}$

Tcp = Expected Project Duration i.e. Duration of the critical path

Scp = Standard Deviation of the Project or Critical path.

3. The required probability of x would be the area under the curve for respective value of z.

P(X) = A(Z).

E. Characteristics of Standard Normal Curve:

1. It is a symmetrical bell shaped curve.

2. The total area under the curve is 1.

3. Because of symmetry, the area to the left and the right of central axis are both equal and is 0.5

4. The maximum value of Z for which area will be 0.5 on either side of axis can be 3.49

5. The area A(z) would mean area between z=0 and the given value of z.

6. Because of symmetry, A(-z) = A(z)

RESOURCE SMOOTHING

It is a network technique used for smoothening peak resource requirement during different periods of the project network. Under this technique the total project duration is maintained at the minimum level. For example, if the duration of a project is 15 days, then the project duration is maintained, but the resources required for completing different activities of a project are smoothened by utilising floats available on non critical activities. These non critical activities having floats are rescheduled or shifted so that a uniform demand on resource is achieved. In other words, the constraint in the case of resource smoothing operation would be on the project duration time. Resource smoothing is a useful technique for business managers to estimate the total resource requirements for various project activities.

In resource smoothing, the time scaled diagram of various activities and their floats (if any), along with resource requirements are used. The periods of maximum demand for resources are identified and non critical activities during these periods are staggered by rescheduling them according to their floats for balancing the resource requirements i.e. the activities having floats are shifted in such a way that the demand for resources is smoothened out.

TIME SCALED NETWORK

In the network diagrams which we have considered, it has been stressed that the length of the individual arrows has no relation to the duration of the activity which arrow represented. It is of course possible to draw the arrows to a time scale, and this can be a very useful method of presentation of networks. In such networks, Critical path is arranged as a straight line with non critical activities above or below it. The dotted horizontal lines represent floats in the activity and the dotted vertical lines represent dummy activities.

RESOURCE LEVELLING:

It is also a network technique which is used for reducing the requirement of a particular resource due to its paucity. The process of resource levelling utilizes the large floats available on non critical activities of the project and thus cuts down the demand on the resource. In resource levelling, the maximum demand of a resource should not exceed the available limit at any point of time. In order to achieve this, non critical activities are rescheduled by utilising their floats. Sometimes, the use of resource levelling may lead to increase in the completion time of the project.

Q-1 A small Project consists of jobs as given in the Table below. Each job is listed with its Normal Time and a Minimum or Crash Time (In days). The Cost (in Rs. per say) of each job is also given to you.

Activity	Normal duration (days)	Minimum Duration (days)	Cost Of Crashing per day (Rs.)
1-2	9	6	20
1-3	8	5	25
1-4	15	10	30
2-4	5	3	10
3-4	10	6	15
4-5	2	1	40

1. What is the Normal Project Length and the Minimum Project Length?

2. Determine the minimum Crashing cost of schedule ranging from normal length down to, and including the Minimum Length schedule. That is, If L = Length of the schedule, find the Costs of Schedules, which are L, L-1, L-2 and so on.

3. Overhead Costs are Rs. 60 per day. What is the Optimum Length Schedule in terms of both Crashing and Overhead Cost? List the Schedule Duration of each job for your solution.

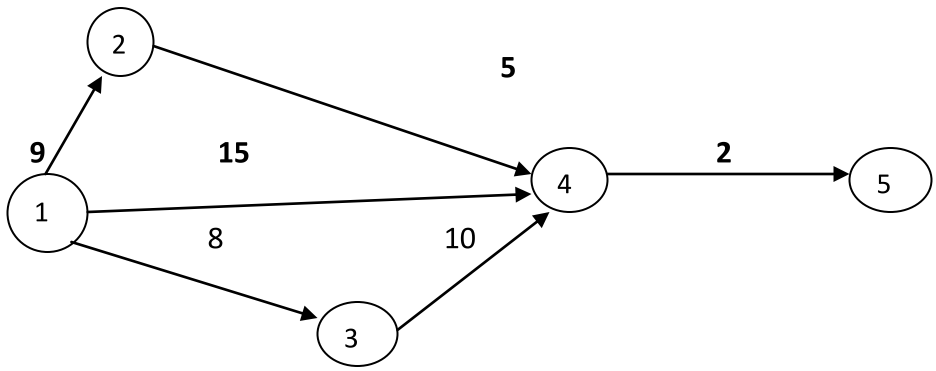
Solution: Note:- SLOPE = Crashing Cost per day = given in the question

