

Proving an Identity

- **Review:** An **identity** is a mathematical statement that is true for all meaningful values of the variable(s) involved.
- When proving an identity, treat each side of the equal sign as its own expression. Do not treat the identity as an equality until it is proven that both sides are equal.

<p>Proving an Identity</p> <ul style="list-style-type: none"> -manipulate each side of the equal sign independently -try to make both sides the same 	<p>Often in trigonometry you are given an equation and asked to show whether or not the statement is true. When proving an identity, remember to only manipulate one side of the proposed equality at a time.</p>
<p>Fundamental Identities</p> $\sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \tan \theta = \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta = 1$	<p>The four basic trigonometric identities will be used often.</p>
<p>Example $\tan x + \cot x = \sec x \csc x$</p> <p>steps:</p> <ol style="list-style-type: none"> Convert to sine and cosine Multiply to get a common denominator Add Trig identity $\begin{aligned} &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} &= \frac{1}{\cos x} + \frac{1}{\sin x} \\ &= \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} && \downarrow \\ &= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} && \\ &= \frac{1}{\sin x \cos x} &= \frac{1}{\sin x \cos x} \end{aligned}$	<p>When proving an identity, it is often useful to draw a line down the middle. Only work on one side of the line at a time.</p> <p>Consider the given example. First express both sides of the identity in terms of sine and cosine.</p> <p>Next, try to manipulate the left side of the identity until it is identical to the right side of the identity.</p> <p>Working on the left side of the equal sign, find a common denominator in order to add $\sin x/\cos x + \cos x/\sin x$. Combine like terms.</p> <p>You can use trig identities to simplify the expression. In this case the numerator, $\sin^2 x + \cos^2 x$, was replaced with the number 1 by the basic identity $\sin^2 x + \cos^2 x = 1$.</p> <p>Now that both sides of the identity are identical, the proof is finished and the stated identity is true.</p>
<p>remember: $\sin^2 x + \cos^2 x = 1$ so, $1 - \sin^2 x = \cos^2 x$</p> <p>Example $\tan x + \sec x = \frac{\cos x}{1 - \sin x}$</p> <p>steps:</p> <ol style="list-style-type: none"> Convert to sine and cosine Add Multiply to get a common denominator Foil $\begin{aligned} &= \frac{\sin x}{\cos x} + \frac{1}{\cos x} && \downarrow \\ &= \frac{\sin x + 1}{\cos x} \cdot 1 && \\ &= \frac{(1 - \sin x)(\sin x + 1)}{(1 - \sin x)(\cos x)} && \\ &= \frac{\cancel{\sin x} + 1 - \cancel{\sin^2 x} - \cancel{\sin x}}{(1 - \sin x)(\cos x)} && \\ &= \frac{\cos^2 x}{(1 - \sin x)(\cancel{\cos x})} && \\ &= \frac{\cos x}{1 - \sin x} &= \frac{\cos x}{1 - \sin x} \end{aligned}$	<p>Draw a line down the middle to separate the equality into two expressions.</p> <p>Change $\tan x + \sec x$ to an expression in terms of sine and cosine. Add like terms.</p> <p>Next, multiply by a well-chosen fraction equal to 1 and simplify.</p> <p>Both sides are the same; therefore the identity must be true.</p>