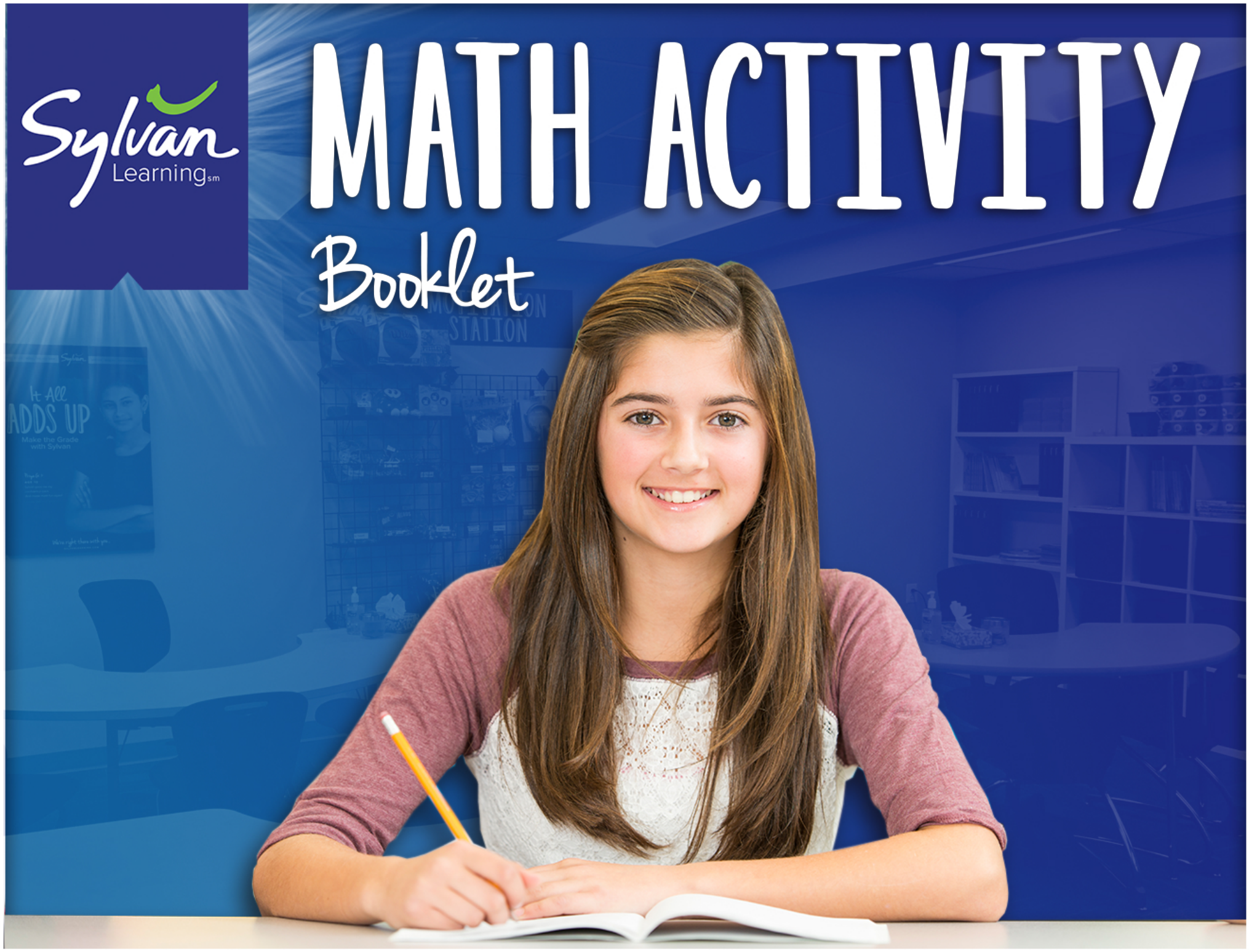
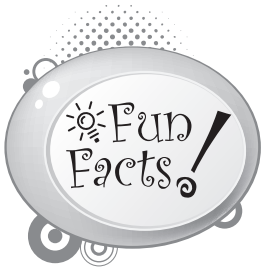
Sylvan Learning sm

MATH ACTIVITY

Booklet



Write five different ways you used math today. For example, did you help your dad do the grocery shopping? Did you have to buy something at the store?



The billionth digit of π is 9.

Daily Math Puzzle

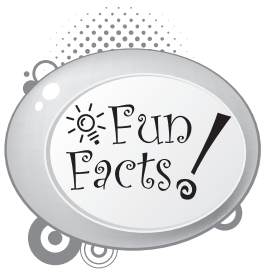
Window Project:
Grades 4–6

Your mom has decided to buy a stained glass window. The artist says that she charges $150.00 per square foot. Your mom wants a square window that is 24 inches on each side. How much will the stained glass window cost?

Solution:

Take a walk around your home or the neighborhood in which you live. Create two math problems based on what you see in your home or neighborhood. For example you might ask:

- How far is it from your kitchen to your bedroom?
- There are six park benches and each one can hold three people.
  How many people can sit on these benches?



The French scientist, Blaise Pascal, has been credited with inventing the very first digital calculator. In 1642, the 18-year old Pascal, son of a French tax collector, invented his numerical wheel calculator called the Pascaline to help his father calculate taxes. The Pascaline had eight movable dials that added up to eight-figure long sums and used base ten. When the first dial (ones column) moved ten notches—the second dial moved one notch to represent the tens column reading of ten—and when the ten dial moved ten notches, the third dial (hundreds column) moved one notch to represent one hundred and so on.

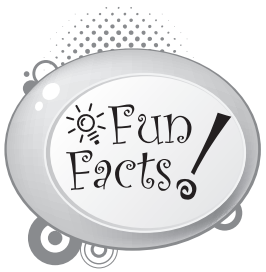
Daily Math Puzzle

Juan's Race:
Grades 6–8

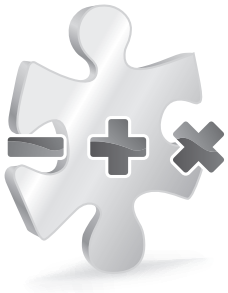
Juan can run 4 km in 30 minutes. At this speed, how far can Juan run in 45 minutes?

Solution:

Write a description of a movie you saw that used or talked about math.



Once upon a time, before calculators or computers existed, humans used a calculating device called the abacus. Abaci, which is plural for abacus, date back to 600 B.C.E. in central Asia and were usually made from wood. The oldest surviving abacus, also called a counting board, is known as the Salamis Tablet, which was used by Babylonians in 300 B.C.E. Today, the abacus is still used in many Japanese schools.



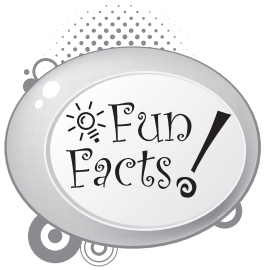
Daily Math Puzzle

Rocket Math:
Grades 9–12

The massive Saturn V rocket was used in the historic Apollo II mission that landed a man on the moon. Suppose that an observer on the ground wants to measure the height of the Saturn V rocket. The observer is at a distance of one kilometer (1000 meters) from the launch pad and measures that the angle between the ground and the top of the rocket is 6.3 degrees. What is the height of the Saturn V rocket?

