

Alternating Series

- An **alternating series** is a series whose terms alternate between positive and negative values.

The alternating series test states that an alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges if the a_n values are positive, decreasing, and approaching zero.

Combining radicals and transcendental functions

EXAMPLE Determine whether $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2\ln n}}$ converges or diverges.

LIMIT $\lim_{n \rightarrow \infty} \frac{1/\sqrt{n}}{1/(\sqrt{n+2\ln n})} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+2\ln n}}{\sqrt{n}}$ Invert and multiply.

$= \lim_{n \rightarrow \infty} \left(\frac{\sqrt{n}}{\sqrt{n}} + \frac{2\ln n}{\sqrt{n}} \right)$ Separate the fractions.

$= 1 + \lim_{n \rightarrow \infty} \frac{2\ln n}{\sqrt{n}} \rightarrow \frac{\infty}{\infty}$

$= 1 + \lim_{n \rightarrow \infty} \frac{2/n}{1/(2\sqrt{n})}$ Use L'Hôpital's rule.

$= 1 + \lim_{n \rightarrow \infty} \frac{4\sqrt{n}}{n}$ Invert and multiply.

$= 1 + \lim_{n \rightarrow \infty} \frac{4}{\sqrt{n}}$ Simplify.

$= 1 + 0 = 1$

Compare the series to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ which diverges because it is a p -series with $p \leq 1$.

$\frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$

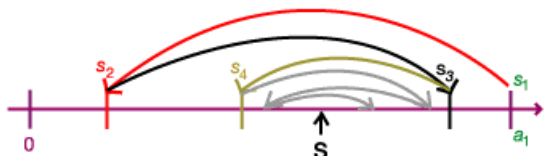
Alternating series get their name from the fact that the terms alternate between being positive and negative.

Raising (-1) to the $n+1$ th power makes it alternate between -1 and $+1$. Make sure that a_n is positive, though.

Q. How do you know if an alternating series converges?

Think about the sequence of partial sums.

$$\begin{aligned}
 s_1 &= a_1 \\
 s_2 &= a_1 - a_2 \\
 s_3 &= a_1 - a_2 + a_3 \\
 s_4 &= a_1 - a_2 + a_3 - a_4
 \end{aligned}$$

For any kind of series, the important information to know is whether it converges or diverges.

Suppose each a_n is smaller than the one before and the signs are alternating. That means that you are either subtracting less than the current sum, or you are adding less than was just subtracted.

Alternating between adding and subtracting means that the partial sums begin to approach some value S . Thus, the series converges if a_n is approaching zero. This is the basis for the **alternating series test**.

The alternating harmonic series

Example! $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \dots$

Q. Does the series converge?

Using the alternating series test, look at $\left\{ \frac{1}{n} \right\}$.

- The terms are all positive, $\frac{1}{n} > 0$.
- The terms are decreasing, $\frac{1}{1} > \frac{1}{2} > \frac{1}{3} > \frac{1}{4} \dots$
- The limit is 0, $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$.

a. Yes, the series converges!

This series, called the alternating harmonic series, consists of the terms of the harmonic series with alternating signs.

There are three steps you have to take when using the alternating series test.

- Check the sign of the non-alternating part of the terms.
- Verify that the non-alternating part is decreasing.
- Make sure the non-alternating parts are approaching zero.

Since all the conditions are met, the series converges by the alternating series test.