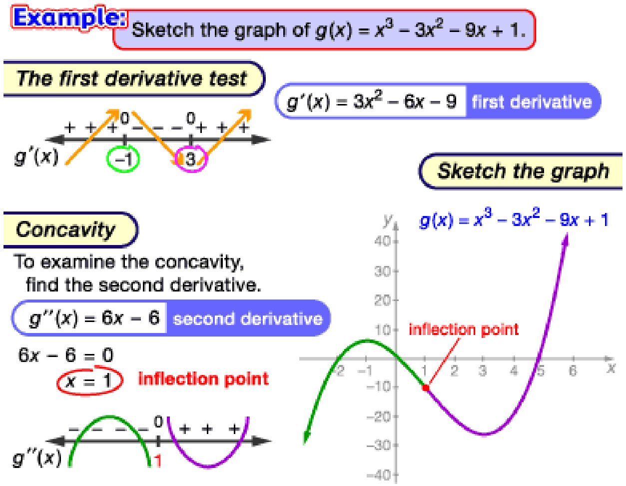
Using the Second Derivative to Examine Concavity

- The second derivative can be used to determine where the graph of a function is **concave up** or **concave down**, and to find **inflection points**.
- Knowing the critical points, local extreme values, increasing and decreasing regions, the **concavity**, and the inflection points of a function enables you to sketch an accurate graph of that function.



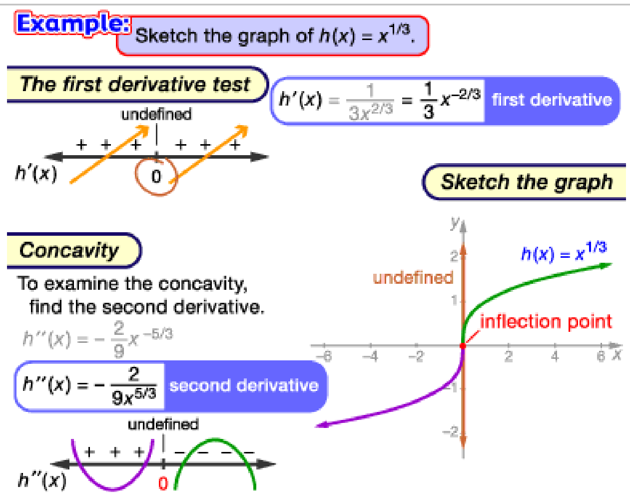
Knowing the **concavity** of a function can help you make a better sketch of its curve. Recall that the graph of a function is **concave up** if the derivative is increasing, and **concave down** if the derivative is decreasing.

To determine the behavior of the derivative, you will need its derivative, i.e. the second derivative of the function.

Set the second derivative equal to zero to determine possible **inflection points**, which are characterized by a change in concavity.

Then make a sign chart for the second derivative. If the second derivative is negative, then the derivative is decreasing and the function is concave down. Similarly, if the second derivative is positive, then the derivative is increasing and the function is concave up.

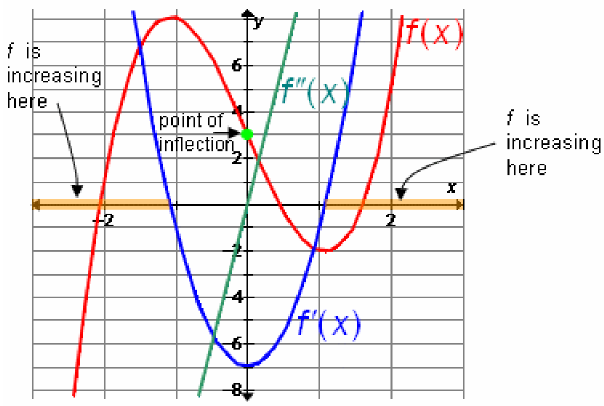
Since the concavity changes at the point where the second derivative equals zero, it is an inflection point, after all.



Once again, take the second derivative of the function in order to determine its curvature.

The second derivative is never equal to zero, but it is undefined at $x = 0$. An inflection point might exist there.

Make a sign chart for the second derivative. Since the function changes from concave up to concave down at $x = 0$, it is an inflection point. Notice that the tangent line at $x = 0$ has an undefined slope.



f is increasing where $f'(x) > 0$.
f is decreasing where $f'(x) < 0$.
f is concave up for $x > 0$ because $f''(x) > 0$ for $x > 0$.
f is concave down for $x < 0$ because $f''(x) > 0$ for $x < 0$.
f has a point of inflection at $x = 0$, where $f''(x) = 0$.

For the function *f* shown in red, examining the graph of its first and second derivatives can lead to knowledge about where the function is increasing and decreasing (first derivative), where it is concave up and concave down, and where there is an inflection point.

The x values where the first derivative is positive (the graph of f' lies above the x-axis) are the x values where f is increasing. Similarly, the x values where the first derivative is negative (the graph of f' lies below the x-axis) are the x values where f is decreasing.

The x values where the second derivative is positive (the graph of f'' lies above the x-axis) are the x values where f is concave up. Similarly, the x values where the second derivative is negative (the graph of f'' lies below the x-axis) are the x values where f is concave down.

At $x = 0$, $f''(x) = 0$, and $x = 0$ is a point of inflection, since f is concave down to the left of $x = 0$ and concave up to the right of $x = 0$.