# Finding Domain and Range

- The **domain** of a function is the set of all $x$-values that can be used as inputs for the function.
- The **range** of a function is the set of all possible $y$-values, or outputs, for a function.
- Values excluded from the domain include values that result in a negative number under a square root sign and values that cause a denominator to equal zero.

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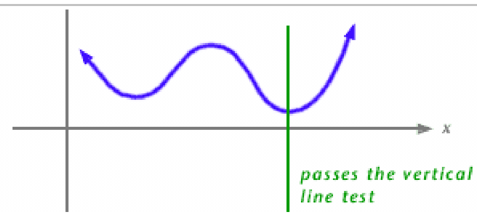

</td>
<td>To be judged a function, an equation must have no more than one $y$-value for every $x$- value. Graphically, this notion means that if you place a vertical line anywhere on the curve, it will intersect the curve in only one point.</td>
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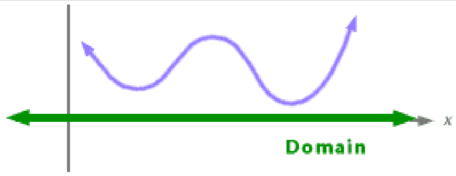

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<td>In this case, the domain is all the real numbers because the curve goes on forever to both the right and the left. As a result, every $x$ there is will have some $y$ on the curve.</td>
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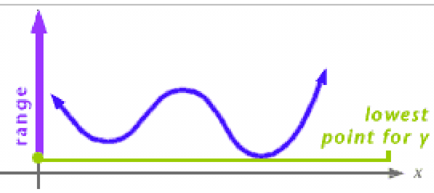

</td>
<td>The range is all the real numbers that are used as $y$-values for a curve. In this example, there will not be any negative $y$ values nor even any positive $y$ values just barely above the $x$-axis.</td>
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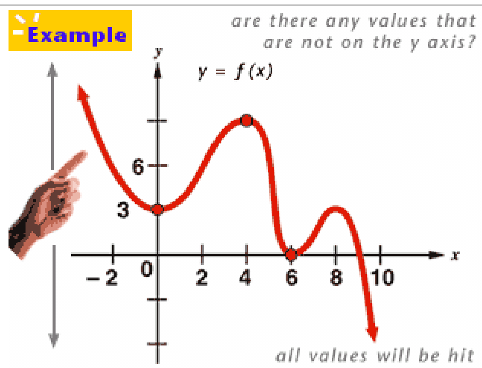

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<td>In this example, the domain is all the values of $x$, and the range is all the values of $y$. There are no values that will be missed or excluded in either direction.<br><br>Domain: $-\infty < x < \infty$<br>Range: $-\infty < y < \infty$</td>
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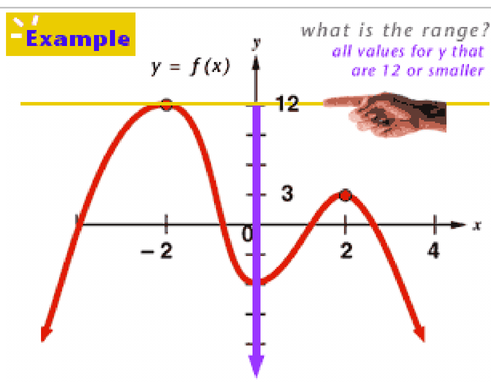

</td>
<td>In this example, the domain will be all the $x$- values. The range, however, will not include any values larger than +12 because the curve does not rise any higher than that point. So, the range is all the $y$-values less than or equal to +12.<br><br>Domain: $-\infty < x < \infty$<br>Range: $y \le 12$</td>
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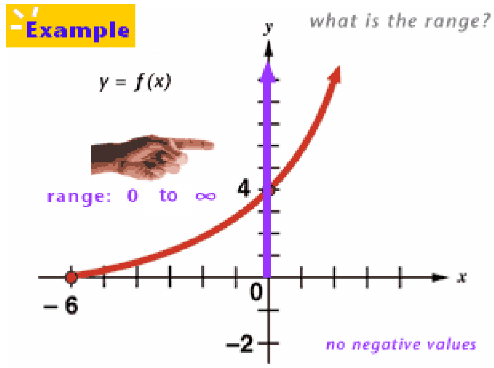
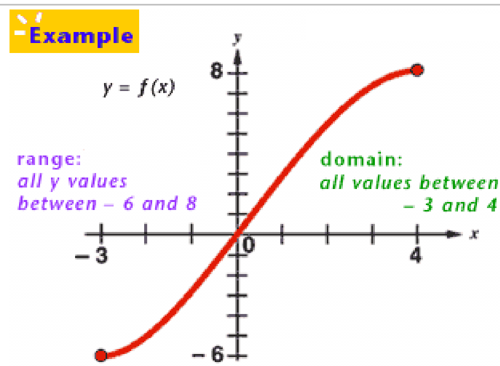