Solution:

Look around your home and find any board games that you recently played with your friends. What is your favorite board game? Write about your favorite board game and describe how it involves math.



A cake can be divided into eight pieces by making three cuts. All you need are two vertical cuts and one horizontal cut. Try it.

Daily Math Puzzle

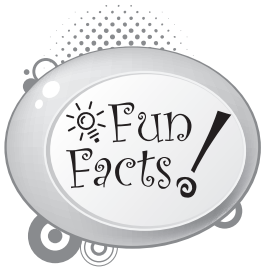
More Stained Glass:

Grades 6–7

Your dad has said he likes the idea of having a stained glass window but he would like a round window. How much will a 24 inch-diameter circular window cost if the artist charges $150.00 per square foot?

Solution:

Think of two careers that are of interest to you. Describe each of these careers, and explain why you would need math to be successful in each one.



A 52-card deck of playing cards contains 2,598,960 possible five-card hands.

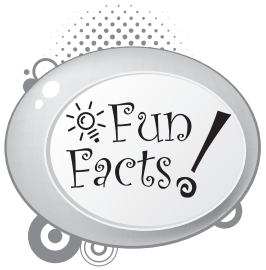
Daily Math Puzzle

Fraction Sums:
Grades 5–7

Add: $\frac{1}{2} + \frac{2}{3}$

Solution:

Imagine a day without math. How would you know how much you could buy with your allowance? How would you tell time? Write about a day without math and how you would do things differently.



If you add the numbers from one to one hundred inclusive $(1 + 2 + 3 + \cdots + 99 + 100)$, the total is 5050.

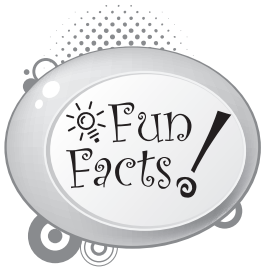
Daily Math Puzzle

Solar System Math:
Grades 9–12

The distance from Earth to the Sun is 1.5×10^{11}m, and the distance from Earth to the Moon is 3.8×10^{8} m. If the Sun and Moon form a 90 degree angle at the Earth, what is the approximate distance from the Sun to the Moon?

Solution:

Think about your allowance. If mom gave you a 20% raise for one month, write about what you would do with the extra money.



A *jiffy* is an actual unit of time that describes $\frac{1}{100}$ of a second.

Daily Math Puzzle

Approximation Action:
Grades 5–8

Estimation is the act of finding an approximate answer, not an exact answer.

(a) Round 512,143 to the hundreds place.

(b) Round 318,205 to the tens place.

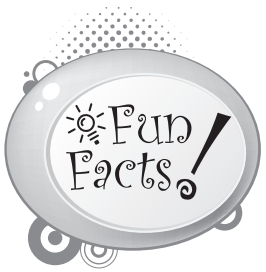
(c) Round 258,723 to the hundred thousands place.

(d) Round 43,602 to the thousands place.

(e) Round 260,211 to the tens place.

Solution:

Tonight, before you go to bed, look at the sky and check the stars. How many different shapes you can see by connecting the stars? Do you remember the names of the different triangles and quadrilaterals? Write about the different shapes you see.



In many languages the word for "inch" is also the word for "thumb" and an inch is roughly the width of a man's thumb. In the fourteenth century, King Edward II of England ruled that one inch equals three grains of barley placed end to end.

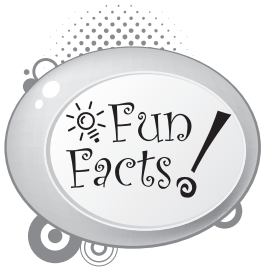
Daily Math Puzzle

Common Multiples:
Grades 6–8

Find the least common multiple of 8 and 12.

Solution:

Talk to your mom and dad and find out how they use math while they are at work. Describe their jobs and explain how they use math.



An inch of snow that falls evenly across one acre of ground is equivalent to about 2,715 gallons of water.

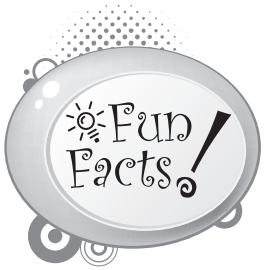
Daily Math Puzzle

Spin Speed:
Grades 9–12

The radius of the earth is (6.3×10^6)m. How fast is a point on the equator moving, given that the earth makes one full revolution in 24 hours?

Solution:

Help your mom or dad make dinner. Pay attention to how math helps you with recipes and measurements. Write about how you used math to make dinner.



If each count took about one second to complete, it would take about 12 days to count to a million and about 32 years to count to a billion.

Daily Math Puzzle

Mean, Mode, Median:
Grades 4–6

The *mean* is another term for the average found by adding a set of numbers and dividing by the quantity of numbers there are in the set.

The *median* is the number in the middle of the set of ordered numbers. To find the median, write the values in order from lowest to highest. Then find the number that is exactly in the middle.

The *mode* is the value that occurs most often in the set.

10, 40, 75, 55, 70, 75, 75, 100, 55, 65, 25

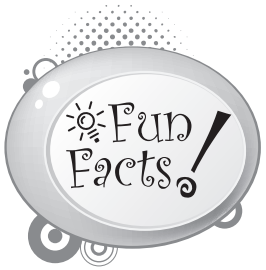
(a) Determine the mean of the given number set.

(b) Determine the median of the given number set.

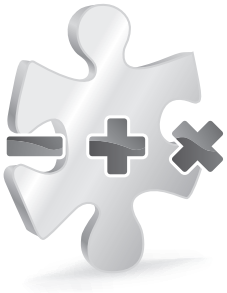
(c) Determine the mode of the given number set.

Solution:

Think about the last test or assignment your teacher returned to you. How would your grade change if you had gotten one more answer wrong? Write about how your teachers use math to do their jobs.



In 1202 Leonardo Pisano, an Italian Mathematician better known as Fibonacci, published an algebra text titled *Liber abaci.* This text contains a now famous problem involving the growth of a population of rabbits. The solution to this problem is called the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …). The Fibonacci sequence can be found in nature in the spirals of sunflowers, shells of the nautilus, and pinecones.



Daily Math Puzzle

All Things Equal:
Grades 6–8

Solve for x:

(a) $6 = x - 2$

(b) $-3x - 5 = 7$

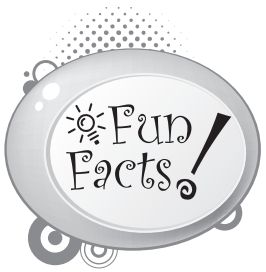
(c) $-106 = 4x + 6x + 4$

(d) $4(7 + 6x) = -260$

(e) $-6x - 3 = 5x - 47$

Solution:

Think about a favorite toy that you would like to buy in a store. If it is on sale for 20% off, how much money would you save? Write about how you use math when you are shopping.



