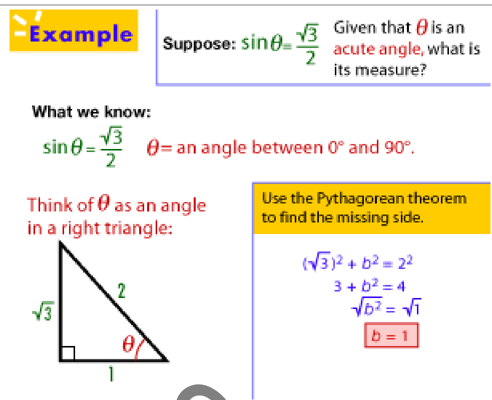
Finding an Angle Given the Value of a Trigonometric Function

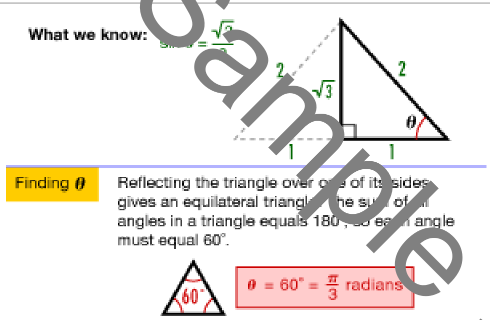
- You can sometimes find the measure of an angle θ given the value of one of its **trigonometric functions** by analyzing a right triangle with θ as one of its angles.



You have learned how to compute **trigonometric functions** given an angle. What about finding an angle given the value of a trig function?

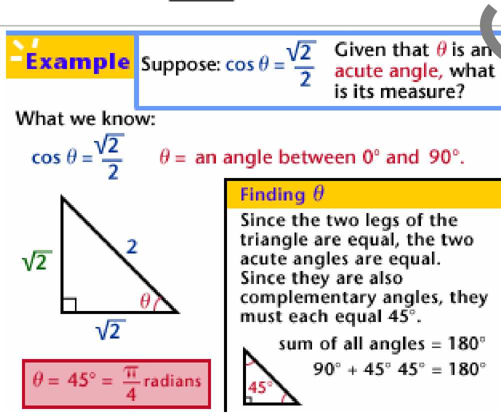
For example, can you find the acute angle θ, given the value of **sin**θ ?

First, sketch a right triangle having θ as an angle. Use the fact that $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$ to label the legs and hypotenuse of the triangle. Then, use the **Pythagorean theorem** to find the length of the missing leg.



Reflecting the triangle over the side of length $\sqrt{3}$ results in an equilateral triangle.

The angles of an equilateral triangle all measure 60°, or π/3 **radians**. Therefore, $\theta = 60°$ or π/3 radians.



Consider this example: find θ if it is an acute angle whose **cosine** is $\sqrt{2}/2$.

Sketch a right triangle with θ as one of its angles. Label the adjacent leg with $\sqrt{2}$ and the hypotenuse with 2. Use the Pythagorean theorem to find that the missing leg.

Since both legs measure $\sqrt{2}$, this right triangle is isosceles. Therefore, $\theta = 45°$ or π/4 radians.

Finding an Angle Given the Value of a Trigonometric Function [Sample Problems]

1)

Suppose $\cot\theta = 1$. Given that θ is an acute angle in a right triangle, what is its measure in radians?

Solution: $\frac{\pi}{4}$

Explanation: By definition, $\text{cotangent}\,\theta = \frac{\text{adjacent}}{\text{opposite}}$. Therefore, let the length of the adjacent side be 1, and let the length of the opposite side be 1. The sides are equal, so this is an isosceles right triangle. Therefore, $\theta = 45° = \frac{\pi}{4}$.

2)

Suppose $\sin\theta = \frac{\sqrt{2}}{2}$. Given that θ is an acute angle in a right triangle, what is its measure in radians?

Solution: $\frac{\pi}{4}$ rad

Explantion: By definition, $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$. Therefore, let the length of the side opposite the angle θ be $\sqrt{2}$, and let the length of the hypotenuse be 2. Use the Pythagorean theorem to find the missing length: $a^2 + b^2 = c^2$; $\left(\sqrt{2}\right)^2 + x^2 = 2^2$; $2 + x^2 = 4$; $x^2 = 2$; $x = \sqrt{2}$. Since both legs of the triangle are equal, the triangle is isosceles. Therefore, $\theta = 45° = \frac{\pi}{4}$ rad.

3)

Suppose $\cos\theta = \frac{\sqrt{3}}{2}$. Given that θ is an acute angle in a right triangle, what is its measure in radians?

<u>Solution</u>: $\frac{\pi}{6}$ rad

<u>Explanation</u>: By definition, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$. Therefore, let the length of the adjacent side be $\sqrt{3}$, and the length of the hypotenuse be 2. Use the Pythagorean theorem to find the missing length. $a^2 + b^2 = c^2$; $\left(\sqrt{3}\right)^2 + x^2 = 2^2$; $3 + x^2 = 4$; $x^2 = 1$; $x = 1$. The sides are not equal, so reflect the triangle across the leg with the greater length to form an equilateral triangle. An equilateral triangle has equal angles of 60°. Since θ is adjacent to the longer leg, it is added to itself to form one of the 60° angles. Divide by 2 to find the original measure: $\frac{60°}{2} = 30° = \frac{\pi}{6}$ rad.