On Cloud Nine®

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Visualizing and Verbalizing® for Math

This is a sample of the *On Cloud Nine®* *Manual*. For more information about the *On Cloud Nine* math program or the *On Cloud Nine Manual*, please visit GanderPublishing.com.

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2

Imagery: The Sensory-Cognitive Connection for Math

Why can't everyone think with numbers? What cognitive processes do some have that others do not? Some individuals easily understand the concepts underlying math processes. They quickly perform math calculations, mentally or on paper, and have an innate sense of whether or not an answer is correct. Math is their friend, something dependable and logical in the world. But for others math is an illogical enemy, filled with random memorization. It's a dragon rearing its head at inopportune times, ready to embarrass or diminish them in the eyes of others. For them, numbers and calculations can be a vast array of steps to memorize. However, for individuals who "get math," the language of numbers turns into imagery and they "see" mathematical relationships. They use an internal language, with imagery, that lets them calculate and verify mathematics, and see its logic.

Mathematics is cognitive processing, thinking, that requires the dual coding of imagery and language. Imagery is fundamental to the process of thinking with numbers. Albert Einstein, the man who illuminated entire aspects of our universe through the theory of relativity, used imagery as the basis for his mental processing and problem solving. Perhaps he summarized imagery's importance best when he said, "If I can't picture it, I can't understand it." Imaging is the basis for thinking with numbers, their functions, and their logic. Even early thinkers recognized the importance of imagery to math. The Greek philosopher Plato said, "And do you not know also that although they [mathematicians] make use of the visible forms and reason about them, they are thinking not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameter . . . they are really seeking to behold the things themselves, which can be seen only with the eye of the mind?"

The relationship of imagery to the ability to think is one of the preeminent theories of human cognition, the Dual Coding Theory (DCT). Allan Paivio, a cognitive psychologist and father of DCT, stated this, "Cognition is proportional to the extent that the coding mechanisms of mental representations (imagery) and language are integrated." Research through the 1970s, 1980s and into the 1990s has validated DCT as a model of human cognition and its practical, as well as theoretical, application to the comprehension of language (Bell, 1991). And mathematics is the essence of cognition.

In the January 15, 1880, issue of the journal *Nature*, Francis Galton, a British anthropologist, published a curious little research finding. As part of a general study of mental imagery, Galton had friends and acquaintances fill out a questionnaire asking them to report on whether they could "see" numbers and if so in what way. Some of them could, it seemed, and usually the numbers were arranged along a line, or a series of lines, that got progressively less distinct and sometimes vanished into the mental distance as the numbers got larger. Sometimes the numbers had color or texture. A very visual male philosopher reported that his numbers had a great deal of personality: "9 is a wonderful being of whom I felt almost afraid…, 8, I took for his wife…6, of no particular sex but gentle and straight-forward…" Galton himself considered his results a curiosity—interesting mainly for the extent to which they showed the tendency of mental traits to run in families.

Not until 1967 did another article in *Nature* provide evidence that all of us have a mental number line of sorts, even those of us for whom it may not be vivid. Two Stanford University psychologists, Robert Moyer and Thomas Landauer, measured the time it took a person to choose the concept of "larger" between two single typed numerals by flipping either a left-hand or right-hand switch. They found that the smaller the difference between the concept of two numbers, the longer it took, such as deciding between 6 and 7 as opposed to 1 and 9. This "distance effect" has been verified again and again, and suggests strongly that the brain converts the numbers into analog magnitudes—line segments, for instance—before comparing them. Choosing between two lines is obviously harder the closer in length they are.

Stanislas Dehaene and Laurent Cohen, neurologists in Paris, have been recently researching math and the brain, and in a series of papers have sketched a rough model of how the brain processes numbers and does simple arithmetic through imagery. All of their evidence, mostly done with brain-lesion patients, shows that the "elementary ability to perceive and manipulate numbers is part of our evolutionary heritage—something we're born with." One of their patients showed that the ability to grasp the meaning of numbers, by translating them into an approximate analog representation of quantity, and the ability to calculate precisely were two different processes occurring at least in part in different regions of the brain.

Math is thinking (dual coding) with numbers, imagery, and language; reading/spelling is thinking with letters, imagery, and language. Both processes require the integration of language and imagery to assist in the foundational and application processes. Dual coding in math, just as in reading, requires two aspects of imagery: symbol/numeral imagery (parts/details) and concept imagery (whole/gestalt). Perhaps the two imagery systems do reside in slightly different areas of the brain.

Numeral Imagery

Visualizing numerals is one of the basic cognitive processes necessary to understanding math. For example, we image the numeral "2" for the concept of two. When we see the numeral "3," we know that it represents the concept of three of something—three pennies, three apples, three horses, three dots. If someone gives us two pennies for the numeral three, we have a discrepancy between our numeral-image for three and the reality of three. The first imagery needed for math is the symbolic or numeral imagery that represents the reality of a number concept.

What does numeral imagery look like? Here's one example. Cecil was very good in math, could think with numbers, arrive at answers in his head, and easily mentally check for mathematical discrepancies in finance or life situations. When he was asked how he could do this, he gave an easy, quick answer that related his math ability to imagery. "I just visualize numbers and their relationships, and certain numbers are in certain colors, and the number line in my head goes specific directions." Cecil could visualize both numerals and concepts, both types of imagery, but the most unusual was his color imagery—he had assigned colors to specific numbers!