1: Paths Table

Path	**Normal Duration (Normal Days)**	**Min. Duration (Crash Days)**	**Duration after Stage**				
			(1)	**(ii)**	**(iii)**	**(iv)**	**(v)**
Path X: 1-2-4-5	9+5+2 =16	6+3+1 = 10	16	16-1=15	15	15-2=13	13-1=12
Path Y: 1-4-5	15+2 = 17	10+1 = 11	17	17-1=16	16-1=15	15-2=13	13-1=12
Path Z: 1-3-4-5	8+10+2 = 20 (Initial CP)	5+6+1 = 12 (Min Duration)	20-3 = 17	17–1=16	16-1=15	15-2=13	13-1=12

Note:- Since Minimum Duration is only 12 days on Path Z, the project can be crashed and brought to min 12 days only.

Network:-



CRASHING PROCESS

Stage	Decision On Crashing	Crash Costs
Stage I	Initial CP is path Z, and Activity with least slope is 3-4.This can be crashed for 10-6 =4 days maximum, but the time gap between Paths Z & Y (i.e. next longest path) is only 3 days .SO, Activity 3-4 is crashed for the permissible period of 3 days (Least)	Rs. 15 per day for 3 days = Rs. 45
Stage II	Paths Y and Z are the CPs (17 days).activities available for crashing are –(1) Common Activity 4-5 (with slope 40) or (2) Separate Activities (with higher slope).So, Common Activity 4-5 is crashed for 1 day (being maximum time reduction possible).	Rs. 40 per day for 1 day = Rs.40

Stage III	Paths Y and Z are the CPs (16 days). Activity combinations available for crashing are – (1) 3-4 & 1-4 (with slope 15 + 30 = 45_ and (2) 1-3 &1-4 (With slope 25+30 = 55). The first combination is chosen due to lower slope and crashed for 1 day.	Rs. 45 per day for 1 day = Rs. 45
Stage IV	All Paths are critical Since Common Activity is fully crashed in stage II, the only possible crashing is to crash 2-4, 1-4 & 1-3 by 2 days (being the least of the maximum time reduction possible for 2-4). Slope is 10+30+25 = 65	Rs.65 per day for 2 days= Rs.130
Stage V	All Paths are critical .The only activities available for crashing are 1-2 (3), 1-4(2) and 1-3 (1) (with maximum time reduction possible in brackets. Each activity is reduced by 1 day (being minimum of the three). Slope is 20+ 30+25 = 75.	Rs.75 per day for 1 day = Rs.75

After this, no further Crashing is possible since Minimum Duration of the Project is 12 days as per Note in WN 2 above.

Costs Table (on a per day basis)

Stage	Duration	Indirect Cost at Rs.60/day	Crash Costs (Activity)	Total Costs
Initial	20 days	Rs. 1,200	Ni;	Rs.1,200 = (normal

				Duration Cost)
I	19 days 18 days 17 days	Rs.1,140 Rs.1,080 Rs.1,020	Rs. 15 (3-4) Rs. 30 (3 – 4) Rs. 45 (3 -4)	Rs.1,155 Rs.1,110 Rs.1,.065
II	16 days	Rs.960	Rs. 85 (4 -5)	Rs.1,045
III	15 days	Rs.900	Rs.130 (3-4 & 1-4)	Rs.1,030=(optimal Duration Cost)
IV	14 days 13 days	Rs.840 Rs.780	Rs.195 (2-4, 1-4 & 1-3) Rs.260 (2-4, 1-4 & 1-3)	Rs. 1,035 Rs.1,040
V	12 days	Rs.720	Rs.335(1-2, 1-4, & 1-3)	Rs.1,055(Minimum Duration Cost)

Activity Schedule of Optimum Duration (15 days)

Activity	1-2	1-3	1-4	2-4	3-4	4-5
Duration	9	8	14	5	6	1

QUESTIONS FOR PRACTICE:

Q-2 The activities involved in a computer installation project are given below. Draw the network.

	Activity	Preceding Activity
A	Physical preparation	None
B	Organisational planning	None
C	Personnel selection	B
D	Equipment installation	A
E	Personnel training	C
F	Detailed system designing	C
G	File conversion	F
H	Establishment of standards	F
I	Program preparation	H
J	Program testing	I
K	Parallel operation	D,E,G,J
L	Finalising documentation	I
M	Follow up	K,L

Q-3 Prepare the network from the following activity data:

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M
Preceding	-	-	A	B,C	D	E	B,C	F	F,G	H,I	B	F,G,K	J, L

Q-4 From the following set of activities, draw the project network. Calculate all floats. Find critical path and calculate the longest duration of the project.