Over two hundred years ago, a young boy, Carl Friedrich Gauss (who grew up to be a great mathematician) applied mathematical patterns to solve a problem after he and his classmates were scolded for misbehaving. Their teacher, hoping to keep Gauss and his classmates quiet for a while, told his students to add all of the numbers from one to one hundred and put the answer on his desk. He thought the assignment would keep them busy for an hour or so. About 30 seconds later, the ten-year-old Gauss tossed his slate (a small chalkboard) onto the teacher's desk with the answer "5050" written on it. The teacher, amazed, asked him how he came up with the answer so quickly. Gauss explained that he noticed that if he added one to one hundred he got 101; he observed the same thing when he added two to 99 and so on until he got to 50 + 51. That's 50 pairs of 101. So he just multiplied 101 by 50 to get 5050.

Daily Math Puzzle

Fraction Fun:
Grades 6–8

Solve:

(a) $\frac{3}{10} + \frac{4}{15} =$

(b) $\frac{11}{56} + \frac{3}{7} =$

(c) $\frac{2}{7} \times \frac{3}{8} =$

(d) $\frac{2}{9} \times \frac{4}{9} =$

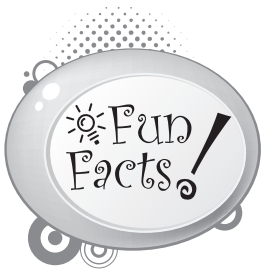
(e) $\frac{1}{8} \div \frac{1}{2} =$

(f) $\frac{2}{9} \div \frac{6}{8} =$

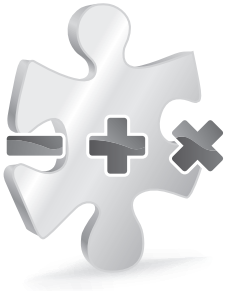
(g) $\frac{2}{3} \div \frac{3}{8} =$

Solution:

Imagine you are taking a trip to see your grandparents. Find out what city they live in and calculate the distance from your house to their house and the time it would take you to drive there if you traveled an average of 45 miles per hour.



Using fingers to perform math computations is called *Chisenbop*.



Daily Math Puzzle

Basic Trigonometry:
Grades 9–12

Right triangle trigonometry focuses on the relationship between an angle and its sides. If one value of a right triangle is known—whether a side length or an angle degree—you can determine the other values using the following trigonometric functions.

Note: Θ is used to represent an angle.

$$\sin\Theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos\Theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan\Theta = \frac{\text{opposite}}{\text{adjacent}}$$

(a) In a right triangle ABC, $m\angle B = 55°30'$, and $b = 6.05$. If c is a right angle, find c and a.

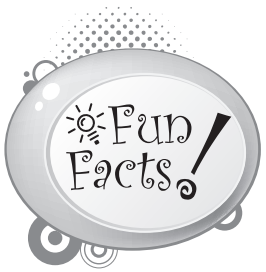
(b) Two sides are given for each of the following right triangles, compute the sin, cos, and tan of the $\angle A$ and $\angle B$. Express the results as common fractions.

I. $a = 9$, $c = 41$ II. $a = 35$, $c = 37$ III. $a = 24$, $b = 7$

(c) In a right triangle $c = 6.3$ and $\tan B = 1.2$. Find the length of sides a and b.

Solution:

Talk to your mother or father about how much your family spends on household expenses. If you were responsible, how would you track of all of these expenses?



Read this definition.

Algebra: From the 825 B.C.E. book, *ilm al-jabr w'al Maga balah* (translated "the science of cancellation and reduction") by the great Iranian Mathematician Mohammed Ibn Musa al-Khowarizmi. After years of bad pronunciation by Europeans, the word became known as *aljabra* and eventually, algebra.

Daily Math Puzzle

Pythagorean Theorem Practice:

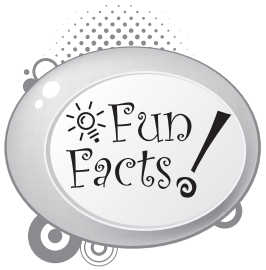
Grades 9–12

The Pythagorean Theorem has many uses. You can use it to verify whether a triangle is a right triangle, or you can use it to find the missing measures of sides. The Pythagorean Theorem states that $a^2 + b^2 = c^2$, where a and b are the two legs, and c is the hypotenuse of the right triangle.

Determine if the numbers $a = 9$, $b = 12$, and $c = 15$ forms a right triangle.

Solution:

Firefighters use math to do their job. Write about how you think they use math at work.



There is no numerical value for the word *zillion*. Look it up in any dictionary! It is simply an indefinite, very large number.

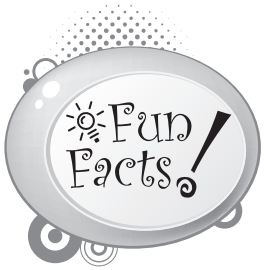
Daily Math Puzzle

Mathematics of Grades:
Grades 9–12

John wants to score at least 80% in his English class. John's teacher has already given three assignments and will be giving one final assignment. John has received the following marks on his assignments, $\frac{15}{20}$, $\frac{17}{25}$, and $\frac{26}{30}$. If the final assignment is worth 25 points, how many points must John earn to have a final grade of 80%.

Solution:

Write about something you enjoy when you work on math homework.



In the American counting system, each time three zeros are placed behind a number, the number name changes. So, a one followed by three zeros is one-thousand, not ten hundred.

Daily Math Puzzle

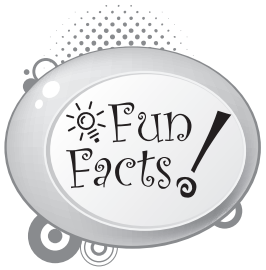
The Big One:
Grade 3 and up

Steps:

1. Think of a number.
2. Add 3 to the number.
3. Multiply that sum by 2.
4. Next, subtract 4 from your last answer.
5. Divide your current answer by 2.
6. Subtract the number you started with from that quotient.
7. Your answer will always be 1.
8. Verify that this process also works for the numbers 4 and 15.

Solution:

Use an online search engine to locate the name of a famous mathematician. Write about his or her life and something important he or she did to help develop the subject of mathematics.



The *googol* is a number followed by 100 zeros or 10^{100}. The *googolplex* is the number followed by a googol of zeros.

Daily Math Puzzle

How Much Loose Change?

Grades 4 and up

Tell a friend that you can tell how much change he has in his pocket if he will show you the final answer to these steps.

Sample: Suppose he counts the change and has 56 cents.

Tell him to:

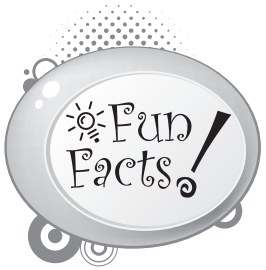
1. Multiply the amount of change he has by 2.
2. Add 3 to the previous product.
3. Multiply the previous sum by 5.
4. Subtract 6 from the last product.