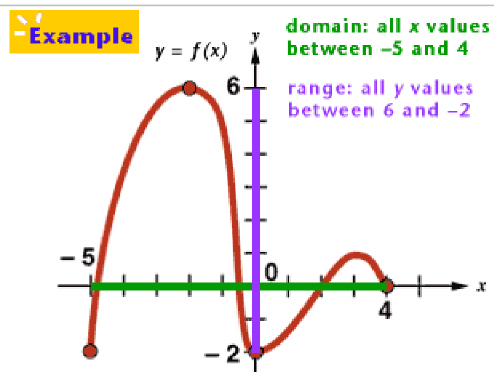

In this example, the domain will be all $x$-values greater than or equal to –6 because the curve stops and does not pass left of –6. The range will be 0 and all positive numbers because the curve does not move below the $x$-axis but does touch the $x$-axis.

Domain: $x \geq -6$
Range: $y \geq 0$



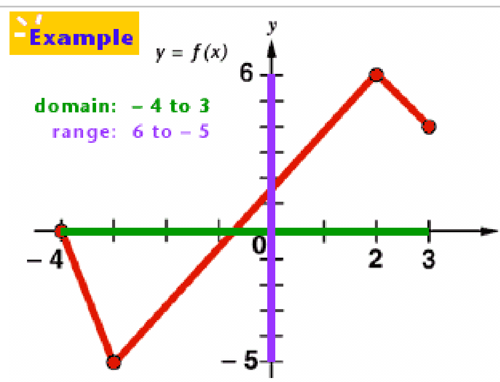

In this example of a curve segment, the endpoints are (–3, –6) and (4, 8). Since the curve is continuous between those two points, all values between the two points are used on the curve. As a result the domain will include all values between –3 and 4. The range will include all values between –6 and 8.

Domain: $-3 \leq x \leq 4$
Range: $-6 \leq y \leq 8$




This curve is continuous and moves between (–5, –2) and (4, 0). The domain is limited to include all values between or equal to –5, the left end of the curve, and +4, the right end of the curve. The range is limited to include all values between or equal to –2, the lowest point on the curve, and +6, the highest point on the curve.

Domain: $-5 \leq x \leq 4$
Range: $-2 \leq y \leq 6$



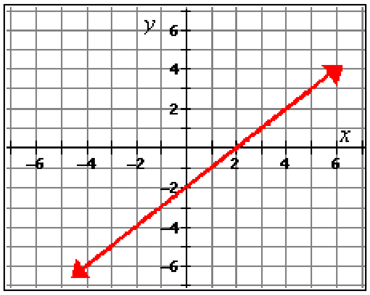

This unusual curve has endpoints at (–4, 0) and (3, 4). That fact will limit its values as shown. Its domain will be all the values between or equal to –4 and +3. Its range will be all the values between or equal to –5 and +6.

Domain: $-4 \leq x \leq 3$
Range: $-5 \leq y \leq 6$

## Finding the Domain and Range of a Function

### Sample problem 1:

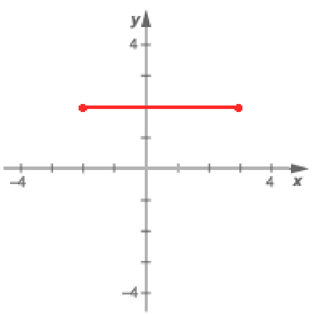
Find the domain and the range of the function.
*Express the answer using interval notation.*




Solution: Domain $(-\infty,\infty)$, Range $(-\infty,\infty)$
Explanation: The arrows on each end of the graph indicate that the graph extends infinitely in each direction. There are no jumps or open circles in the graph. So, since all $x$-values are included in the graph and all $y$-values are included in the graph, the domain is all real numbers and the range is all real numbers. Express all real numbers using interval notation.
So, the domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

### Sample problem 2:

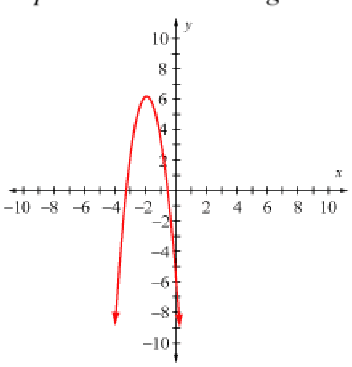
Find the domain of the function graphed below.




Solution: $-2 \le x \le 3$
Explanation: The domain is defined as all possible values of $x$. The smallest $x$ value included in the line is $-2$ and the largest is 3. The graph is continous (contains no breaks), so the domain is all values between $-2$ and 3, including $-2$ and 3, or $-2 \le x \le 3$.