"What color is the number 14?" he was asked.

His eyes went up, and in all seriousness, he said, "Light blue."

Puzzling. "Well, how about the number 3?"

Cecil, eyes up again, said, "Reddish pink."

"How about the number 88?"

Cecil, smiling, eyes up, said, "That one is kind of a purple."

Thinking he might have made them up on the spot, a check months later revealed the same colors! Though Cecil may have experienced other areas of difficulty in his life, he was a wizard at card games and math. Just as easy as breathing, he could compute math mentally, though he only had a third grade education. He saw his numbers in a certain linear pattern of straight to the left. His son, Rod, who graduated in math from college, had similar imagery for math. Though Rod's numerals weren't in a specific color, his number line had a definite pattern in his head, with turns and twists providing him with an imagery base for the numerals. Asked what he used it for, he said, "I don't know, and I don't know how it got there, but it exists at an unconscious level. It is a representation of number relationships that are a part of my internal math structure." Why some have innate imaging ability, and others do not, indeed may be related to a genetic propensity, just as Galton suspected.

Chronological relationships appear in our mind for a number line, the days of the week, the months in the year. *Imagery—both numeral and concept—is our sensory system's way of making the abstract real.* It is a means to vicariously experience math.

Concept Imagery

While imaging numerals is important to mathematical computation, another aspect of imagery is equally as important: *concept imagery*. Understanding, problem solving, and computing in mathematics requires the ability to process the gestalt (whole)—another form of imagery. Sometimes children or adults can visualize numerals, the parts, but can not bring those parts to a whole; just as they can sometimes visualize words but not bring those words to a whole to form concepts. Mathematical skill requires the ability to get the gestalt, see the big picture, in order to understand the process underlying mathematical logic.

"Concept imagery is the ability to image the gestalt (whole)," Bell (1991). Concept imagery is a primary factor basic to the process involved in oral and written language comprehension, language expression, and critical thinking. It is the sensory information that connects us to language and thought. In

On Memory and Recollection, Aristotle wrote, "It is impossible even to think without a mental picture." And Thomas Aquinas wrote, "Man's mind cannot understand thoughts without images of them."

However, many individuals have weakness in creating mental images and thereby have weakness in reading comprehension, oral language comprehension, expressive language, following directions, and logical thinking. Researchers in reading and imagery have produced direct evidence linking reading and mental imagery as well as studied the relationship of imagery to prior knowledge and thinking processes (Stemmler 1969; Richardson 1969; Paivio 1971, 1986; Marks 1972; Sheehan 1972; Levin 1973, 1981; Pressley 1976; Sadoski 1983; Kosslyn 1983; Tierney and Cunningham 1984; Peters and Levin 1986).

While weakness in imagery may cause problems in reasoning and math, strength in imagery is a foundation for math. Cecil, like many others for whom math is easy, will tell you they calculate math quickly because they "see" relationships. They use an internal imaged number line and integrate numeral and concept imagery for simple and complex problem solving. "Forty-six plus seven is easy because it takes four to get to fifty and that leaves three, so the answer is fifty-three." For them, the language of word problems is easily converted to mathematical solutions.

Whether math skill is a genetic gift or not—and the answer is not the purpose of this book—imagery can be developed and applied to math with the formula: *concrete experiences to imagery to computation*.

On Cloud Nine: Concrete to Imagery to Computation

While imagery is the link to mathematical processing, math's roots are in the realm of the concrete. Numbers can be experienced and the relationship of those numbers can be concretized (made concrete) by using manipulatives. What appears abstract, numbers (squiggles) that work together, can be experienced and imaged to concreteness. Indeed, because of its concrete roots, math can be safer than decoding, spelling, or language comprehension.

The *On Cloud Nine* math program moves through three basic steps to develop mathematical reasoning and computation: (1) manipulatives to experience the realness of math, (2) imagery and language to concretize that realness in the sensory system, and (3) computation to apply math to problem solving.

Concrete
to
Imagery
to
Computation

Concrete experiences, manipulatives, have been used for many years in teaching math (Stern, 1971). However, children and adults have often experienced success with manipulatives, but failure in the world of computation (NCTM, 1989; Moore, 1990; Papert, 1993). They had what has often been described as "application problems."

However, manipulatives can be used to concretize imagery which can then be applied to computation. Imagery is the link. Arnheim (1966) wrote, "Thinking is concerned with the objects and events of the world we know...When the objects are not physically present, they are represented indirectly by what we remember and know about them. In what shape do memory and knowledge deliver the needed facts? In the shape of memory images, we answer most simply. Experiences deposit images." *On Cloud Nine* manipulatives deposit images. They concretize numbers and mathematical concepts, and place that realness into imagery as a base to draw from for computation.

To bring concept and numeral imagery to a conscious level as the missing link in math instruction, *On Cloud Nine* integrates and applies the principles of Bell's programs: *Visualizing and Verbalizing for Language Comprehension and Thinking (*V/V*)* and *Seeing Stars: Symbol Imagery for Phonemic Awareness, Sight Words, and Spelling (*SI*)*. As individuals become familiar with the concrete manipulatives, they are *questioned* and *directed* to consciously transfer the experienced to the imaged. They image the concrete and attach language to their imagery. The integration of imagery and language is then applied to computation. They develop the sensory-cognitive processing to understand and use the logic of mathematics in mental and written computation.