His change will be the first two digits of this answer.

Verify that this process also works for 24 cents.

Solution:

Have you ever gone to the drugstore with your mom or dad to buy cough medicine? The person behind the counter who gives you your medicine is a pharmacist. How does a pharmacist use math? Write about two ways.



Zero is considered an even number.

Daily Math Puzzle

Age and Money Trick:

Grades 5 and up

Tell a friend that with this trick you can determine her age and how much change (less than one dollar) she is carrying. For example, suppose her age is 15 and she has 35 cents in change.

Tell her to:

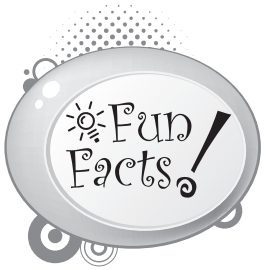
1. Multiply her age by 4.
2. Add 10 to the product.
3. Multiply the sum by 25.
4. Subtract the number of days in a year, 365, from this product.
5. Add to the total the change she has.
6. Add 115 to the previous total.

The first two digits in the answer are her age (15). The second two digits are the amount of change (35).

Verify that this process also works for an age of 20 and change totalling 24 cents.

Solution:

How would you use math if you were helping your parents in the garden? For example, if you have 12 different flowers to plant and a limited amount of garden space, how far apart should the plants be spaced? Explain how you would use math in the garden.



The distance from home plate to pitcher's box is 60 feet and 6 inches.

Daily Math Puzzle

When is your Birthday?

Grades 4 and up

You can tell a person's birthday if he follows these steps: (For the example, supposed his birthday is July 10)

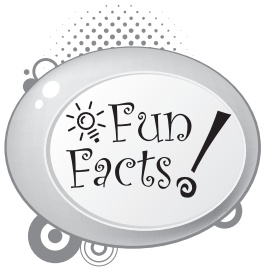
1. Multiply the number of the month by 5.
2. Add 7 to the product.
3. Multiply the sum by 4.
4. Add 13 to the answer.
5. Multiply the last sum by 5.
6. Add the day of the month to your answer.

Have the person tell you his answer to step six. Mentally subtract 205. The first digit(s) shows the month, and the last two digits show day of the month. It always works.

Verify that this process also works for a birthday on January 7th.

Solution:

Write about a book you read that used or talked about math.



In ancient times, the foot was 11 $\frac{1}{42}$ inches. Today it is 12 inches, the length of the average man's foot.

Daily Math Puzzle

What's Your Address?

Grades 5 and up

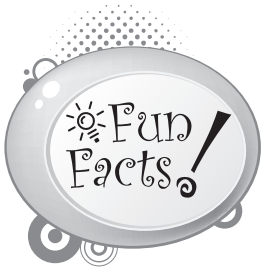
1. Double your house number.
2. Add the number of days in a week to your answer.
3. Multiply that sum by 50.
4. Add your age to the total.
5. Subtract the number of days in a year from your last result.
6. Add 15 to your answer.

The tens and units digit of the answer are your age; the other digits are your house number. Try it.

Verify that this process also works for an age of 40 and a house number of 2701.

Solution:

What's your favorite sporting event to watch? As you watch the event, write about how the athletes and coaches use math to do their job.



Any number, squared, is equal to one more than the product of the numbers on either side of it. For example, $5^2 = 25$ and $4 \times 6 = 24$.

Daily Math Puzzle

The Lost Dollar:

Grades 8–12

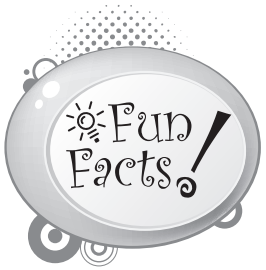
Mrs. Bea Fuddled had $50 in her bank account. Below is a list of her withdrawals and the corresponding balances.

1st withdrawal	$20	Balance	$30
2nd withdrawal	$15	Balance	$15
3rd withdrawal	$9	Balance	$ 6
4th withdrawal	$6	Balance	$ 0
Totals	$50		$51

Why the difference, and what happened to the extra dollar? Can you solve this one?

Solution:

Write about your favorite kind of math.



If 111,111,111 is multiplied by itself, the result is all of the digits in ascending to descending order, or 12,345,678,987,654,321.

Daily Math Puzzle

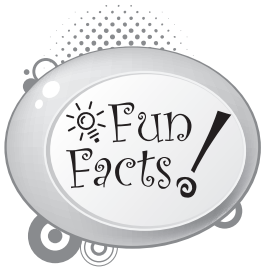
Book Sale:
Grades 6–12

You are posed with the problem of buying exactly 100 books for exactly $100.

Books may be bought for prices of $10 each, $3 each, or 50¢ each. How are you going to solve this problem?

Solution:

Write about ways to make your math homework more fun. For example, would you count with jellybeans instead of your fingers or a calculator? Would you write problems using characters from your favorite story or TV show?



If you have three quarters, four dimes, and four pennies, you have $1.19. You also have the largest amount of money in coins possible without being able to make change for a dollar.

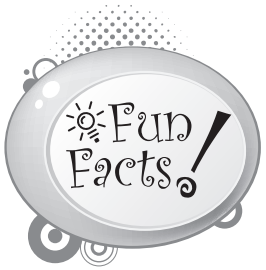
Daily Math Puzzle

Life or Death:
Grades 7–12

An old king, wishing to be rid of one of his advisors, put two pieces of paper in a hat. He told a judge present that if the advisor drew out the scrap marked "Death," he must die. The old king had written "Death" on both scraps of paper. However, when the crafty advisor showed the judge one piece of paper, the judge decided in his favor. How did the advisor outwit the old king?

Solution:

Help your friend with her homework. Write the directions to solving one of the math problems you have. Take turns and work together to solve all of the problems.



A three dimensional object with 20 faces or sides is called an icoshedron.

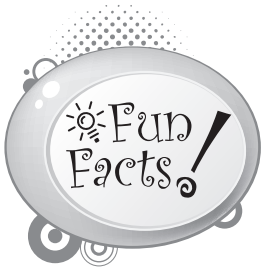
Daily Math Puzzle

Frustrated Frog:
Grades 6–12

A well is ten feet deep. A frog climbs up five feet during the day but falls back four feet during the night. Assuming that the frog starts at the bottom of the well on day one, on which day does he reach the top?

Solution:

What would the world look like without numbers? Write about some things that would be different. For example, your house wouldn't be number 86 on the block—it might be Yellow Square.



The oldest known unit of measurement (mentioned in the Bible when Noah was building the ark) is the *cubit*—the distance from the elbow to the thumb knuckle.

Daily Math Puzzle

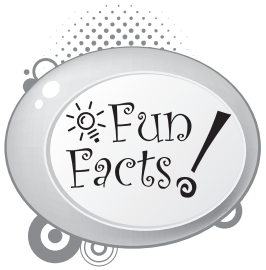
To Be or Not To Be:
Grades 9–12

You are in a room with two doors and two judges. Walk out of one door, and you will win a grand prize. Walk out the other, and you forfeit all winnings. One judge always tells lies. One judge always tells the truth.