### Sample problem 3:

Find the domain and the range of the function.
*Express the answer using interval notation.*



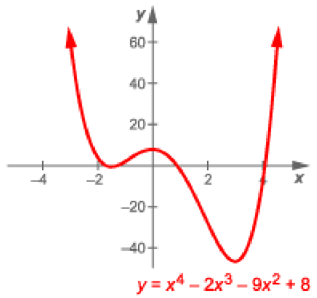
Solution: Domain $(-\infty, \infty)$; Range $(-\infty, 6]$

Explanation: Notice that there are no open circles or jumps in the graph. The domain is the set of all possible $x$-values for the function. The arrows at each end of the graph indicate that the function extends infinitely to the left and to the right. Therefore, all real numbers are possible $x$-values for the function. Thus, the domain is all real numbers, or using interval notation, $(-\infty, \infty)$.

The range is the set of all possible $y$-values for the function. The $y$-values extend from the greatest possible value of $y = 6$ then infinitely in the negative direction. Therefore, all real numbers between $-\infty$ and 6 are included in the range of the function. Thus, the range is all numbers less than or equal to 6, or using interval notation, $(-\infty, 6]$.

**Sample problem 4:**

Find the domain of the function $y = x^4 - 2x^3 - 9x^2 + 8$.

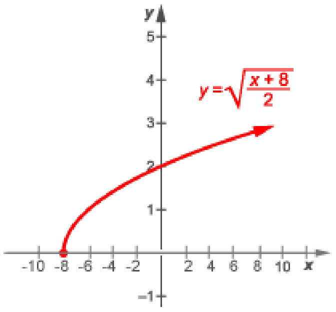



Solution: $-\infty < x < \infty$

Explanation: In the graph the arrows indicate that the graph extends infinitely in both the negative and positive directions, thus the possible $x$ values are all real numbers. Additionally, in the equation, $x$ is defined for all real numbers. Therefore, the domain is $-\infty < x < \infty$.

**Sample problem 5:**

Given $f(x) = \sqrt{\dfrac{x+8}{2}}$, use this equation and the graph to find the domain and range of the function $f$.




Solution: Domain: $x \geq -8$; Range: $y \geq 0$

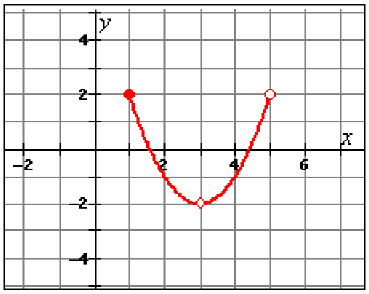
Explanation: In the graph the possible values of $x$ are any value $-8$ or larger. Thus, the domain is $x \geq -8$.

The possible $y$-values are all non-negative numbers. In the equation $f(x) = \sqrt{\frac{x+8}{2}}$, the square root cannot be negative, so the smallest $y$-value is 0 and there is no positive limit. Therefore, the range is $y \geq 0$.

**Sample problem 6:**

Find the range of the function.

*Express the answer using interval notation.*



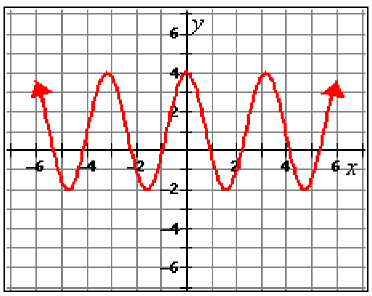

Solution: $(-2,2]$

Explanation: The range is the set of all possible $y$-values for the function.
Notice that there is an open circle at the point $(3,-2)$. An open circle indicates that the point is not included in the graph. So, the $y$-value associated with this point, $-2$, is excluded from the range. There are no points in the graph with a $y$-value that is less than $-2$. The $y$-values extend in a positive direction from $-2$ to, and including, 2. Thus, the range is $(-2,2]$.

**Sample problem 7:**

Find the domain of the function.

*Express the answer using interval notation.*



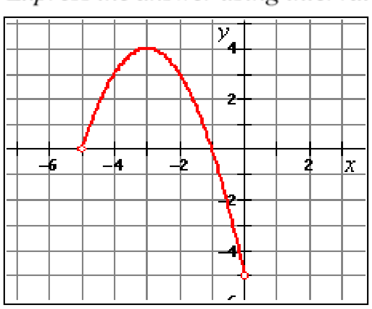

Solution: $(-\infty,\infty)$

Explanation: The domain is the set of all possible $x$-values for the function.
The arrows at each end of the graph indicate that the function extends infinitely to the left and to the right. Notice that there are no open circles or jumps in the graph. Therefore, all real numbers are possible $x$-values for the function. Thus, the domain is all real numbers, or using interval notation, $(-\infty,\infty)$.