Activity	1-2	1-4	2-3	3-5	3-8	4-8	5-6	5-8	6-7	7-8	7-9	8-9	9-10
Duration (days)	4	36	2	15	10	2	4	9	9	9	8	20	20

Q-5 The number of days of total float (TF), earliest start times (EST) and duration in days are given for some of the following activities:

Activity	**TF**	**EST**	**Duration**
1-2	0	0	
1-3	2		
1-4	5		
2-4	0	4	
2-5	3		5
3-6	2	12	
4-6	0	12	
5-7	3		
6-7		23	
6-8	2		
7-8	0	23	
8-9		30	6

(a) Draw the network

(b) List the paths with their corresponding durations and state when the project can be completed.

Q-6 The following activities and their duration are given:

Activity	1-2	1-3	1-4	2-5	2-4	3-4	3-6	4-7	5-7	6-7	6-8	7-8
Duration	9	10	6	8	7	5	12	20	8	7	7	6

After 15 days of working the following is observed for the network of an erection job:

(i) Activities 1-2, 1-3, 1-4 and 3-4 were completed as scheduled.

(ii) Activity 2-4 is in progress and will take 3 more days for completion.

(iii) Activity 3-6 is in progress and will take 18 more days for completion.

(iv) Activity 4-7 will be completed 20 days from today.

(v) Activity 6-7 will take 12 days to complete instead of the scheduled 7 due to some problem.

(vi) Activity 6-8 will take 5 days to complete instead of the scheduled 7 due to added workforce.

Update the network in the light of the above information and determine the critical path and revised project duration.

Q-7 The following data pertains to the network given below. The given times are in days and the costs are in rupees. Crash the project to find the optimum duration and the optimum project cost if the indirect cost is Rs 160 per day.

	Normal		**Crash**	
Activity	**Time**	**Cost**	**Time**	**Cost**
1-2	3	360	2	400
2-3	6	1440	4	1620
2-4	9	2160	5	2380
2-5	7	1120	5	1600
3-4	8	400	4	800
4-5	5	1600	3	1770
5-6	3	480	2	760
		7560		

Q-8 A project is composed of activities having the following time duration in weeks:

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
Optimistic time	1	1	2	1	2	2	3
Most Likely time	1	4	2	1	5	5	6
Pessimistic time	7	7	8	1	14	8	15

(i) Draw the project network.

(ii) Find the expected duration and variance for each activity.

(iii) Calculate the standard deviation, the variance and the duration of the critical path.

(iv) What is the probability that the project will be completed:

- **a.** Exactly on due date?
- **b.** At least three weeks before than expected?
- **c.** No more than three weeks later than expected?

(v) If the project due date is 18 weeks find the probability of not meeting the due date?

(vi) What due date has 90% chance of being met?

(vii) What is the chance of completing the activity 3-5 one week earlier than expected?

(viii) Find the event variances

Q-9 A firm has been contracted to deliver extruding presses to a multinational company. The contract price negotiated is contingent upon meeting a specified delivery time with a bonus offered for early delivery. The firm has the following cost and time information:

Time in weeks					Cost in Rs	
Activity	**To**	**Tm**	**Tp**	**Crash**	**Normal**	**Crash**
1-2	1	3	5	1	5000	9000
2-3	1	4	7	3	8000	14000

2-4	1	3	5	2	4000	6000
2-5	5	8	11	7	5000	6000
3-6	2	4	6	2	3000	5000
4-6	5	6	7	4	2000	3600
5-7	4	5	6	4	10000	14000
6-7	1	3	5	1	7000	10600
Total normal cost					44,000	

The contract terms provide the following contract amounts:

Delivery time (weeks)	16	15	14	13	12
Contract amount (Rupees)	62,000	62,500	65,000	70,000	72,500

On the basis of this data what delivery schedule do you recommend the company to apply?

Q-10 The following details for a project are given:

Activity	1-2	1-3	1-4	2-6	3-5	5-6	4-6
Days	4	2	8	6	4	1	1
No. of men required per day	2	3	5	3	2	3	8

1. Draw network and find critical path.

2. What is the peak requirement of manpower? On which day it will occur?

3. If maximum labour available on any day is 10, when can the project be completed?