If you could ask one judge one question to help get out of the room safely, what would you ask?

Solution:

April is Mathematics Education Month. In what ways could you celebrate the month of April?



A stack of $1 bills one mile high would be worth 14 million dollars.

Daily Math Puzzle

Send More Money:
Grades 6–12

The following cryptic message is one of the most widely-known mathematical puzzles. The object is to determine the digits represented by the letters.

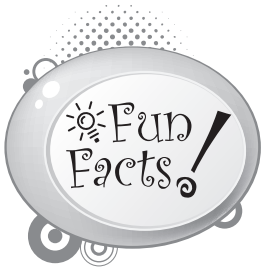
1. Assign some values for the letters in the words *SEND* and *MORE*. Try to identify patterns.
2. Write the basic facts where the addends are different and the sum is no more than 10.
3. Keep track of which digits you have used.
4. Try this easier problem. It has more than one solution.

```
 SEND
+MORE
MONEY
```

```
 OND
+SIX
 777
```

Solution:

Describe how you think a sportscaster on television would use math during her day.



The expression "a hairbreadth away" actually measures $\frac{1}{48}$ of an inch.

Daily Math Puzzle

7–11 Story Problem:

Grades 8–12

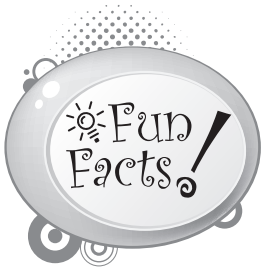
A guy walks into a 7–11 store and selects four items to purchase. The clerk at the counter informs the gentleman that the total cost of the four items is $7.11. He was completely surprised that the cost was the same as the name of the store.

The clerk informed the man that he simply multiplied the cost of each item and arrived at the total. The customer calmly informed the clerk that the items should be added and not multiplied. The clerk then added the items together and informed the customer that the total was still exactly $7.11.

What are the exact costs of each item?

Solution:

Think about all of the places where you see numbers during the day. Write a story about how numbers are a part of your day.

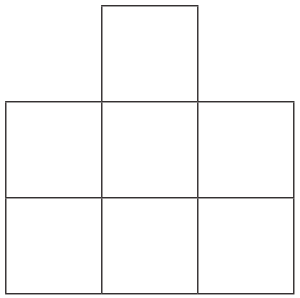


In 1974, Lynn Nolan Ryan, a pitcher for the California Angels, threw the fastest pitch ever recorded. It was 100.9 miles per hour.

Daily Math Puzzle

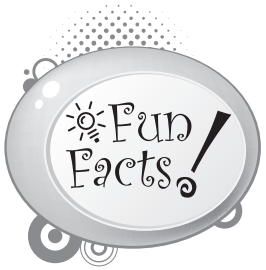
Moving Sticks:
Grades 7–12

Form five squares of the same size by moving just four sticks into new positions.



Solution:

What is your favorite number and why?



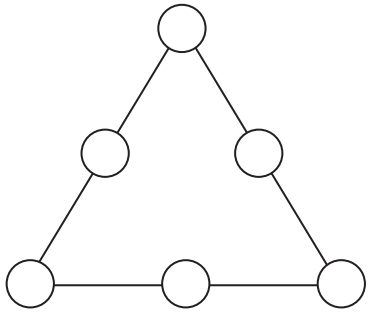
The billionth digit of π is 9.

Daily Math Puzzle

Magic Fractional Triangle:

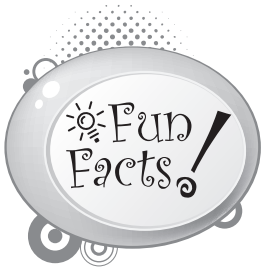
Grade 3 and up

Use each fraction $\left(\frac{0}{6}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}\right)$ only once, so that each side of a triangle adds to one.



Solution:

Think about your favorite fictional character (Harry Potter, the Hardy Boys, or Nancy Drew). How does he or she use math?

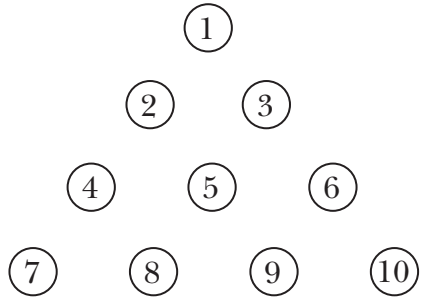


If it were possible to drive through space at 75 mph (120 km/h), you could reach the sun in a little over 142 years.

Daily Math Puzzle

Bottoms Up:
Grades 4–12

Form a triangle with ten pennies as shown. Moving only three pennies, reverse the triangle so that it points down instead of up. Good Luck!!



Solution:

Sylvan Learning

STUDY SKILLS MATH SAT/ACT PREP WRITING READING HOMEWORK HELP

Math Puzzle Answers

1. **The square stained glass window will cost: $600.00.**

Hint: Break this down into steps:

Step One: How many feet are in 24 inches? *24in = 2 ft*

Step Two: Find the area of the square. A = 2 *ft* × 2 *ft* = 4 *sq ft*

Step Three: Multiply the area by $150.00.

4 *ft* × $150 *per square foot* = $600

2. **Juan can run 6 km in 45 minutes.**

Hint: Use the equation:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Step One: Calculate Juan's speed.

Distance = 4 km

time = 30 minutes

$$\text{Juan's speed} = \frac{4 \text{ km}}{30 \text{ min}} = 0.1\overline{3} \ \frac{\text{km}}{\text{min}}$$

Step Two: Knowing Juan's speed, we can determine how far he runs in any given time. Multiply his speed by the time he spends running. If he spends 45 minutes running at $0.1\overline{3} \ \frac{\text{km}}{\text{min}}$, the distance can be found as follows.

$$\text{Distance} = 0.1\overline{3} \ \frac{\text{km}}{\text{min}} \times 45 \text{ minutes} = 6 \text{ km}$$

*Another way of calculating the answer is by breaking the information into smaller pieces. Juan runs 4*km* in 30 *minutes*. Therefore, he runs 2 *km* in 15 *minutes* and 6 *km* in 45 *minutes*.

3. **The Saturn V rocket is about 110 m (more than 300 feet) high.**

Hint: This is a trigonometry problem that makes use of the tangent function. Remember that the tangent is the ratio of the opposite side to the adjacent side of a right triangle.

Step One: Draw a right triangle with a base *d* and a height *h*, where *d* is distance of the observer from the launch pad and *h* is the height of the rocket. The angle between the base and the hypotenuse of the triangle is *a* = 6.3 degrees.