**Sample problem 8:**

Find the range of the function.

*Express the answer using interval notation.*

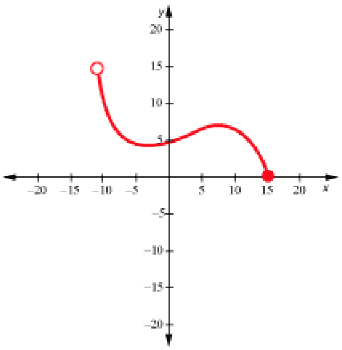


Solution: $(-5,4]$

Explanation: The range is the set of all possible $y$-values for the function. Notice that there is an open circle at the left end of the graph, or the point $(-5,0)$. An open circle indicates that the point is not included in the graph. However, the $y$-value associated with this point, 0, is included in the range because the graph passes through a point that does include 0, the point $(-1,0)$. The least possible $y$-values extend from, but do not include, $-5$. The greatest possible $y$-value is 4. There are no jumps or open circles between $-5$ and 4. Therefore, all real numbers between $-5$ and 4, but not including $-5$, are included in the range of the function. Thus, the range is $(-5,4]$.

**Sample problem 9:**

Find the domain and range of the function graphed below.



Solution: Domain: $-10 < x \leq 15$

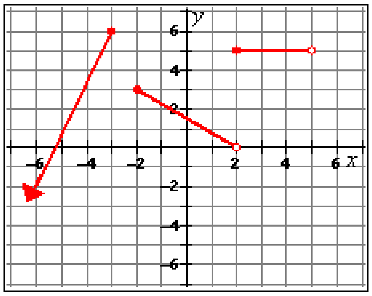
Range: $0 \leq y < 15$

Explanation: The open circle at $(-10, 15)$ indicates that the point is not included in the domain/range. The domain (possible $x$-values) extends from $-10$ to 15, including 15, but not $-10$, or $-10 < x \leq 15$. The range (possible $y$-values) extends from 0 to 15, but not including 15, or $0 \leq y < 15$.

**Sample problem 10:**

Find the domain of the function.

*Express the answer using interval notation.*



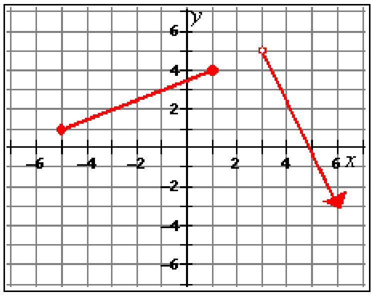
Solution: $(-\infty,-3]\cup[-2,5)$

Explanation: The domain is the set of all possible $x$-values for the function. The arrow at the left end of the graph indicates that the function extends infinitely to the left, or in a negative $x$ direction. Notice that there is a jump between the $x$-values $-3$ and $-2$. So, the values between $-3$ and $-2$ are excluded from the domain. However, $-3$ and $-2$ are included in the domain because the points $(-3,6)$ and $(-2,3)$ are made with closed circles. The graph may appear to make a second jump when $x$ is 2, but the $x$-value 2 is included because the point $(2,5)$ is a closed circle. The domain extends to 5, but does not include 5. Thus, the domain is $(-\infty,-3]\cup[-2,5)$.

**Sample problem 11:**

Find the domain and the range of the function.

*Express the answer using interval notation.*




Solution: Domain $[-5,1]\cup(3,\infty)$, Range $(-\infty,5)$

Explanation: Determine all of the $x$-values included in the points in the graph to find the domain. The point $(-5,1)$ gives the least value in the domain. Notice that $-5$ is included in the domain since the circle at $(-5,1)$ is closed. The graph continues to the point $(1,4)$, and again, the value 1 is included in the domain. The graph then makes a jump to the point $(3,5)$. Therefore, the $x$-values between 1 and 3 are not included in the domain. Furthermore, 3 is not included because the circle at $(3,5)$ is open. The arrow indicates that the graph continues infinitely in a positive $x$ direction. So, all real $x$-values greater than 3 are also included in the domain. Thus, the domain includes $-5 \le x \le 1$ and $3 < x < \infty$. Use the symbol for union to express the domain using interval notation. So, the domain is $[-5,1]\cup(3,\infty)$.

Determine all of the $y$-values included in the points in the graph to find the range. The arrow indicates that the graph continues infinitely in a negative $y$ direction. The point with the greatest $y$-value is $(3,5)$. However, the range does not include 5 because the circle at $(3,5)$ is open. All $y$-values up to 5 are included in the graph. So, the range is $-\infty < y < 5$, or using interval notation, $(-\infty,5)$. Remember, a parenthesis is always used with infinity.