Finding the Value of a Logarithmic Function

- When calculating logs, think of the expression in terms of the exponential form. Ask yourself "what exponent would I need to make this statement true."
- A log written without a "b" value is the **common log**.
  y* = log*x
  In a common log, the "b" value is understood to be 10. In other words, $y = \log x$ is equivalent to the statement $10^y = x$.

More log Problems Example solve the log $\log_6 36 = 2$ $6^? = 36$	To solve logs, convert them to exponential form and see if you know what exponent you will need. 6 raised to what power is 36? $6^2 = 36$. The second power!
hard ~~More~~ log Problems Example solve the log $\log_4\left(\frac{\sqrt[3]{4}}{2}\right)$ write everything as $\sqrt{4}$ change roots to exponents $\boxed{\frac{\sqrt[3]{4}}{\sqrt{4}} = \frac{4^{1/3}}{4^{1/2}} = 4^{1/3-1/2} = 4^{-1/6}}$ $\log_4 \frac{\sqrt[3]{4}}{2} = \log_4 4^{-1/6} = ?$ $\Rightarrow 4^? = 4^{-t}$, so $? = \frac{-1}{6}$ $\boxed{\log_4 \frac{\sqrt[3]{4}}{2} = \frac{-1}{6}}$	In some problems, you will need to simplify pieces of the logarithm in order to see a good answer. You can't see what the question is asking for when the expressions aren't simplified. Make it your first priority to try to reduce them so you can work the problem. Notice that this radical expression simplifies to $4^{-1/6}$. Substituting that into the log problem makes it a lot easier: what exponent do you raise 4 to if you want $4^{-1/6}$? To –1/6!
really hard ~~More~~ log Problems solve the log $6^{\log_6 28}$ · express the exponential expression as a logarithmic expression $6^{\log_6 28} = ?$ $\log_6 ? = \log_6 28$ $? = 28$ $\boxed{6^{\log_6 28} = 28}$	In this problem, the log statement is the exponent. Look for a way to simplify things. The statement $\log_6 28$ equals some value: $\log_6 28 = ?$ or in exponential form: $6^? = 28$. With this information, you realize you are looking for the exponent for 6 that produces 28.

Finding the Value of a Logarithmic Function [Sample Problems]

1)

Find the value of the logarithmic expression $\log_2 32$ without using a calculator.
Solution: 5
Explanation: Express the logarithmic equation $\log_2 32 = x$ as an exponential equation: $2^x = 32$. To solve for x, express each side with a common base: $2^x = 2^5$, so $x = 5$. Thus, $\log_2 32 = 5$.

2)

Find the value of the logarithmic expression $\log_5 \frac{1}{625}$ without using a calculator.
Solution: -4
Explanation: Express the logarithmic equation $\log_5 \frac{1}{625} = x$ as an exponential equation: $5^x = \frac{1}{625}$. Express each side with a common base and solve for x.

$$\begin{aligned} 5^x &= \frac{1}{625} \\ 5^x &= 625^{-1} && \textit{Negative Exponent Property} \\ 5^x &= \left(5^4\right)^{-1} && \textit{Express 625 as a power of 5.} \\ 5^x &= 5^{-4} && \textit{Power of a Power Property} \\ x &= -4 \end{aligned}$$

3)

Find the value of the logarithmic expression $\log_{10} 0.0001$ without using a calculator.
Solution: -4
Explanation: Express the logarithmic equation $\log_{10} 0.0001 = x$ as an exponential equation using 10 as the base: $10^x = 0.0001$. To solve for x, express each side with a common base: $10^x = 10^{-4}$, so $x = -4$. Thus, $\log_{10} 0.0001 = -4$.

4)

Simplify. $15^{\log_{15} 0.8}$
Solution: 0.8
Explanation: Let the expression be equal to x. Then, apply the law of logarithms, $b^y = x \Rightarrow \log_b x = y$, to simplify: $15^{\log_{15} 0.8} = x \Rightarrow \log_{15} x = \log_{15} 0.8$. Because of the equality, it can be seen that $x = 0.8$. Thus, $15^{\log_{15} 0.8} = 0.8$.

5)

Find the value of the logarithmic expression $\frac{1}{4}\log_7 \sqrt[3]{7^2}$ without using a calculator.

Solution: $\frac{1}{6}$

Explanation: Express the logarithmic equation $\log_7 \sqrt[3]{7^2} = x$ as an exponential equation: $7^x = \sqrt[3]{7^2}$.
To solve for x, express the radical as a rational exponent.

$$7^x = \sqrt[3]{7^2}$$

$$7^x = \left(7^2\right)^{\frac{1}{3}} \quad \textit{Express the radical as a rational exponent.}$$

$$7^x = 7^{\frac{2}{3}} \quad \textit{Power of a Power Property}$$

$$x = \frac{2}{3}$$

Now, simplify the expression: $\frac{1}{4}\log_7 \sqrt[3]{7^2} = \frac{1}{4}\left(\frac{2}{3}\right) = \frac{1}{6}$.

6)

Find the value of the logarithmic expression $\log_8\left(\frac{\sqrt[5]{16}}{2}\right)$, without using a calculator.

Solution: $-\frac{1}{15}$

Explanation: Begin by simplifying the expression inside the parentheses:

$\frac{\sqrt[5]{16}}{2} = \frac{16^{\frac{1}{5}}}{2} = \frac{\left(2^4\right)^{\frac{1}{5}}}{2} = \frac{2^{\frac{4}{5}}}{2} = 2^{\frac{4}{5}-1} = 2^{-\frac{1}{5}}$. So, $\log_8\left(\frac{\sqrt[5]{16}}{2}\right) = \log_8\left(2^{-\frac{1}{5}}\right)$.

Express the logarithmic equation $\log_8\left(2^{-\frac{1}{5}}\right) = x$ as an exponential equation: $8^x = 2^{-\frac{1}{5}}$.

To solve for x, express each side with a common base: $2^{3x} = 2^{-\frac{1}{5}}$; $3x = -\frac{1}{5}$; $x = -\frac{1}{15}$.