Step Two: The tangent of the angle *a* is equal to the height of the rocket, *h*, divided by the distance, *d*, and so we can write the equation:

$$\text{Tan } a = \frac{h}{d}$$

Step Three: We know that *a* = 6.3 degrees and *d* = 1,000 *m*, and so we solve for *h* by multiplying both sides by 1,000:

$$\text{Tan } 6.3 = \frac{h}{1{,}000}$$

$$1{,}000 \times \tan 6.3 = 1{,}000 \times \frac{h}{1{,}000}$$

$$1{,}000 \times \tan 6.3 = h$$

$$h = 1{,}000 \times \tan 6.3$$

Step Four: Calculate using a scientific calculator. Make sure your calculator is set to degrees and not radians.

$\tan 6.3° \approx 0.11$

Step Five: Calculate the height h:

$h \approx 1{,}000 \times 0.11$

$h \approx 110$ *m*

4. **The circular stained glass window will cost: $471**

Hint: You first need to know the circle's area. Area ($A = \pi r^2$). Use 3.14 as an approximation of π. Remember that the radius is half of the diameter.

Step One: How many feet are in 24 inches? *24in = 2 ft*

Step Two: Find the radius of the circle. 2 *ft* ÷ 2 = 1 *ft*

Step Three: Find the area of the circle. $A = \pi (1 \text{ ft})^2 \approx 3.14$ *sq ft*

Step Four: Multiply the area by the price per square foot.

3.14 *sq ft* × $150 *per square foot* = $471

5. **The sum is $1\frac{1}{6}$.**

Hint: You must first find a common denominator before adding the fractions.

Step One: Find the least common denominator of 2 and 3.

Step Two: Use the common denominator to write equivalent fractions. $\frac{1}{2} = \frac{3}{6}$ and $\frac{2}{3} = \frac{4}{6}$

Step Three: Add the fractions and write your answer as a mixed number. $\frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$

6. The distance from the Sun to the Moon is approximately 1.5×10^{11} *m*.

Hint: Use the Pythagorean theorem, $a^2 + b^2 = c^2$.

Step One: Draw a right triangle with Earth, Moon, and Sun as the vertices. Earth should be at the right angle.

Step Two: Label the side between Earth and Sun with the letter *a*, the side between Earth and Moon *b*, and the side between Sun and Moon as *c*. Side *c* is the hypotenuse of the right triangle.

Step Three: We know $a = 1.5 \times 10^{11}$ *m*, and $b = 3.8 \times 10^{8}$ *m*. Substitute these values into the Pythagorean theorem and solve.

$$(1.5 \times 10^{11})^2 + (3.8 \times 10^{8})^2 = c^2$$
$$(2.25 \times 10^{22}) + (14.44 \times 10^{16}) = c^2$$
$$(2.25 \times 10^{22}) + (0.00001444 \times 10^{22}) = c^2$$
$$(2.25 \times 10^{22}) \approx c^2$$
$$1.5 \times 10^{11} \approx c^2$$

7. The numbers rounded to the given place are:

a) 512,100
b) 318,210
c) 300,000
d) 44,000
e) 260,210

Round:	512,143 to hundreds	318,205 to tens	258,723 to hundred thousands	43,602 to thousands	260,211 to tens
Step One: Underline the digit in the place named.	512,<u>1</u>43	318,2<u>0</u>5	<u>2</u>58,723	4<u>3</u>,602	260,2<u>1</u>1
Step Two: If the digit to the right of the underlined digit is 5 or greater, add one to the underlined digit. If it is less than 5, keep the underlined digit the same.	4 < 5 Keep the underlined digit the same.	5 = 5 Add 1 to the underlined digit.	5 = 5 Add 1 to the underlined digit.	6 > 5 Add 1 to the underlined digit.	1 < 5 Keep the underlined digit the same.
Step Three: Write the number with zeros to the right of the underlined digit.	512,100	318,210	300,000	44,000	260,210

8. The Least Common Multiple is 24.

Hint: Multiples of 8 are the same as counting by 8's.

Step One: List some multiples of 8. They are 8, 16, 24, 32, 40, 48…

Step Two: List some multiples of 12. They are 12, 24, 36, 48, 60, 72…

Step Three: 24 and 48 are common multiples in both lists, but the smaller number, 24, is the Least Common Multiple.

9. A point on Earth's equator is moving at approximately 1,648,500 meters per hour.

Hint: Use the equation speed = distance ÷ time.

Step One: The distance that a point on the equator travels in one full revolution is equal to the circumference of the earth. ($C = 2\pi r$) Use 3.14 for the approximate value of π.

$C \approx 2 \times 3.14 \times (6.3 \times 10^{6})$

$C \approx 39{,}564{,}000$

Step Two: The time is given as 24 hours. Divide the distance traveled by the time.

speed $\approx 39{,}564{,}000 \div 24$

speed $\approx 1{,}648{,}500$ *meters per hour*

10. The mean of the number set is about 58.636, the median of the number set is 65, and the mode is 75.

Step One: To find the mean, add the numbers in the set and divide by 11.

$645 \div 11 = 58.\overline{63}$

Step Two: To find the median, arrange the numbers in order from lowest to highest and determine the number that is exactly in the middle.

10, 25, 40, 55, 55, <u>65</u>, 70, 75, 75, 75, 100

Step Three: To find the mode determine which number occurs most frequently in the set of numbers. The number 75 occurs three times in the set of numbers.

10, 25, 40, 55, 55, 65, 70, <u>75</u>, <u>75</u>, <u>75</u>, 100

11. The solutions are shown below:

a) $x = 8$ **d) $x = -12$**

b) $x = -4$ **e) $x = 4$**

c) $x = -11$

Hint: Remember that what is done to one side of an equation must also be done to the other side of the equation.

Step One: Simplify both sides of the equation using Order of Operations and substitution to combine like terms.

Step Two: Use the properties of equality to add or subtract the same quantity from both sides of an equation.

Step Three: Use the properties of equality to multiply or divide both sides of an equation by the same value.

a)
$$\begin{aligned} 6 &= x - 2 \\ 6 + 2 &= x - 2 + 2 \\ 8 &= x \\ x &= 8 \end{aligned}$$

b)
$$\begin{aligned} -3x - 5 &= 7 \\ -3x - 5 + 5 &= 7 + 5 \\ -3x &= 12 \\ \frac{-3x}{-3} &= \frac{12}{-3} \\ x &= -4 \end{aligned}$$

c)
$$\begin{aligned} -106 &= 4x + 6x + 4 \\ -106 &= 10x + 4 \\ -106 - 4 &= 10x + 4 - 4 \\ -110 &= 10x \\ \frac{-110}{10} &= \frac{10x}{10} \\ -11 &= x \\ x &= -11 \end{aligned}$$

d)
$$\begin{aligned} 4(7 + 6x) &= -260 \\ 28 + 24x &= -260 \\ 28 + 24x - 28 &= -260 - 28 \\ 24x &= -288 \\ \frac{24x}{24} &= \frac{-288}{24} \\ x &= -12 \end{aligned}$$

e)
$$\begin{aligned} -6x - 3 &= 5x - 47 \\ -6x - 3 - 5x &= 5x - 47 - 5x \\ -11x - 3 &= -47 \\ -11x - 3 + 3 &= -47 + 3 \\ -11x &= -44 \\ \frac{-11x}{-11} &= \frac{-44}{-11} \\ x &= 4 \end{aligned}$$

12. The solutions are shown below:

a) $\frac{17}{30}$ **b) $\frac{5}{8}$** **c) $\frac{3}{28}$** **d) $\frac{8}{81}$** **e) $\frac{1}{4}$** **f) $\frac{8}{27}$** **g) $1\frac{7}{9}$**

Step One: Recall the rules for operations with fractions including the use of common denominators for addition and subtraction of fractions and reciprocals for division.

Step Two: Find the sum, difference, product, or quotient as indicated.

Step Three: Simplify the final answer.

a) $\frac{3}{10} + \frac{4}{15} = \frac{9}{30} + \frac{8}{30} = \frac{17}{30}$

b) $\frac{11}{56} + \frac{3}{7} = \frac{11}{56} + \frac{24}{56} = \frac{35}{56} = \frac{5}{8}$

c) $\frac{2}{7} \times \frac{3}{8} = \frac{6}{56} = \frac{3}{28}$

d) $\frac{2}{9} \times \frac{4}{9} = \frac{8}{81}$

e) $\frac{1}{8} \div \frac{1}{2} = \frac{1}{8} \times \frac{2}{1} = \frac{2}{8} = \frac{1}{4}$

f) $\frac{2}{9} \div \frac{6}{8} = \frac{2}{9} \times \frac{8}{6} = \frac{16}{54} = \frac{8}{27}$

g) $\frac{2}{3} \div \frac{3}{8} = \frac{2}{3} \times \frac{8}{3} = \frac{16}{9} = 1\frac{7}{9}$

13. The solutions are shown below:

a) $c = 7.34$ and $a = 4.16$

b)

	$\sin A = \cos B$	$\cos A = \sin B$	$\tan A$	$\tan B$
I.	$\frac{9}{41}$	$\frac{40}{41}$	$\frac{9}{40}$	$\frac{40}{9}$
II.	$\frac{35}{37}$	$\frac{12}{37}$	$\frac{35}{12}$	$\frac{12}{35}$
III.	$\frac{24}{25}$	$\frac{7}{25}$	$\frac{24}{7}$	$\frac{7}{24}$

c) $a = 4.03$ and $b = 4.84$

Hints: Be sure your calculator is in degree mode. To find the measure of an angle use $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. When given 2 sides of a right triangle, the Pythagorean Theorem ($a^2 + b^2 = c^2$) can be used to find the missing side.

a) Use the sin ratio to find the value of c.

$\sin 55°30' = \frac{6.05}{c}$

$c(\sin 55°30') = 6.05$

$c = 6.05/(\sin 55°30') \approx 7.34$

Use the cos ratio to find the value of a.

$\cos 55°30' = \frac{a}{7.34}$

$a = 7.34(\cos 55°30') \approx 4.16$

b) Using the Pythagorean Theorem to find the missing side.

I. $9^2 + b^2 = 41^2$

$81 + b^2 = 1681$

$b^2 = 1600$

$b = 40$

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{41}$ $\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{40}{41}$

$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{40}{41}$ $\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{9}{41}$

$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{40}$ $\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{40}{9}$

II. $35^2 + b^2 = 37^2$

$1225 + b^2 = 1369$

$b^2 = 144$

$b = 12$

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{35}{37}$ $\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{37}$

$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{37}$ $\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{35}{37}$

$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{35}{12}$ $\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{12}{35}$

III. $24^2 + 7^2 = c^2$

$576 + 49 = c^2$

$625 = c^2$

$b = 25$

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{24}{25}$ $\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{7}{25}$

$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{7}{25}$ $\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{24}{25}$

$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{24}{7}$ $\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{7}{24}$

c) $B = \tan^{-1}(1.2) = 50.19°$

$\cos 50.19° = \frac{a}{6.3}$ $\sin 50.19° = \frac{b}{6.3}$

$a = 6.3(\cos 50.19°) \approx 4.03$ $b = 6.3(\sin 50.19°) \approx 4.84$

14. Yes, it is a right triangle.

Hint: Be sure to substitute the longest length for the hypotenuse, c.

$9^2 + 12^2 = 15^2$

$81 + 144 = 225$

$225 = 225$

This is a true statement. So, the lengths of the sides satisfy the Pythagorean Theorem.

15. John needs an 88% on the final assessment.

Hint: Determine the number of points possible and find 80% of that amount.

The total number of points possible is 20 + 25 + 30 + 25 = 100. John needs a total of 80 points to earn an 80% for the course. So far he has earned 15 + 17 + 26 = 58. He needs to earn 22 more points to have a total of 80 points. That means on the last assessment he will need 22 out of 25 points or 88%.

16. The solutions are shown below.

Hint: Show your steps in an ordered manner.

1. Think of a number.	4	15
2. Add 3 to the number.	4 + 3 = 7	15 + 3 = 18
3. Multiply that sum by 2.	7 x 2 = 14	18 x 2 = 36
4. Next, subtract 4 from your last answer.	14 – 4 = 10	36 – 4 = 32
5. Divide your current answer by 2.	10 ÷ 2 = 5	32 ÷ 2 = 16
6. Subtract the number you started with from that quotient.	5 – 4 = 1	16 – 15 = 1
7. Your answer will always be 1.	1	1

17. The solution is shown below.

Hint: Show your steps in an ordered manner.

1. Multiply the amount of change you have by 2.	24 x 2 = 48
2. Add 3 to the previous product.	48 + 3 = 51
3. Multiply the previous sum by 5.	51 x 5 = 255
4. Subtract 6 from the last product.	255 – 6 = 249
5. Your change is the first two digits of the answer.	24

18. The solution is shown below.

Hint: Show your steps in an ordered manner.

1. Multiply her age by 4.	20 x 4 = 80
2. Add 10 to the product.	80 + 10 = 90
3. Multiply the sum by 25.	90 x 25 = 2250
4. Subtract the number of days in a year, 365, from this product.	2250 – 365 = 1885
5. Add to the total the change, less than one dollar, she has.	1885 + 24 = 1909
6. Add 115 to the previous total.	1909 + 115 = 2024 (Age = 20, change = 24)

19. The solution is shown below.

Hint: Show your steps in an ordered manner.

1. Multiply the number of the month by 5.	1 x 5 = 5
2. Add 7 to the product.	5 + 7 = 12
3. Multiply the sum by 4.	12 x 4 = 48
4. Add 13 to the answer.	48 + 13 = 61
5. Multiply the last sum by 5.	61 x 5 = 305
6. Add the day of the month to your answer.	305 + 7 = 312
7. Mentally subtract 205.	312 – 205 = 107 (1 is January, 07 is 7th)

20. The solution is shown below.

Hint: Show your steps in an ordered manner.

1. Double your house number.	2701 x 2 = 5402
2. Add the number of days in a week to your answer.	5402 + 7 = 5409
3. Multiply that sum by 50.	5409 x 50 = 270,450
4. Add your age to the total.	270,450 + 40 = 270,490
5. Subtract the number of days in a year from your last result.	270,490 – 365 = 270,125
6. Add 15 to your answer.	270,125 + 15 = 270,140 House number is 2701. Age is 40.

21. There is no logical reason why the withdrawal column and the balance column have to total the same amount. There is no error here. Only the withdrawal column should total $50.

22. One possible solution is to purchase 94 books at 50 cents, one book at $3, and five books at $10.

Hint: Spending exactly $100 will require purchasing an even number of 50 cent books.

If only 50 cent books were purchased, then 200 of them could be purchased for $100. For each $3 book purchased there will need to be 6 fewer 50 cent books for the total cost to remain at $100. For each $10 book purchased, there will need to be 20 fewer 50 cent books purchased for the total cost to remain at $100.

Start with purchasing 200 of the 50 cent books. If we purchase one $3 book, then only 194 of the 50 cent books could be purchased for the cost to remain at $100. If we then chose to purchase five of the $10 books, then we would need to purchase 100 fewer of the 50 cent books. This would give a result of 94 books at 50 cents, one book at $3 and five books at $10.

An equation that could be used to represent this situation is 0.50A + 3B + 10C = 100, where A is the number of 50 cent books purchased, B is the number of $3 books purchased and C is the number of $10 books purchased. Notice that 0.50(94) + 3(1) + 10(5) = 47 + 3 + 50 = 100.

23. The advisor drew one of the scraps of paper and tore it up announcing that it was his choice. He then showed the other scrap of paper to the judge. If the king was fair, he would have written "Death" on one scrap and "Life" on the other. Since he wrote "Death" on both, the scrap that was not torn up will say "Death". If the judge assumes the king was fair, then the word "Life" should be on the torn up scrap of paper. The torn up scrap of paper was the advisor's choice and so he goes free.

24. The frog reaches the top on the sixth day.

Hint: Draw a vertical number line numbered from 0 to 10 to represent the well and act out the given scenario using the number line.

He advances on foot per day for the first five days. On the fifth day he reaches as high as nine fee but falls back to five feet. Then on the sixth day he reaches the top.

25. Point to one door and ask one judge, "Will the other judge tell me that when I walk through this door I will win the grand prize?" If the answer is no, choose the door you are pointing too. If the answer is yes, pick the other door.

Hint: Organize the possible outcomes in a chart.

The chart below shows the answers by each judge when asked the question "Will the other judge tell me that when I walk through this door I will win the grand prize?"

If you point to the door that leads to the grand prize, the liar will say "No" because the truth-teller would say "Yes, the door leads to the grand prize". If you point to the door that leads to the grand prize, the truth-teller will say "No" because that is what the liar's answer will be. If you point to the door that leads to a forfeit, the liar will say "Yes" because the truth-teller would say "No, the door leads to a forfeit". If you point to the door that leads to a forfeit, the truth-teller will say "Yes" because that is what the liar's answer will be.

		Outcome	
		Grand Prize	Forfeit
Judge's Statements	Liar	No	Yes
	Truth-teller	No	Yes

By looking at the chart we can see if either judge says "No", the door that is being pointed to should be selected. If either judge says "Yes", then the other door should be chosen.

26. The answer is O = 0, M = 1, Y = 2, E = 5, N = 6, D = 7, R = 8 and S = 9.

Hint: Start with the left-most column and work toward the right.

Step 1: Looking at the left-most column, M must be the number 1. If the sum of S and M was 9 or less, there would be no need for a number in the left-most column. Therefore, M is 1 and the sum of S and M must be between 10 and 18, inclusive.

Step 2: Since M = 1, then S + M = S + 1. Since S + M must be between 10 and 18, inclusive, S must be 9. Any number smaller than 9 would not produce a sum of S and M between 10 and 18.

Step 3: S + M = 9 + 1 = 10. This means that O = 0.

Step 4: In the third column from the left E + O = N. Replace the letter 0 with the number 0. E + 0 = N. Since E and N are not the same number, N+R in the fourth column would need to sum to 10 or more. This means E + 1 = N.

Step 5: Try E = 2 and N = 3. But that would not result in E + N being more than 10. Try E = 3 and N = 4 or E = 4 and N = 5. Again, this would not result E + N being more than 10. E = 5 and N = 6 would work.

Step 6: Fill in what we know so far. The digits that remain are 2, 3, 4, 7 and 8.

```
  SEND        956D
 +MORE       +10R5
 MONEY       1065Y
```

6 + R must be 15. However, R cannot be 9, because S = 9. This means the ones column must sum to more than 10. So that 1 + 6 + R = 15 and R = 8.

Step 7: The digits that remain are 2, 3, 4 and 7. D + 5 must sum to 10 or more. Therefore, D cannot be 2, 3 or 4. If D = 7, then Y = 2.

The final solution is:

```
  SEND        9567
 +MORE       +1085
 MONEY       10652
```

Here is the solution to the second part:

```
 ONE
+SIX
 777
```

Think of all the digits that sum to 7. 0 and 7, 1 and 6, 2 and 5, 3 and 4.

One possible solution is 123 + 654 = 777.

27. One possible answer is $3.16, $1.25, $1.20 and $1.50.

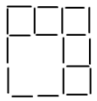
Hint: Call the individual prices a,b,c and d and write two equations involving these variables.

abcd = 7.11 and a + b + c + d = 7.11.

Let's think of the factors of 7.11 or $\frac{711}{100}$. This factors as $\frac{(3^2)(79)}{(2^2)(5^2)}$.

Let's multiply the numerator and denominator by 2 and 5. This gives $\frac{(3^2)(79)(2)(5)}{(2^2)(5^2)(2)(5)}$ Breaking apart the factors gives $\left(\frac{79}{5^2}\right)\left(\frac{5}{2^2}\right)\left(\frac{2\cdot 3}{5}\right)\left(\frac{3}{2}\right)$ or $(3.16)\,(1.25)\,(1.20)\,(1.50)$. These numbers multiply to 7.11 and sum to 7.11.

28. The answer is



Hint: Move the dotted lines.

29. The answer is 0/6, 1/6 and 2/6 at the corners. Place 5/6 between 0/6 and 1/6. Place 4/6 between 0/6 and 2/6. Place 3/6 between 1/6 and 2/6.

Hint: For the fractions to sum to 1, the numerators must sum to 6.

To keep the sum along each side balanced, the first 3 numbers 0/6, 1/6 and 2/6 should be placed at the corners. Once these are placed add two numbers on one side and subtract the result from 1. This will determine the number to place along the side. For example, 0/6 + 1/6 = 1/6 and 1 – 1/6 = 5/6. Place 5/6 between 0/6 and 1/6.

30. Make the following moves:

a) Move 7 to the left of 2

b) Move 10 to the right of 3

c) Move 1 centered below 8 and 9

Hint: Think about keeping the middle portion of the triangle in place and moving the corners.